Boğaziçi University, Department of Economics
Spring 2012
EC 206 MICROECONOMICS II
FINAL - Answer Key
04.06.2012, Monday

• Do not forget to write your full name, student number and section on the top.

• Turn off your cell phone and put it away. During the exam if you are seen with a cell phone, on or off, 20 points will be taken off from your exam immediately.

• Put away all your lecture notes, books, etc.

• There are 6 questions and 15 pages in the exam. Make sure you have them all.

• Please answer all of the questions in the space provided for each question.

• Show your work.

• You have 150 minutes.

GOOD LUCK!!
1. (14 pts) The famous film makers, Joel and Ethan Coen (the Coen Brothers) who are highly acknowledged with their films Barton Fink (Palme d’Or winner in 1991), Fargo (Best Director winner in Cannes Film Festival in 1996), The Big Lebowski (Golden Bear nominee in 1998), are about to start to shoot their new movie. They have two screenplays at hand, one is a comedy (C) and the other is a thriller (T), however they are having hard time to decide which one to pick (both screenplays are very good because they wrote them together). They decide to do the following: Each brother will independently and simultaneously pick one of the screenplays. If they both pick the same, then they will direct and produce it together and both will be credited as the director and the producer, and each will get a payoff of 9. If they don’t pick the same screenplay, each will direct (on his own) the screenplay he’s picked and will serve as the only producer for the other screenplay. In this case Joel gets 0, and Ethan gets a payoff \( k \) if he is the director of the comedy, \( 2k \) if he is the director of the thriller, \( 2 \) if he is the producer of the comedy and \( 3 \) if he is the producer of the thriller, where \( k \in \{3, 5\} \). Nature selects \( k \) with equal probability and tells Ethan his \( k \) but does not tell Joel. All of the above is common knowledge among the brothers.

(a) (2 pts) What is the strategy space for Ethan?

**Answer:** \( S_E = \{CC, CT, TC, TT\} \) where the first action is for \( k = 3 \) and the second action is for \( k = 5 \). For instance, \( CT \) means \( C \) if \( k = 3 \) and \( T \) if \( k = 5 \).

(b) (2 pts) What is the strategy space for Joel?

**Answer:** \( S_E = \{C, T\} \)

(c) (10 pts) Find the set of pure strategy Bayesian Nash equilibria.

**Answer:** The game is as follows:

<table>
<thead>
<tr>
<th>Ethan/Joel</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>9,9</td>
<td>k+3,0</td>
</tr>
<tr>
<td>T</td>
<td>2k+2,0</td>
<td>9,9</td>
</tr>
</tbody>
</table>

When \( k = 5 \), for Ethan \( T \) strictly dominates \( C \). So, we can eliminate, \( CC \) and \( TC \). Then, the Bayesian game, after calculating the expected payoffs, is reduced to

<table>
<thead>
<tr>
<th>Ethan/Joel</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>21/2, 9/2</td>
<td>15/2, 9/2</td>
</tr>
<tr>
<td>TT</td>
<td>20/2, 0</td>
<td>18/2, 9</td>
</tr>
</tbody>
</table>

Those in bold show the best responses. Thus, the Bayesian Nash equilibria are \{\((CT,C),(TT,T)\)\}.
2. (10 pts) In their thriller screenplay, the Coen Brothers, include the following scene: The two main characters, Steve and Peter, play the following version of the Russian roulette: There is a rifle which can hold up to 15 bullets. They load it with only one bullet and make sure it is in the end, so that the first 14 shots will go blank. Then they alternate in taking turns: whenever it’s a player’s turn he has to pull the trigger either once, or twice or three times, and each time one pulls the trigger he has to point the rifle at his own head! Steve takes the first turn. Assuming both Steve and Peter are masters of game theory, who will stay alive? Explain.

**ANSWER:** If Steve leaves exactly 5 shots for Peter, then whatever Peter does Steve can always leave exactly 1 shot for Peter in the next round. Then, to leave exactly 5 shots, Steve leaves exactly 9 shots for Peter. When there are 9 shots left, whatever Peter does Steve can always leave exactly 5 shots for Peter in the next round. Then to leave exactly 9 shots, Steve leaves 13 shots. When there are 13 shots left, whatever Peter does Steve can always leave exactly 9 shots for Peter in the next round. Thus, if Steve pulls the trigger only twice, then there are 13 shots left. When there are 13 left for Peter,

- If Peter pulls once, then Steve pulls three times and leaves exactly 9 shots for Peter.
- If Peter pulls twice, then Steve pulls twice and leaves exactly 9 shots for Peter.
- If Peter pulls three times, then Steve pulls once and leaves exactly 9 shots for Peter.

Likewise, when there are 9 shots left for Peter,

- If Peter pulls once, then Steve pulls three times and leaves exactly 5 shots for Peter.
- If Peter pulls twice, then Steve pulls twice and leaves exactly 5 shots for Peter.
- If Peter pulls three times, then Steve pulls once and leaves exactly 5 shots for Peter.

Then, again when there are 5 shots left for Peter,

- If Peter pulls once, then Steve pulls three times and leaves exactly 1 shot for Peter.
- If Peter pulls twice, then Steve pulls twice and leaves exactly 1 shot for Peter.
- If Peter pulls three times, then Steve pulls once and leaves exactly 1 shot for Peter.

And, Peter has to pull the trigger and he is dead! Steve stays alive!
3. (18 pts) In their comedy screenplay, the Coen Brothers, tell the story of an undergraduate student, Jeff, who is on his way to the final exam of his favorite course, Cinematography. Jeff, on his way to the exam, realizes he has forgotten to submit a homework for his least favorite course, Game Theory. The exam place and the office of the game theory professor are on two different campuses. So he has two options: either deliver the homework after the exam (option L) and incur a penalty for being late, or give the homework to this random student who happens to be around (option S). This student, Walter, can then either deliver the homework on time (D) or throw it away in the nearest trash can (T). For Jeff, the payoff is 1 if the homework is delivered on time, 0 if delivered late, and -1 if not delivered. Jeff doesn’t know Walter well, so he thinks Walter can be either a nice guy with probability p or a jerk with probability 1 − p. The payoff for a nice Walter is x if he delivers, and y if he throws it away. The payoff for a jerk Walter is y if he delivers, and x if he throws it away. Assume x > y. Walter receives payoff 0 in case Jeff doesn’t choose option S. All of the above is common knowledge.

(a) (4 pts) Draw the extensive form of this game. (Hint: You may want to start with Nature playing!)

**ANSWER:**

Note that the strategy sets are $S_{Jeff} = \{L, S\}$ and $S_{Walter} = \{DD, DT, TD, TT\}$.
(b) (7 pts) For every \( p \), find a pure strategy Perfect Bayesian equilibrium.

**ANSWER:**
Walter picks D if he is nice, and picks T if he is jerk. Thus, his equilibrium strategy has to be \( DT \). Then, for Jeff, the expected payoffs from L and S are given as follows.

\[
EU_J(L) = 0 \\
EU_J(S) = p \cdot 1 + (1-p) \cdot (-1) = 2p - 1
\]

Thus, Jeff picks S if \( 2p - 1 > 0 \), otherwise he picks L. That is, for \( p > 1/2 \) he picks S and for \( p < 1/2 \) he picks L. When \( p = 1/2 \) he is indifferent between S and L. Thus, the set of PBE for different values of \( p \) is

For \( p > 1/2 \), \( PBE = \{(S, DT)\} \).
For \( p = 1/2 \), \( PBE = \{(S, DT), (L, DT)\} \).
For \( p > 1/2 \), \( PBE = \{(L, DT)\} \).
(c) (7 pts) Now assume that $p = 3/4$. Find all pure strategy Bayesian Nash equilibria. Are they also Perfect Bayesian equilibrium?

**ANSWER:**

If $p = 3/4$ then the Bayesian game, after calculating the expected payoffs, is

<table>
<thead>
<tr>
<th>Jeff/Walter</th>
<th>DD</th>
<th>DT</th>
<th>TD</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>0, 0</td>
<td>0, 0</td>
<td>0, 0</td>
<td>0, 0</td>
</tr>
<tr>
<td>S</td>
<td>1, (3x+y)/4</td>
<td>1/2, x</td>
<td>-1/2, y</td>
<td>-1, (x+3y)/4</td>
</tr>
</tbody>
</table>

Then, the set of Bayesian Nash equilibria is $BNE = \{ (L, TD), (L, TT), (S, DT) \}$

Note that the first two, $(L, TD), (L, TT)$, are not PBE!!
4. (24 pts) Ethan and Joel are at the set of their new film. They give a break for the film crew. Before they start to shoot the next scene, they decide to play the following game, just to relax a little bit.

where the first payoff number is for Ethan and the second one is for Joel.

(a) (2 pts) Write down the strategy set for Ethan.

**ANSWER:**

\( S_{\text{Ethan}} = \{CY, CN, BY, BN, SY, SN\} \).

(C stands for Cannes, B stands for Berlin, S stands for Sundance, Y is for Yes and N is for No.)

(b) (2 pts) Write down the strategy set for Joel.

**ANSWER:**

\( S_{\text{Joel}} = \{\text{Director}, \text{Producer}\} \)
(c) (7 pts) Find all pure strategy Subgame Perfect Nash equilibria.

**ANSWER:**
There is only one subgame: the game itself. The normal form is

<table>
<thead>
<tr>
<th>Ethan/Joel</th>
<th>Director</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>CY</td>
<td>4,4</td>
<td>4,4</td>
</tr>
<tr>
<td>CN</td>
<td>4,4</td>
<td>4,4</td>
</tr>
<tr>
<td>BY</td>
<td>5,1</td>
<td>2,3</td>
</tr>
<tr>
<td>BN</td>
<td>5,1</td>
<td>3,0</td>
</tr>
<tr>
<td>SY</td>
<td>5,1</td>
<td>2,3</td>
</tr>
<tr>
<td>SN</td>
<td>1,0</td>
<td>2,3</td>
</tr>
</tbody>
</table>

The Nash equilibria of the game above are (CY, Producer), (CN, Producer) and (BN, Director).
Thus, SPNE = \{ (CY, Producer), (CN, Producer), (BN, Director) \}
(d) (9 pts) Find all pure strategy Perfect Bayesian equilibria.

ANSWER:

Fix Y for Ethan. Then Producer is strictly dominant for Joel. Then, Ethan’s optimal action is C. Then q = 1, but when q = 1, Y is not the optimal, thus Ethan is not sequentially rational. So there is no PBE with Ethan playing Y.

Fix N for Ethan. Both Producer and Director can be optimal for Joel.

Fix Producer for Joel. Then Ethan’s optimal action is Cannes. Then, both information sets are off the equilibrium path. R4 puts no restriction on p but it requires q = 1. When q = 1, for Ethan N is optimal, so Ethan is sequentially rational. For Producer to be optimal for Joel we can pick p = 0 (in fact for any p ≤ 3/4, Producer is optimal for Joel). So when p = 0, Joel is also sequentially rational. Thus we have a PBE: (CN, Producer; p=0, q=1).

Fix Director for Joel. Then Ethan’s optimal action is Berlin. Then, p = 1 and q = 0. But when q = 0, for Ethan N is not optimal, so Ethan is not sequentially rational. So there is no such PBE.

Thus, PBE = {(CN, Producer; p = 0, q = 1)}
(e) (4 pts) Is there any pure strategy Subgame Perfect Nash equilibrium which is not a Perfect Bayesian equilibrium, but satisfies R1, R2 and R3 (the first three requirements for Perfect Bayesian equilibrium we have covered in class)?

**ANSWER:**
Yes, there are 2 SPNE which satisfy R1, R2 and R3, yet they are not PBE!! Both \((CY, Producer; any p, q = 0)\) and \((BN, Director; p = 1, q = 1)\) satisfy R1, R2 and R3 but fail to satisfy R4, hence they are not PBEs.
The Coen Brothers have their own film production company, called Mike Zoss Productions (MZP) which has been credited on their films from O Brother, Where Art Thou? onwards. Besides producing their own films, they also provide support for young promising film makers through MZP. Right now, they are interviewing a young film maker, Rezan Yeşilbaş, who has applied to MZP with his first full-length film project. Rezan received the Palme d’Or Short Film with his short Sessiz - Be Deng, the second bit of his Trilogy of Women. The Coen Brothers have heard this success of Rezan at Cannes Film Festival and they also know that Rezan has been working as the assistant of the acclaimed Turkish director Zeki Demirkubuz since 2008. So, it’s no surprise that the Coen Brothers think that he is good and they want to provide funding for his project, but they are just not sure whether Rezan is a genius (G) film maker who has the potential to be one of the greatest film makers of his time, or someone who is just good but not genius (NG). They think that he is a genius with probability $\frac{1}{4}$ and that he is not a genius with probability $\frac{3}{4}$ before the interview. In the interview, they ask him the following question: Will you need a funding of $\text{100K}$ dollars (Low Budget) or $\text{500K}$ dollars (High Budget)? After hearing the answer they decide to fund him either $\text{100K}$ or $\text{500K}$. All of the above is common knowledge. Considering the Coen Brothers as one player, the payoffs are as follows: If Rezan is genius and asks for $\text{100K}$, then both players get $\text{3}$ if the brothers fund him for $\text{100K}$, and both get $\text{5}$ if the brothers fund him for $\text{500K}$. If Rezan is a genius and asks for $\text{500K}$, then both players get $\text{2}$ if the brothers fund him for $\text{100K}$, and both get $\text{4}$ if the brothers fund him for $\text{500K}$. If Rezan is good but not a genius, then both get $\text{1}$ if the brothers fund him for $\text{100K}$, and both get $\text{0}$ if the brothers fund him for $\text{500K}$.

(a) (4 pts) Draw the extensive form game.

**ANSWER:**

![Game Diagram](image-url)
(b) (8 pts) Find the set of separating Perfect Bayesian equilibria in pure strategies.

ANSWER:
For (100, 500) for Rezan, that is, 100 if G, 500 if NG, it must be \( p = 1 \) and \( q = 0 \). Then, for Coen Brothers (500, 100) is optimal, that is 500 if 100, 100 if 500. Then there is no deviation by any type. Thus, \(((100,500),(500,100); \ p=1 \ , \ q=0)\) is a separating equilibrium.

For (500, 100) for Rezan, that is, 500 if G, 100 if NG, it must be \( p = 0 \) and \( q = 1 \). Then, for Coen Brothers (100, 500) is optimal, that is 100 if 100, 500 if 500. Then there is no deviation by any type. Thus, \(((500,100),(100,500); \ p=0 \ , \ q=1)\) is a separating equilibrium.
(c) (10 pts) Now, instead, imagine that the Coen Brothers think that Rezan can be of three types: Average (A), good but Not Genius (NG), and Genius (G), with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively. When Rezan is type G or type NG, all the payoffs are the same as in part (a). If Rezan is Average type, then Rezan gets 2 if funded 100K and gets 3 if funded 500K, and brothers get -2 if they fund him for 100K, and -3 if they fund him for 500K. Find the set of pooling Perfect Bayesian equilibria in pure strategies.

**ANSWER:**

Fix $(100,100,100)$.

Then, $p = 1/4$ and $r = 1/2$. The expected payoffs for the Coen Brothers are

$EU(100) = \frac{1}{4} \cdot 3 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot (-2) = \frac{3}{4}$

$EU(500) = \frac{1}{4} \cdot 5 + \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot (-3) = \frac{1}{2}$

Thus, Coen Brothers pick 100. For the information set that is off-the-equilibrium path (when Rezan sends 500), suppose Coen Brothers pick 500. But then G-type deviates to 100 ($4 > 3$). Thus it must be 100 in this information set. Then there is no deviation by any type. And $q = 0$ and $t = 1$ make 100 optimal in this information set. Thus we get a pooling PBE: $(100,100,100)$, $(100,100)$; $p=1/4$, $r=1/2$, $q=0$, $t=1$).

Fix $(500,500,500)$.

Then, $q = 1/4$ and $t = 1/2$. The expected payoffs for the Coen Brothers are

$EU(100) = \frac{1}{4} \cdot 2 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot (-2) = 1/2$

$EU(500) = \frac{1}{4} \cdot 4 + \frac{1}{2} \cdot 0 + \frac{1}{4} \cdot (-3) = 1/4$

Thus, Coen Brothers pick 100. But then the G-type deviates to 100 regardless of what the Coen Brothers pick at the off-the-equilibrium information path (when Rezan send 100), since $3 > 2$ and $5 > 2$. Thus, no such pooling PBE exists.
After funding Rezan Yeşilbaş a certain amount for his first feature film and hoping that he’ll turn out to be a great film maker, the Coen Brothers go back to producing their own films. Every year the two brothers co-produce a new film and also each brother decides whether to have an extra production on his own or not to have any extra job. If both choose not to do any extra work, they each get a payoff of 10. If they both choose to do an extra job, both receive a payoff of 3 only. If one brother chooses to do an extra job and the other does not, then the one who does an extra job receives 12, and the other brother receives 0 (probably because he is so upset). So, the stage game they play every year is

<table>
<thead>
<tr>
<th></th>
<th>Ethan/Joel</th>
<th>No Extra</th>
<th>Extra</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Extra</td>
<td>10,10</td>
<td>0,12</td>
<td></td>
</tr>
<tr>
<td>Extra</td>
<td>12,0</td>
<td>3,3</td>
<td></td>
</tr>
</tbody>
</table>

Suppose that the above stage game between the two brothers is repeated an indefinite number of times: At the end of each stage (each year), the game is played once more with probability \( r \in (0,1) \). If at some year they stop playing, from next year on, all payoffs are zero. Suppose there is no discounting. Write down a strategy profile the outcome of which is \((\text{NoExtra}, \text{Extra}), (\text{Extra}, \text{NoExtra}), (\text{NoExtra}, \text{Extra}), (\text{Extra}, \text{NoExtra}), ...\), and find the set of \( r \) values for which the strategy profile you have is a Subgame Perfect Nash equilibrium.

**ANSWER:** Consider the following strategies:

- \( s_E = \text{NoEx at } t = 1 \). Then, if it’s been (NoEx,Ex), (Ex,NoEx),... so far, then Ex at \( t = 2n \) and NoEx at \( t = 2n + 1 \) (where \( n \geq 1 \) is an integer), otherwise start playing Ex and play Ex forever.
- \( s_J = \text{Ex at } t = 1 \). Then, if it’s been (NoEx,Ex), (Ex,NoEx),... so far, then NoEx at \( t = 2n \) and Ex at \( t = 2n + 1 \) (where \( n \geq 1 \) is an integer), otherwise start playing Ex and play Ex forever.

3 classes of subgames. In the ones that start with (Ex,Ex), \( s_E \) and \( s_J \) induce a Nash equilibrium. In the other two we have the following conditions:

1. Those starting with (NoEx,Ex):
   - No deviation brings
     \[ 0 + 12r + 0 + 12r^3 + ... = \frac{12r}{1-r} \]
   - One shot deviation brings
     \[ 3 + 3r + 3r^2 + ... = \frac{3}{1-r} \]
   - So we need \( \frac{12r}{1-r} \geq \frac{3}{1-r} \), that is, \( r \geq \frac{1}{3} \)

2. Those starting with (Ex,NoEx):
   - No deviation brings
     \[ 12r + 0 + 12r^2 + ... = \frac{12}{1-r} \]
   - One shot deviation brings
     \[ 10 + 3r + 3r^2 + ... = 10 + \frac{3r}{1-r} \]
   - So we need \( \frac{12}{1-r} \geq 10 + \frac{3r}{1-r} \), that is, \( r \geq \frac{1}{3} \). This is satisfied by any \( r \).

Thus for any \( 1 \geq r \geq \frac{1}{3} \), the strategies given above, \( s_E \) and \( s_J \) give us a SPNE.