Consider the following system of equations and the solution \((x, y; u, v) = (2, 1; 2, -1)\) to it.

\[
\begin{align*}
  u^2 - v^2 - x^3 + y^2 + 4 &= 0 \\
  2uv + v^2 - 2x^2 + 3y^4 + 8 &= 0
\end{align*}
\]

(a) Show that the system above can be solved locally around \((x, y; u, v) = (2, 1; 2, -1)\) by implicitly defined functions.  

(b) Compute \(\frac{\partial x}{\partial u}\) and \(\frac{\partial y}{\partial u}\) at \((2, 1; 2, -1)\).

**Answer:**

(a) The Jacobian matrix is given by

\[
\left( \begin{array}{cc}
-3x^2 & 2y \\
-4x & 12y^3
\end{array} \right)_{(2,1;2,-1)} = \left( \begin{array}{cc}
-12 & 2 \\
-8 & 12
\end{array} \right)
\]

The determinant of \(\left( \begin{array}{cc}
-12 & 2 \\
-8 & 12
\end{array} \right)\) is \(-128 \neq 0\). Thus, we can apply implicit function and conclude that the system can be solved locally around \((2, 1; 2, -1)\).

(b) We have

\[
\left( \begin{array}{c}
\frac{\partial x}{\partial u} \\
\frac{\partial y}{\partial u}
\end{array} \right) = \left( \begin{array}{cc}
3x^2 & -2y \\
4x & -12y^3
\end{array} \right)^{-1} \left( \begin{array}{c}
2u \\
2v
\end{array} \right) = \frac{1}{8xy-36x^2y^2} \left( \begin{array}{cc}
-12y^3 & 2y \\
-4x & 3x^2
\end{array} \right) \left( \begin{array}{c}
2u \\
2v
\end{array} \right)
\]

Hence,

\[
\frac{\partial x}{\partial u} = \frac{1}{8xy-36x^2y^2}(-24y^3u + 4yv) \quad \text{and} \quad \frac{\partial y}{\partial u} = \frac{1}{8xy-36x^2y^2}(-8xu + 6x^2v).
\]

Now calculate these two at \((2, 1; 2, -1)\).