Boğaziçi University, Department of Economics  
Spring 2014  
EC 206 MICROECONOMICS II  
MIDTERM - ANSWER KEY  
03.04.2013, Thursday, 18:00

• Do not forget to write your full name, student number and registered section on the top.

• Turn off your cell phone and put it away. During the exam if you are seen with a cell phone, on or off, 50 points will be taken off from your exam immediately.

• Put away all your lecture notes, books, etc.

• There are 4 questions and 6 pages in the exam. Make sure you have them all.

• Please answer all of the questions in the space provided for each question.

• Show your work.

• You have 100 minutes.

GOOD LUCK!!
1. (24 pts) Consider the three player normal form game below.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>(0,1,2)</td>
<td>(0,2,1)</td>
<td>(1,2,0)</td>
<td>(1,0,2)</td>
</tr>
<tr>
<td>M</td>
<td>(2,0,1)</td>
<td>(1,0,2)</td>
<td>(0,1,2)</td>
<td>(0,1,2)</td>
</tr>
<tr>
<td>D</td>
<td>(1,0,2)</td>
<td>(2,0,1)</td>
<td>(2,0,1)</td>
<td>(2,0,1)</td>
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<tr>
<td>L</td>
<td>(3)</td>
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(a) (12 pts) Is this game weak dominance solvable? If it is, find the solution. If not, find the final reduced game.

**ANSWER:**

- For player 1, U is weakly dominated by D.
- For player 2, X and W are weakly dominated by Z.
- For player 3, R is weakly dominated by L.

We eliminate U, X, W and R at the same time.

In the reduced game, For player 1, M is weakly dominated by D.
- For player 2, Y is weakly dominated by Z.

Thus the weak dominance solution is \((D, Z, L)\).

(b) (12 pts) Find the set of rationalizable strategy profiles.

**ANSWER:**

There is no never-a-best-response strategy. Thus every strategy profile is rationalizable. See the best responses above in bold.
2. (22 pts) There are three players. Each player simultaneously picks an integer between 1 and 100, inclusive. If the sum of the three integers is less than or equal to 100, then each player gets exactly her own integer as her payoff. If the sum of the three integers is larger than 100, then the one who picked the largest integer is eliminated and gets 0 payoff, and each of the remaining two players gets her own integer as her payoff as long as the sum of these two players’ integers is less than or equal to 100, otherwise they also get 0 payoff. When the sum of the three integers is larger than 100 with two or three players picking the same integer which is the largest, then the one who is eliminated is chosen with equal probability. Find all pure-strategy Nash equilibria.

**ANSWER:**

**case 1:** $x_1 + x_2 + x_3 < 100$: Any player $i$ profitably deviates to $x_i + 1$.

**case 2:** $x_1 + x_2 + x_3 = 100$:

**case 2.1:** $x_1 < x_2 < x_3$: Player 1 profitably deviates to $x_1 + 1$.

**case 2.2:** $x_1 = x_2 = x_3$: not possible! $x_i = 33.33$ not an integer!

**case 2.3:** $x_1 = x_2 < x_3$: No deviation if $x_1 = x_2 = 33$ and $x_3 = 34$. For every other profile player 1 profitably deviates to $x_1 + 1$.

**case 2.4:** $x_1 < x_2 = x_3$: Player 1 profitably deviates to $x_1 + 1$.

**case 3:** $x_1 + x_2 + x_3 > 100$:

**case 3.1:** $x_1 < x_2 < x_3$: If $1 < x_1 < 50$ then player 3 profitably deviates to $x_1 − 1$. If $x_1 = 1$, then player 3 profitably deviates to $x_2 − 1$. If $x_1 > 50$, then player 3 profitably deviates to $100 − x_1$.

**case 3.2:** $x_1 = x_2 = x_3 = x$: Except $(100,100,100)$, in every other profile in this case there is a profitable deviation. If $x > 50$, any player profitably deviates to $100 − x$. If $x \leq 50$, then any player profitably deviates to $x − 1$.

**case 3.3:** $x_1 = x_2 < x_3$: Player 3 profitably deviates to 1.

**case 3.4:** $x_1 < x_2 = x_3$: Player 3 profitably deviates to 1.

Thus there are only 4 Nash equilibria: $(33, 33, 34), (33, 34, 33), (34, 33, 33)$ and $(100, 100, 100)$
3. (34 pts) There are two firms \( A \) and \( B \). \( A \) has a quality level \( k_A \) and \( B \) has a quality level \( k_B \), where \( k_A \) and \( k_B \) are given and publicly known. Suppose given these quality levels, each firm simultaneously picks its own price, \( A \) picks \( p_A \) and \( B \) picks \( p_B \), where \( p_A, p_B \in [0, \infty] \). For any given quality level marginal cost of producing any unit is zero. Each firm is maximizing own profit. Consumers are uniformly distributed over the interval \([0, 1]\), that is, at each point/location in \([0, 1]\) there is a consumer. The consumer located at \( \theta \in [0, 1] \) has a utility function \( u = 50 + \theta k_i - P_i \) if she buys from firm \( i \), for \( i = A, B \). Each consumer demands at most 1 unit and buys from the firm which gives her a higher and non-negative utility. For instance, a consumer located at \( \theta = 0.3 \) buys from \( A \) if \( 50 + 0.3 k_A - P_A > 50 + 0.3 k_B - P_B \) and \( 50 + 0.3 k_A - P_A \geq 0 \). If a consumer is indifferent between the two firms, then she flips a fair coin to decide.

(a) (12 pts) Suppose \( k_A > k_B \). Find the pure-strategy Nash equilibrium price levels as functions of \( k_A \) and \( k_B \).

**ANSWER:**

The consumer who is indifferent between firm \( A \) and firm \( B \) is at the location \( \theta^* \) where

\[
50 + \theta^* k_A - P_A = 50 + \theta^* k_B - P_B
\]

Solving this we get \( \theta^* = \frac{P_A - P_B}{k_A - k_B} \). All consumers with \( \theta > \theta^* \) will buy from firm \( A \), and all consumers with \( \theta < \theta^* \) will buy from firm \( B \). Thus firm \( A \)'s share is \( (1 - \theta^*) \) and firm \( B \)'s share is \( \theta^* \).

Firm \( A \) solves

\[
\max_{P_A} P_A (1 - \theta^*) = P_A (1 - \frac{P_A - P_B}{k_A - k_B})
\]

The first order condition with respect to \( P_A \) is given by

\[
P_A = \frac{P_B + k_A - k_B}{2} \tag{1}
\]

Similarly, firm \( B \) solves

\[
\max_{P_B} P_B \theta^* = P_B \frac{P_A - P_B}{k_A - k_B}
\]

The first order condition with respect to \( P_B \) is given by

\[
P_B = \frac{P_A}{2} \tag{2}
\]

Solving (1) and (2) we get

\[
P_A^* = \frac{2}{3} (k_A - k_B)
\]

\[
P_B^* = \frac{1}{3} (k_A - k_B)
\]
(b) (10 pts) Suppose \( k_A = 3 \) and \( k_B = 1 \). Find the reduced strategy sets for each player after eliminating never-a-best-response pure-strategies for exactly three rounds.

**ANSWER:**

The best response functions when \( k_A = 3 \) and \( k_B = 1 \) are

\[
P_A = \frac{P_B}{2} + 1 \quad \quad P_B = \frac{P_A}{2}
\]

In the first round, for player A, \( P_A < 1 \) and \( P_A > 53 \) are never-a-best-response. For player B, \( P_B > 51 \) are never-a-best-response.

In the second round, for player B, \( P_B < \frac{1}{2} \) and \( P_B > 26.5 \) are never-a-best-response. For player A, \( P_A > 26.5 \) are never-a-best-response.

In the third round, for player A, any \( P_A < \frac{5}{4} \) and \( P_A > \frac{14}{25} \) are never-a-best-response. For player B, \( P_B > \frac{13}{25} \) are never-a-best-response.

Thus, after eliminating never-a-best-response strategies for three rounds, we get \( S_A = \left[ \frac{5}{4}, \frac{14}{25} \right] \) and \( S_B = \left[ \frac{1}{2}, 13.25 \right] \).

(c) (12 pts) Now suppose that the firms simultaneously choose own quality level from the set \([0, 10]\). After quality levels are determined and become publicly known, they simultaneously pick prices as in above. The cost of choosing a quality level \( k \) is given by

\[ c(k) = \frac{k^2}{2} \]

Find the pure-strategy Nash equilibrium quality levels.

**ANSWER:**

Now, we know, \( P_A^* = \frac{2}{3}(k_A - k_B) \) and \( P_B^* = \frac{1}{3}(k_A - k_B) \). Note that \( \theta^* = \frac{P_A^*-P_B^*}{k_A-k_B} = 1/3 \).

Then, the profit for firm A is given by

\[
\Pi_A(k_A, k_B) = P_A^*(1 - (1/3)) - \frac{k_A^2}{2} = \frac{2}{3}(k_A - k_B) \frac{2}{3} - \frac{k_A^2}{2} = \frac{4}{9}(k_A - k_B) - \frac{k_A^2}{2}
\]

Then, the first order condition with respect to \( k_A \) is

\[
\frac{4}{9} - k_A = 0, \quad \text{that is, } k_A^* = \frac{4}{9}.
\]

Similarly, the profit for firm B is given by

\[
\Pi_B(k_A, k_B) = P_B^*(1/3) - \frac{k_B^2}{2} = \frac{1}{3}(k_A - k_B) \frac{1}{3} - \frac{k_B^2}{2} = \frac{1}{9}(k_A - k_B) - \frac{k_B^2}{2}
\]

Then, the first order condition with respect to \( k_B \) is

\[
\frac{1}{9} - k_B, \quad \text{which is always negative, thus we need to look at the corner solutions: if } k_B = 0 \text{ then } \Pi_B = \frac{1}{3}, \quad \text{and if } k_B = 10, \text{ then } \Pi_B < 0. \quad \text{Thus, } k_B^* = 0.
\]

Note that this is the solution when \( k_A > k_B \). If, however, \( k_B > k_A \), then we get \( k_A^* = 0 \) and \( k_B^* = \frac{4}{9} \).

Thus, we have two equilibria, \((k_A^* = \frac{4}{9}, k_B^* = 0)\) and \((k_A^* = 0, k_B^* = \frac{4}{9})\).
4. (20 pts) Find the set of all mixed strategy Nash equilibria in the two player normal form game below. (Hint: Anything
dominated?)

\[
\begin{array}{ccc}
 & L & C & R \\
U & (2,4) & (3,0) & (1,1) \\
M & (3,0) & (1,3) & (2,2) \\
D & (5,1) & (0,2) & (4,1) \\
\end{array}
\]

**ANSWER:**

Note that a mix of L and C strictly dominates R, and a mix of U and D strictly dominates M.

Thus the reduced game is

\[
\begin{array}{cc}
L(q) & C(1 - q) \\
U(p) & (2,4) & (3,0) \\
D(1 - p) & (5,1) & (0,2) \\
\end{array}
\]

First note that there are no pure strategy Nash equilibrium.

Now note that \( p^* = 1/5 \) makes player 2 indifferent between L and C. And, \( q^* = 1/2 \) makes player 1 indifferent between
U and D.

Thus, there is a unique Nash equilibrium: \( (p^* = 1/5, q^* = 1/2) \), that is, \((1/5, 0, 4/5), (1/2, 1/2, 0))\) is the only Nash
equilibrium.