1- Three friends are trying to decide which dvd to watch. Below, you can see the utilities they get from 3 different alternatives. Assume that they will watch only one movie.

<table>
<thead>
<tr>
<th>Tepenin Ardi</th>
<th>Yozgat Blues</th>
<th>Bir Zamanlar Anadolu’da</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ali</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Ahmet</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Mehmet</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

Suppose that they decide to watch Yozgat Blues and implement the following transfer scheme: Ali pays 1 to Mehmet and Ahmet pays 2 to Mehmet (Observe that this is a budget-balanced transfer). Based on this answer the following:

a- Find the final utilities for all.

b- Find another alternative (a dvd to watch and a budget-balanced transfer scheme) that Pareto dominates the choice described above.

Now suppose that they decide to watch Bir Zamanlar Anadolu’da and implement the following transfer scheme: Ali pays 3 to Ahmet and 1 to Mehmet (Observe that this is a budget-balanced transfer). Based on this answer the following:

c- Find the final utilities for all.

d- Find another alternative (a dvd to watch and a budget-balanced transfer scheme) that Pareto dominates the choice described above.

e- Describe the outcome in detail if we use a VCG mechanism to pick a movie and a transfer scheme.

2- Let the marriage market be composed of 4 men and 4 women. Below you can see the preferences of these men and women:

\[ P(m_1) = w_1, w_3, w_2, w_4 \]
\[ P(m_2) = w_3, w_2, w_1 \]
\[ P(m_3) = w_4, w_1, w_2, w_3 \]
\[ P(m_4) = w_1, w_2, w_4, w_3 \]
\[ P(w_1) = m_1, m_3, m_2, m_4 \]
\[ P(w_2) = m_3, m_2, m_1, m_4 \]
\[ P(w_3) = m_1, m_2, m_4, m_3 \]
\[ P(w_4) = m_2, m_1, m_3, m_4 \]

**a-** Assume that men propose. Use the deferred acceptance algorithm to find a stable matching.

**b-** Assume that women propose. Use the deferred acceptance algorithm to find a stable matching.

**c-** Find a matching which is not stable.

**3-** Assume you’re bidding in a second price sealed bid auction. Give an example for each one of the following cases:

**a-** Bidding less than your valuation causes losing the auction when you can win and make profit.

**b-** Bidding more than your valuation causes getting negative final utility.

**c-** Bidding something not equal to your valuation is the same having a bid equal to your valuation.

**4-** Show with an example (different than the one used in class) that a one-sided matching problem might not have a stable outcome.

**5-** Think of 2 firms choosing quantities \( (q_1, q_2) \) in Cournot competition. The market demand is given by \( P = a - Q \). The marginal cost for firm 1 is \( c_1 \) and the marginal cost for firm 2 can be either \( c_L \) with probability \( \theta \) and \( c_H \) with probability \( 1 - \theta \). Firm 1 can not observe the marginal cost of firm 2 and the marginal costs are ordered as \( c_H > c_1 > c_L \).

**a-** Find the Bayes-Nash equilibrium of this game \( \{q_1, q_2(c_L), q_2(c_H)\} \).

**b-** Interpret the effect of \( \theta \) on \( q_1, q_2(c_L) \) and \( q_2(c_H) \).

**c-** Interpret the effect of \( c_l \) and \( c_h \) on \( q_1 \).

**6-** A buyer and a seller take part in a double auction. Assume that the valuations for both parties are distributed uniformly in \([0, 1]\).

**a-** Find a Bayes-Nash equilibrium where both the buyer and the seller are using a cut-off strategy. Simulate your model with 100 pairs with random
valuations (use www.random.org to obtain random values). Calculate the efficiency, average buyer surplus and average seller surplus.

b- Repeat the procedure above for the linear equilibrium we computed in class.

c- Is it possible to have an equilibrium where the buyer and seller use a cut-off strategy and the cut-off values are different for the buyer and the seller? Prove.

7- Two players \((i, j)\) take part in a common-value auction for a single object. Each player gets a private signal \((\gamma_i \text{ for player } i \text{ and } \gamma_j \text{ for player } j)\) and these values are distributed uniformly between 0 and 1. The value of the object is the same for both players and it is given by \(v = \gamma_i + \gamma_j\). The bidding functions for these players are given by \(b_i(\gamma_i)\) and \(b_i(\gamma_j)\). The winner and the payment is determined as in first price sealed-bid auction.

a- Is it possible to have an equilibrium where \(b_i(\gamma_i) = \gamma_i\) and \(b_j(\gamma_j) = \gamma_j\)?

8- Two players \((i, j)\) take part in a public good provision game. The player simultaneously choose to contribute (or not) towards the provision of the good. In other words, each player has two possible strategies: contributing or not contributing \((s_i, s_j \in \{C, N\})\). The value of the public good is 1 for both players and the good is provided if at least one these players choose to contribute. Contribution is costly and these costs \((\delta_i, \delta_j)\) are both uniformly distributed between 0 and 2. Costs are private information.

a- Find a Bayes-Nash equilibrium for this game.