NAME and SECTION:

Consider the two player normal form game below.

\[
\begin{array}{ccc}
\alpha & (1, 2) & (2, 0) & (2, 1) \\
\beta & (1, 0) & (3, 2) & (1, 0) \\
\delta & (2, 1) & (2, 1) & (3, 2) \\
\end{array}
\]

1. Is there any rationalizable pure strategy profile which is not a Nash equilibrium? If yes, find all.

2. Is there any pure strategy Nash equilibrium which is not rationalizable? If yes, find all.

3. Find all pure and mixed strategy Nash equilibria. (Hint: You can first reduce the game by eliminating the strictly dominated strategies.)

**ANSWER:**

1. For player 1, \(\alpha\) is a NBR strategy, and for Player 2, there is no NBR strategy. Eliminate \(\alpha\) and reduce the game. In the reduced game, \(x\) is a NBR strategy for player 2, and for player 1 there is no NBR strategy. Eliminate \(x\) and reduce the game. There is no other NBR strategy. The rationalizable pure strategy profiles are then \(\{\beta y, \beta z, \delta y, \delta z\}\). The pure strategy Nash equilibria are \(\{\beta y, \delta z\}\). Thus, the rationalizable pure profiles that are not Nash are \(\{\beta z, \delta y\}\).

2. No. Any Nash equilibrium is rationalizable.

3. For player 1, \(\alpha\) is strictly dominated by the following mix strategy, \(\sigma_1 = (0, 1/3, 2/3)\), where \(\beta\) is played with 1/3 probability and \(\delta\) is played with 2/3 probability. Eliminate \(\alpha\). Then in the reduced game, for player 2, \(x\) is strictly dominated by \(\sigma_2 = (0, 1/2, 1/2)\), where \(y\) and \(z\) are both played with 1/2 probability. Eliminate \(x\). Then the final reduced game is

\[
\begin{array}{ccc}
y (q) & z (1-q) \\
\beta (p) & (3, 2) & (1, 0) \\
\delta (1-p) & (2, 1) & (3, 2) \\
\end{array}
\]

Let’s check the indifference conditions: \(EU_1(\beta, q) = EU_1(\delta, q)\) implies \(3q + 1 - q = 2q + 3(1 - q)\), that is, \(q = 2/3\). And, \(EU_2(y, p) = EU_2(z, p)\) implies \(2p + 1 - p = 2(1 - p)\), that is, \(p = 1/3\). Thus, the set of all pure and mixed Nash equilibria is \(\{\beta y, \delta z, (\left(0, \frac{1}{3}, \frac{2}{3}\right), (0, \frac{2}{3}, \frac{1}{3})\}\}.\)