There are five candidates entering an election. There are three possible positions available at the political spectrum: Left, Right and Center, L,C,R in short. Think of the political spectrum to be a unit interval, [0,1], and L,C and R correspond to points 0, 1/2 and 1, respectively. One of the candidates is already located at the position R, point 1 (so this candidate’s position is fixed and not a player in the game). The other four candidates are simultaneously choosing among the three available positions. There are 100 voters uniformly distributed over the political spectrum. Each voter votes for the candidate that is closest to him/her. If there is more than one candidate that is closest to a voter, then the voter votes for one of these voters with equal probability. Each voter is maximizing own share of votes. Find the set of pure-strategy Nash equilibria, if any.

**ANSWER:** To make it easier to look at all possible cases, consider the possible distributions of the candidates over the three points. Denote the case where there are x candidates at 0, y candidates at 1/2 and z candidates at 1 with (x,y,z). Note that (x,y,z) is not a strategy profile!

**case1:** 4 candidates choose the same point:

(4,0,1), (0,4,1), (0,0,5): One candidate deviates to an empty point. (75 > 50, 25 > 75/4 and 75 > 100/5, respectively.)

**case2:** 3 candidates choose the same point:

(3,0,2): A candidate at point 0 deviates to 1/2. (50 > 75/3)

(0,3,2): A candidates at point 1 deviates to the empty point. (25 > 25/2)

(3,1,1): A candidate at point 1 deviates to 1/2. (75/2 > 25)

(1,3,1): No deviation! Candidate at point 0 gets 25. If he deviates to 1/2 then receives 75/4, not profitable. If he deviates to point 1 receives 12, again not profitable. A candidate at point 1/2 receives 50/3. If he deviates to either of the other two points, he receives 12,5, which is not profitable. Thus, any distribution of this form gives a pure-strategy Nash equilibrium.

(0,1,4), (1,0,4): One of the 4 candidates located at the same point deviates to the empty point. (25 > 25/4 and 50 > 75/4, respectively.)

**case3:** 2 candidates choose the same point:

(2,1,2), (2,2,1): A candidate at point 0 deviates to 1/2. (50/2 > 25/2 and 50/3 > 25/2, respectively.)

(2,0,3), (1,2,2): A candidate at point 1 deviates to 1/2. (50 > 75/3 and 50/3 > 25/2, respectively.)

(0,2,3), (1,1,3): A candidate at point 1 deviates to 0. (25 > 25/3 and 25/2 > 25/3, respectively.)

The cases are exhaustive. Therefore \( NE = \{(s_1, s_2, s_3, s_4) : \text{exactly one } s_i \text{ is 0, exactly three } s_i \text{ is 1/2} \} \)