1. Suppose that there are 3000 buyers distributed over a 1 km beach such that 1000 buyers are distributed evenly over the west half of the beach, and 2000 buyers are distributed evenly over the east half of the beach. Each buyer incurs a transportation cost $t$ per km. Each buyer buys only one ice-cream. There are two ice cream sellers, $A$ and $B$. $A$ is located at the west end of the beach, and $B$ is located at the east end. Seller $A$ has a marginal cost given by $MC_A = 1$ TL and seller $B$ has a marginal cost given by $MC_B = 2$ TL. There are no fixed costs. Sellers are simultaneously setting their prices, $P_A$ and $P_B$.

(a) Find the location, $\hat{x}$, of the buyer who is indifferent between buying from seller $A$ and buying from seller $B$, as a function of $P_A$, $P_B$, and $t$.

(b) Suppose that the prices are such that $\hat{x}$ is in the west half of the beach. Write the profit of each seller as a function of $P_A$, $P_B$, and $t$.

2. Consider the location model we discussed, where the two firms first pick their location simultaneously and then compete in prices. Is minimal differentiation (locating at the same point, that is, $a + b = 1$) an equilibrium? If yes show that it is an equilibrium and find the respective prices. If not, show that there is a deviation.

3. Consider the circular city model we discussed in class. Recall that the market is circular and there is free entry. Suppose that the transportation cost to travel a distance $x$ is $TC(x) = 0.5x^2$. All firms have a marginal cost equal to $c = 2$ TL and a fixed entry cost $f = 100$ TL.

(a) Find the equilibrium price for a given the number of firms, $n$.

(b) Find the equilibrium number of firms.

(c) Find the socially optimal number of firms. Compare it to the one you found in (b) and comment.

4. Nispetiye avenue is best described as the interval $[0, 1]$. Two kebab places serving identical iskender are located at the edges of the avenue. Antep Iskender ($A$) is located at the southwest end of the avenue, and Bursa Iskender ($B$) is located at the northeast end of the avenue. Both $A$ and $B$ have a marginal cost equal to $c > 0$. Iskender consumers are uniformly distributed on the avenue (on the interval $[0, 1]$), where at each point on the interval lives one consumer. Each consumer buys one iskender from the restaurant in which the price plus the transportation cost is the lowest. On Nispetiye Avenue, however, there is a constant wind (Lodos) blowing from the northeast end toward the southwest.
end, hence the transportation cost for consumer who travels to the northeast is $w$ TL per unit of distance, and only 1TL for a consumer who travels to the southwest, where $w > 1$. Transportation costs are linear. Let $p_i$ denote the price of an iskender at restaurant $i$, where $i = A, B$. Let $\hat{x}$ (the distance to $A$) be the location of the consumer who is indifferent to whether he/she eats at Antep Iskender or Bursa Iskender.

(a) Find $\hat{x}$ as a function of $p_A, p_B$ and $w$.

(b) How does $\hat{x}$ change with respect to $w$? Provide some intuition.

(c) Find the equilibrium prices, $p^*_A$ and $p^*_B$, as a function of $w$ and $c$.

(d) Suppose $w = 2$ and $c = 1$. Find $\hat{x}$. Is it closer to $A$ or $B$? Why?