Do not forget to write your full name, student number and section on the top.

Turn off your cell phone and put it away. During the exam if you are seen with a cell phone, on or off, 20 points will be taken off from your exam immediately.

Put away all your lecture notes, books, etc.

There are 5 questions and 10 pages in the exam. Make sure you have them all.

Please answer all of the questions in the space provided for each question.

Show your work!

You have 150 minutes.

GOOD LUCK!!
1. (18 pts) Behzat Ç., the head of the murder investigation unit in Ankara Police Department, wants to buy a red 1974 made Volkswagen Beetle for her daughter as a birthday present. He goes to the second hand car market and finds a nice looking, handsome red Beetle. Behzat agrees with the seller on the following terms: They will both name one of the two possible prices, 1000TL or 4000TL, simultaneously. If they name the same price, then trade occurs at that price. If they name different prices and Behzat names the higher price, then trade occurs at the average price, 2500TL. If they name different prices and the seller names the higher price, then trade does not occur. If trade occurs at price $P$, then Behzat’s payoff is $V - P$ and the seller’s payoff is $P$. If trade does not occur, then Behzat’s payoff is 0 and the seller’s payoff is $R$. For Behzat, $V$ can take two values, it’s either 600 with probability $1/3$, or 4200 with probability $2/3$. For the seller, $R$ can take two values, it’s either 1200 or 1500 with equal probabilities. Behzat learns his $V$ value and the seller learns his $R$ value but they do not observe the other’s value.

(a) (4 pts) Write down the strategy sets for both Behzat and the seller.

(b) (14 pts) Find the set of pure strategy Bayesian Nash equilibrium.
2. (20 pts) Harun and Cevdet, who are in Behzat’s investigation team, are bargaining over a cake of size 12. In period 1, Harun offers \( m_1 \in [0, 12] \) to Cevdet. Cevdet accepts or rejects. If it is rejected, Cevdet offers \( m_2 \in [0, 12] \) to Harun in period 2. Harun accepts or rejects. If this offer is also rejected, Harun makes an offer \( m_3 \in [0, 12] \) to Cevdet in period 3, who then accepts or rejects it. If the period 3 offer is rejected the cake is given to Eda (who is not a player here), and Harun receives zero payoff while Cevdet receives a payoff of -1 (since he gets beaten up by Harun). If an offer \( m_t \) is accepted in period \( t \), the player making the offer receives payoff \((3/4)^{t-1}(12 - m_t)\), and the receiver of the offer gets \((3/4)^{t-1}m_t\), where \(3/4\) is their common discount factor.

(a) (4 pts) Describe the strategy set for Cevdet.

(b) (16 pts) Find the unique pure strategy Subgame Perfect Nash equilibrium. What will be Cevdet’s payoff in the equilibrium?
3. (22 pts) Behzat Ç. is investigating a murder crime scene with his team. The only evidence they find is a diagram given below. Little they know, it’s an extensive form game with three players.

(a) (12 pts) Find the set of pure strategy Perfect Bayesian equilibrium.
(b) (6 pts) Find the set of pure strategy Subgame Perfect Nash equilibrium in which player 3 plays $u$. Are they also Perfect Bayesian equilibrium?
(c) (4 pts) Is there any strategy profile which satisfies the first three requirements of Perfect Bayesian equilibrium (R1, R2 and R3), but not a Perfect Bayesian equilibrium?
4. (24 pts) Based on the evidence in the crime scene, Behzat Ç. and his team start to chase the notorious criminal Ercüment Çözer to arrest as a suspect. However, Ercüment Çözer is either protected by some very powerful bureaucrat or not protected at all, with probabilities $p$ and $1 - p$, respectively. When Behzat’s team is about to corner Ercüment Çözer, Ercüment makes a phone call and asks for either a helicopter or a van to come and pick him up. Then, in a few minutes whichever he asked for comes and picks him up. Behzat’s team, after observing what picked him up, either keeps chasing him or stops chasing. The payoffs for Behzat are as follows: Behzat gets 0 whenever he stops chasing. He gets -2 whenever he keeps chasing and Ercüment is in a helicopter. He gets 1 if he keeps chasing and Ercüment is in a van and protected. He gets 2 if he keeps chasing and Ercüment is in a van and not protected. The payoffs for Ercüment are as follows: If Ercüment is protected and asks for a helicopter, he gets a payoff of 3 regardless of Behzat’s action. If Ercüment is not protected and asks for a helicopter, he gets a payoff of 1 regardless of Behzat’s action. If Ercüment is protected and asks for a van, then he gets 3 if Behzat stops chasing, and gets -1 if Behzat keeps chasing. If Ercüment is not protected and asks for a van, then he gets 2 if Behzat stops chasing, and gets -2 if Behzat keeps chasing.

(a) (4 pts) Draw the extensive form game.
(b) (10 pts) For any given $p$, find the set of pure strategy pooling Perfect Bayesian equilibrium.
(c) (10 pts) For any given $p$, find the set of pure strategy separating Perfect Bayesian equilibrium.
5. (16 pts) Hayalet (H) and Akbaba (A), the two other guys from Behzat’s team, are two roommates sharing an apartment. They play the following game for T periods. Assume that both Hayalet and Akbaba have the same discount factor \( \delta \in (0, 1) \).

\[
\begin{array}{ccc}
\text{(H)/(A)} & X & Y & Z \\
A & 1.4 & 1.3 & 9.2 \\
B & 0.0 & 5.4 & 0.0 \\
C & 1.1 & 0.0 & 8.8 \\
\end{array}
\]

(a) (4 pts) Suppose \( T = 2 \). How many subgames are there in the repeated game? How many strategies does Akbaba have?

(b) (12 pts) Suppose now \( T = \infty \), that is the game is infinitely repeated. Consider the following grim-trigger strategies:

For Hayalet: At \( t = 1 \), play C. At any \( t > 1 \), play C if (C,Z) has been played in all of the previous periods, and play A otherwise.

For Akbaba: At \( t = 1 \), play Z. At any \( t > 1 \), play Z if (C,Z) has been played in all of the previous periods, and play X otherwise.

For what values of \( \delta \) do these strategies constitute a Subgame Perfect Nash equilibrium?