Motivation

- Financial markets are becoming ever more integrated which result in frequent comovements of stock price indices in emerging and developed countries. (Shiller, 1989; Epps, 1979; Bekaert, Hodrick and Zhang, 2009; Morana, 2008, etc.) Coupled with the fact that fundamentals differ substantially among emerging and developed markets, the finding suggests that high correlation of asset prices cannot be explained solely by the fundamentals themselves.

While bubbles are frequently cited as plausible factors explaining the stock price movements within a single market, little attention has been paid to how bubbles originating in one country affect prices in another country.

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Can we document the existence of intrinsic bubbles in emerging markets?
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Can we establish a casual link between bubbles in an advanced economy and bubbles in an emerging market economy?
Emerging market portfolios are held to a large extent by international investors therefore the positive and negative effects of market sentiments in one country might be transferred to another via the chains of asset positions. Bubbles might start and collapse on more than one market. Any bubble emanating from one country might spill over to another either simultaneously or with a time lag.
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First study about the empirical link between bubbles in a large and a relatively small economy.

Produce time series of bubble component of equity prices in an emerging market.
ISE100 and S&P500: Some Evidence

![Graph showing ISE100 and S&P500 indices over time from 1986 to 2010.](image)

- **ISE100(TL)**
- **ISE100(US$)**
- **S&P500**

The graph compares the value of ISE100 in Turkish Lira (TL) and US dollars (US$) with the S&P500 index from 1986 to 2010.
Show casual links between US and Turkish bubbles.
Results

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- Show casual links between US and Turkish bubbles.
- We identify the shape and magnitude of bubble formation during major financial crises.
- Show that Sigma Point Kalman Filter (SPKF) performs better than previous methods in predicting price fluctuations.
Methodology

- We develop a methodology to measure bubbles in line with the traditional rational bubbles literature. Next, we determine the directional causality of the bubbles between USA and Turkey as a case in point.
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The success of our strategy relies on the premise that we can successfully capture the size of the bubbles by our proposed methodology.
Rational bubbles are generated by extraneous events or rumors and driven by self-fulfilling expectations which have nothing to do with the fundamentals. Examples: Blanchard and Waston (1982), Blanchard (1979), and Flood and Garber (1980). Empirical results on this type of bubble are inconclusive. Diba and Grossman (1988) and Barsky and De Long (1993) fail to reject the null hypothesis of no bubbles, while West (1987) and Wu (1997) reach the opposite conclusion.
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Intrinsic bubbles are driven by fundamentals alone. First introduced in Froot and Obstfeld (1991). Empirically, they find significant evidence to support their model based on estimations for the US market. Ma and Kanas (2004) present further strong empirical evidence to support the intrinsic bubble model using a longer data period, from 1871 to 1996.
Consider the following standard present value model with risk neutral agents:

\[ P_t = \frac{1}{1 + r_t} E_t(P_{t+1} + d_t) \]

\( P_t \) : the real stock price at time \( t \)
\( D_t \) : the real dividend paid at time \( t \)
\( r_t \) : required rate of return
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The solution to the above equation is given by

\[ P_t = \left( \frac{1}{1 + r_t} \right)^2 E_t (P_{t+2} + D_{t+1}) + \frac{1}{1 + r_t} D_t = \ldots = \]

\[ \sum_{i=0}^{\infty} \left( \frac{1}{1 + r_t} \right)^i E_t D_{t+i} + \lim_{i \to \infty} \left( \frac{1}{1 + r_t} \right)^i E_t (P_{t+i}) \quad (1) \]

Fundamentals \quad Bubble
If the transversality conditions hold, i.e. \( \lim_{i \to \infty} \left( \frac{1}{1+r_t} \right)^i E_t(P_{t+i}) = 0 \) or if \( E_t(P_{t+i})/P_t \leq 1 + r_t \) then \( P_t = \sum_{i=0}^{\infty} \left( \frac{1}{1+r_t} \right)^i E_t D_{t+i} \).
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- In this setup the bubble is not due to wrong pricing of the asset but it is a basic component of its price.
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- Both dividends and prices tend to have unit roots.
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- Standard assumptions of this literature: fixed returns due to constant discount rate, constant dividend growth (or constant dividend process), risk neutral consumers, constant dividend price ratio, non-negative bubbles. These assumptions make the estimation of bubbles problematic since the model of fundamentals is too simplistic.
Intrinsic Rational Bubbles

Uncertainty about fundamentals: \( \sum_{i=0}^{\infty} \left( \frac{1}{1+r_t} \right)^i E_tD_{t+i} \). Froot and Obstfeld (1991) assumption of a constant random walk with drift is shown to be invalid by Drifill and Sola (1998).
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For each time the hypothesis of bubbles is not rejected, there might be other fundamental processes that explain the price volatility.
Our Assumptions

- Bubbles exist but unlike previous literature bubbles can either be positive or negative. Stocks can be both overvalued or undervalued. As a result of this, non-negativity constraint is not imposed to the model estimation.
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Our Assumptions

- Bubbles exist but unlike previous literature bubbles can either be positive or negative. Stocks can be both overvalued or undervalued. As a result of this, non-negativity constraint is not imposed to the model estimation.
- Dividend process is subject to regime shifts.
- State space setup is similar to Wu(1997) but bubbles are assumed to be stochastic and feature time variable parameters. The growth rate of the bubble is not necessarily constant. Bubbles are not observable but extracted from data by an unscented Kalman filter.
We take the following present value model with variable discount rates as a departure point.

\[ P_t = E_t \sum_{s=0}^{\infty} \exp(\sum_{j=0}^{s} r_t + j) D_t + s(1) \]

where \( r_t \) is the discount rate, \( D_t \) is the real dividend per share, \( P_t \) is the real price of a share at the beginning of period and \( E_t \) is the expectations operator. The discount rate satisfies:

\[ e^{r_t} = E_t [P_t + 1 + D_t] P_t \]
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The solution to the above specification includes two terms, the rational bubble component and the fundamental component.
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The problem with estimating equation (3) is that when there are sufficiently negative bubbles, stock prices can be negative. Since stocks can be disposed at no cost, negative bubbles in this specification are a priori theoretically ruled out.
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Dividing (2) by $D_{t-1}$, by taking logs and rewriting $\log D_t = d_t$, one can log-linearize expression (4) around a steady-state constant growth rate for dividends $g = \Delta d_{t+j}$ for $j = 0, 1, 2...$ and a constant discount rate $r$ to obtain a particular linear solution.
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$$p^p_t = d_{t-1} - E_t \sum_{i=0}^{\infty} e^{-i(r-g)}(r - d_{t+i}) - h$$ (5)
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$$p_t^P = d_{t-1} - E_t \sum_{i=0}^{\infty} e^{-i(r-g)}(r - d_{t+i}) - h$$

where $h = \log(\exp(r - g) - 1) - (r - g)/(1 - \exp(r - g))$. This is the market fundamental solution where the price growth is determined by future dividend growth. The general solution includes a bubble term, $b_t$:
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Log-linearized Model

- $b_t$ satisfies

$$\begin{align*}
Et(b_t + 1) &= e^t(\alpha) \\
&= b_t
\end{align*}$$

for $t = 0, 1, 2, ...$

The general solution can be written in differences as:

$$\Delta p_g t = \Delta p_p t + \Delta b t$$

$\Delta b t$ is unobservable, therefore we treat it as a state vector and estimate it optimally in a parametric state-space setup.

Since both dividend process, $\Delta p_p t$ and prices are observable, $\Delta p_g t$ ex-post, we treat them as measurement vectors.

We define a dividend and a bubble process both of which are subject to regime shifts and a price equation that serves as a signal process along with the dividend equation. The resulting state-space model is given as follows:
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- We define a dividend and a bubble process both of which are subject to regime shifts and a price equation that serves as a signal process along with the dividend equation. The resulting state-space model is given as follows:
\[ \Delta p_t = \Delta d_t + F_t \Delta Y_t + \Delta b_t + \varepsilon_t \quad (9) \]

\[ \Delta Y_t = A_t + (C_t - 1) Y_{t-1} + \nu_t \quad (10) \]

\[ \Delta b_t = (\rho_t - 1) b_{t-1} + \gamma_t \Delta d_t + \eta_t \quad (11) \]

$F_t, A_t, C_t$ : time variable coefficients which follow a random walk process.

$\rho_t = r_t - g$ where $r_t$ is $AR(1)$. $\rho_t$ captures stochastic growth of the bubble process

$A_t$ : captures regime shifts in the dividend process,

$\gamma_t$ captures non-linear effect of dividends on bubble formation
Model

\[ Y_t = (d_t, d_{t-1}, \ldots, d_{t-h+1})' \] is a \( h \) vector and
\[
C_t = \begin{pmatrix}
\phi_{1,t} & \phi_{2,t} & \ldots & \phi_{h-1,t} & \phi_{h,t} \\
1 & 0 & \ldots & 0 & 0 \\
0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 1 & 0
\end{pmatrix}
\]
and \( F \) is of the same size.

is an \( h \times h \) matrix, \( m = (1, 0, 0, \ldots, 0) \) is a \( h \)-row vector, and
\[
F_t = mC_t(l - C_t)^{-1}[l - (1 - \rho_t)(l - \rho_tC_t)^{-1}] \]
is also a \( h \)-row vector and \( I \) is an \( h \times h \) identity matrix. Innovations \( \eta_t \) and \( \nu_t \) are assumed to be independent, serially uncorrelated and to have zero mean and finite variance \( \sigma^2_{\eta} \) and \( \sigma^2_{\nu} \), respectively. where \( \xi \) is \( N(0, 0.01) \).
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Data

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- The data employed for Turkey has been taken from DataStream and ISE. All data is consolidated on monthly basis.

- Real stock prices are the nominal Standard and Poor’s (S&P) 500 and ISE100 indices, deflated by the Consumer Price Index (CPI). Real dividends are the nominal dividends deflated by the CPI.
Algorithm

1) Obtain the order of the log-dividend process (ARIMA (2,1,0))
2) Run the unscented Kalman Filter and extract the bubble component.
3) Run causality tests on bubbles.

- The UKF is founded on the intuition of Julier & Uhlmann, (2004) that it is easier to approximate a probability distribution than it is to approximate an arbitrary nonlinear function or transformation. UKF belongs to a bigger class of filters called Sigma-Point Kalman Filters.
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- Extended Kalman Filter propagates the state distribution through the first order linearization of the nonlinear system. As a result of that the posterior mean and covariance could be corrupted.

- UKF uses a deterministic sampling approach so this problem is eliminated naturally. UKF estimates a nonlinear function of a random variable through a linear regression between n points drawn from the prior distribution of the random variable and it is a derivative free alternative to EKF.
Table 1. ARIMA (2,1,0) Parameters.

Initial values for dividend process parameters.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$A_{1,1}$</th>
<th>$C_{1,1}$</th>
<th>$C_{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey</td>
<td>-0.005</td>
<td>0.104</td>
<td>-0.063</td>
</tr>
<tr>
<td>USA</td>
<td>0.001</td>
<td>0.828</td>
<td>-0.102</td>
</tr>
<tr>
<td>World</td>
<td>0.002</td>
<td>0.050</td>
<td>-0.036</td>
</tr>
</tbody>
</table>

Table 2. Root Mean Square Error: Comparison with Alternative Models.

<table>
<thead>
<tr>
<th></th>
<th>This Model</th>
<th>Wu(1997)</th>
<th>Intrinsic Bubbles</th>
<th>Simple Present Value</th>
</tr>
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<tr>
<td>RMSE%</td>
<td>4.02</td>
<td>4.33</td>
<td>21.38</td>
<td>39.97</td>
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</table>
Figure 1. Bubble Percentages of Fundamental Price Changes % (US)
Figure 2. Bubble Percentages of Fundamental Price Changes % (TUR)
A closer look at some episodes

Figure 3. Great Depression (US)
Figure 4. Black Monday (US)
A closer look at some episodes

Figure 5. The 2008-09 Episode (US)
A closer look at some episodes

Figure 6. Crisis of 2001 (TR)

- Actual Price
- Bubble
A closer look at some episodes

Figure 7. The 2008-09 Episode (TR)
Table 3. Granger causality tests: p-values.

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Lags :</th>
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<th>5</th>
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<tbody>
<tr>
<td>BTR (\rightarrow) BUSA</td>
<td></td>
<td>0.791</td>
<td>0.574</td>
<td>0.451</td>
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<td>0.009</td>
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<td>BTR (\rightarrow) BWORLD</td>
<td></td>
<td>0.916</td>
<td>0.991</td>
<td>0.984</td>
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</tbody>
</table>

\(\rightarrow\): does not Granger cause
Bubble tests

- Survey of econometric tests by Gurkaynak (2008)
Bubble tests

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Bubble tests

- Survey of econometric tests by Gurkaynak (2008)


Intrinsic bubble tests where bubbles are at least partly determined by dividends. Froot and Obstfeld (1991) and Drifill and Sola (1998), Schaller and Van Norden (2002). Drifill and Sola (1998) capture the main problem with bubble literature: The model with no bubbles and non-linear fundamental processes and the model with intrinsic bubbles but linear fundamental process have equal power. Bidarkota (2007) assumes a random walk for dividends but extends the innovations to dividend process to the family of stable distributions.

Estimating Unobserved Bubbles
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Estimating Unobserved Bubbles

Producing times series of intrinsic bubbles. Wu (1997), Assumes ARIMA(2,1,0) for the dividend process. Prone to the same problems.
Lau et al. (2005) use a Kalman filter estimate bubbles for Taiwan, Singapore, Korea and Malaysia under the classical assumptions. An important caveat with using the rational valuation model in emerging markets:
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- Dividend policy: Do all cash flows filter through dividend payout? If not, the bubbles are overestimated.
Conclusions

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- Bubbles are more pronounced during crises.
- There is a clear spill-over of bubbles from US to Turkey.
Further Research

- Repeat the exercise with European data
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- Revisit the causality relationship