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1 Introduction

Banks regulation aims to discipline banks and promote financial stability. It is based on three pillars, capital requirements, market discipline, and supervisory review process. Under Basel I, banks are required to hold capital for credit and market risk. Since operational losses have a significant effect on the economy, it has become a crucial point for bank's regulation. Under basel II, banks must also compute capital charge for operational risk. Operational risk is defined as 'the risk of direct or indirect loss resulting from inadequate or failed systems or from external events'. It includes internal fraud, external fraud, employment practices and workplace safety, clients, products and business practices, damage to physical assets, business disruption, and system failure and execution. Actually, capital charge for operational risk can be computed using Basic Indicator Approach, Standardized Approach, or Advanced Measurement Approach. Under the BIA, banks must hold capital for operational risk equal to the average over the previous three years of a fixed percentage (denoted alpha) of positive annual gross income (a proxy of the level of operational risk exposure).Under the STA , a variant of the BIA, banks activities are divided into eight business lines: corporate finance, trading and sales, retail banking, commercial banking, payment and settlement, agency services, asset management, and retail brokerage. For each business line, the corresponding capital charge is computed by multiplying gross income by a factor (denoted beta) assigned to that business line. The total capital charge is obtained by summing capital charges for each business. Importantly, the first two approaches doesn't truly reflect bank's exposure to operational risk. Indeed, operational capital charge doesn't take into account banks efforts to manage and mitigate operational risk. The capital charge under AMA is more risk sensitive[1] For each business line and loss type, the capital charge is equal to a percentage (gamma) of expected loss. Expected losses is equal to the product of an exposure indicator (specified by

[1]For example: Basel II clause, 677. Under the AMA, a bank will be allowed to recognize the risk mitigating impact of insurance in the measures of operational risk used for regulatory minimum capital requirements. The recognition of insurance mitigation will be limited to 20% of the total operational risk capital charge calculated under the AMA.

the supervisor), by the probability of losses event, by losses given event.

The inclusion of operational risk into capital requirements has been severely criticized. Unlike credit or market risk, there is no excessive risk taking related to operational risk. Moreover, banks typically hold cash in excess of the required capital in order to absorb future operational losses[2]. Hence, imposing operational risk capital would be counter productive if it does enhance banks' incentives to manage and mitigate operational risk. However, there is no evidence that capital charge, computed under basel II, will give banks incentives to reduce their exposure to operational risk. Moreover the coexistence of the three approaches may aggravate the problem.

In this paper, we design a continuous time model to address the issue of operational risk capital requirements. It is a first attempt to quantify capital charge for operational risk based on an incentive approach. We consider a bank that faces two types of risks: credit risk and operational risk. We represent credit risk by a Brownian motion and operational risk by a Poison process with constant intensity. The bank can reduce the frequency of operational losses by exerting a costly effort[3]. The regulator exerts random audit in order to observe bank's effort and capital. She closes the bank as soon as its capital falls below the required level. The manager chooses the dividend policy and operational risk exposure to maximize the expected discounted value of future dividend payments.

First, we analyze the case when there is no charge for operational risk. We find that the bank holds capital in excess of the required credit risk capital and chooses to reduce its operational risk exposure only when its well capitalized. When bank's capital is near the liquidation threshold, the marginal value of internal funds is too high, therefore it is not optimal to pay cash in order to reduce operational risk exposure, i.e, the bank chooses keep cash inside to insure itself against credit risk. This gives a rationale for operational

[2]See Berger *et al*

[3]Basel II clause 663.(a) The bank must have an operational risk management system with clear responsibilities assigned to an operational risk management function. The operational risk management function is responsible for developing strategies to identify, assess, monitor and control/mitigate operational risk; for codifying firm-level policies and procedures concerning operational risk management and controls

risk capital requirements. However, operational risk capital requirements would be counter productive if it does not encourage banks to monitor and control operational risk.

Second, we analyze bank's behavior under operational risk capital requirements. The regulatory capital charge is chosen in order to reduce the moral hazard problem. We compute the minimal capital charge such that good banks (banks complying with regulation) always reduce their exposure to operational risk. We find that BIA does not really create incentives to reduce operational risk exposure. Then, coherent with the spirit of Basel II, we also compute capital charges that relate to operational risk exposure. We show that AMA creates real incentives to manage operational risk.

The paper is organized as follows. Section 2 briefly reviews related literature. Section 3 sets up the model. Section 4 analyzes the behavior of the bank when there are no operational risk capital requirements. Section 5 characterizes operational risk capital charges that insure that banks complying with regulation always exert effort.

2 Related Literature Review

The literature on bank regulation is huge[4]. Here we briefly review two branches of the literature that are closely related to our work. The first strand of literature analyzes credit capital requirements. In this literature, bank's capital is exogenous. Decamps et al (2002) analyzes the articulation between market capital requirements, supervisory review and market discipline. They compute the minimal capital requirement that pushes banks to monitor their investment. They show that market discipline helps reducing the required capital if the moral hazard problem is not so big. Fries et al (1997) examines the interactions between optimal closure rules and bail out subsidiary policies. They derive a closure rule that minimizes expected discounted social bankruptcy costs and the costs of monitoring banks that continue to operate. Bhattacharaya *et al* (2002) assume unobservable bank's capital and random audit. They compute optimal closure rule that eliminates risk shifting incentives for

[4]For a recent review of literature see Santos (2001) or Stolz (2002)

well capitalized banks. Dangl and Lehar (2004) compare Value at Risk based risk capital requirements and Basel Building Approach when regulator makes random audit. They conclude that Value at Risk based risk capital requirements give well capitalized banks stronger incentives to reduce risk. These works models bank'asset as a geometric Brownian motion and allow for endogenous bankruptcy. In presence of deposit insurance protection, equity holders may find it profitable to increase asset risk, i.e, value function is convex.

A second branch of literature tries to endogenize capital and considers liquidity management. Robertson (1996) analyzes optimal business and dividend policy when firm's cash reserves as a drift Brownian motion. The firm pays dividend in excess of a certain barrier and switch to a more profitable/risky technology as cash reserves increase. Milne and Whalley (2001) extend this work to endogenize bank's capital. They assume fixed costs for bank's recapitalization . They find that capital requirements may explain the credit crunch. Kuppo *et al* (2008) introduces delay in recapitalization process. They show that market risk requirements is counterproductive. Hoojgard and Taksar (2004) apply this model to analyze optimal dividend and reinsurance policy of an insurance company. Belhaj (2006) models operational risk by jumps with fixed size. He finds that operational risk may push banks to take more risk and to reduce its capital cushion. Rochet and Villeneuve (2006) and Belhaj (2009) analyze the problem of insurance against shocks of an industrial firm. Shocks have fixed size in Rochet and Villeneuve (2004) and are exponentially distributed in Belhaj (2009). They show that only cash rich firms chooses to insure against shocks. ,

3 The model

We start by giving the mathematical formulation of the problem. (Ω, F, P) is a probability space, $\{F_t\}_{t>0}$ is a filtration, $\{w_t\}_{t\geq 0}$ is a standard Brownian motion adapted to the filtration and $\{N_t\}_{t\geq 0}$ and $\{A_t\}_{t\geq 0}$ are two independent Poisson processes. The filtration F_t represents the information available at time t and all decisions made are based upon this information.

Consider a bank who holds a portfolio of illiquid assets (loans) B. The bank's cash reserves evolve according to

$$m_t = m_0 + (\mu t + \sigma w_t - \sum_{i=1}^{N_t} Y_i)B, \tag{3.1}$$

where m_0 is the initial cash reserves level, μ is the expected cash flow per unit of time, and σ is the volatility of the cash flow[5]. We normalize bank's asset to 1, thereafter bank cash reserves represents the ratio cash reserves to asset book value. We denote the intensity of the Poisson process N_t by $\overline{\lambda}$ and we assume that the stochastic jumps Y_i are independent and have exponential distribution ($e^{-\delta y}$). The Brownian motion corresponds to credit risk and represents small movements of the cash reserves for small period[6]. The Poisson process represents operational risk. Here to keep the model tractable, we assume that cash reserves are not remunerated, In practise banks invest cash in a portfolio market yielding capital charge for market risk.

The main objective of the corporate operational function is to develop policies, procedures and practices to ensure that operational risk is appropriately identified, measured, monitored and controlled. In practise, banks can mitigate their operational risk through appropriate internal processes and controls. Such activities include internal auditing, penalties, rewards, or duplication of processes. Here, we assume that the bank can exert effort to reduce the frequency of operational losses, it costs c per unit of time. When the bank exerts effort it reduces the intensity of operational risk to a level $\underline{\lambda}$ $(= \overline{\lambda} - \Delta\lambda)$. The decision to make effort is made before knowing the realization of audit and operational losses. We assume that effort is socially optimal, ie, the cost of effort is less than the expected reduction in operational losses, $c < \frac{\Delta\lambda}{\delta}$. Let e_t denotes the bank decision to make effort or not,

$$e_t = \left\{ \begin{array}{c} 1 \text{ when the bank exerts effort} \\ 0 \text{ otherwise} \end{array} \right\}.$$

. Further, we assume that bank's effort is not observable. This will give rise to a moral

[5]The drifted Brownian process can be seen as a limiting process of poisson processes

[6]In practise, the credit risk may also contains jumps. Here to keep the model simple, we assume that there are no jumps in credit risk.

hazard problem. Operational risk charge will be designed in order to reduce the moral hazard problem. Finding the optimal contract is beyond the scope of this work. An optimal contract may need more than capital requirements[7]. We will only focus on the design operational risk capital requirements.

We model the random audit (as in Merton 1978) by a poisson process A_t with intensity ψ. The probability that an audit takes place in a period of time dt is equal to ψdt. We assume that the probability of an audit is independent of the result of previous audit,

$$dA_t = \left\{ \begin{array}{c} 1 \text{ with probability } \psi dt \\ 0 \text{ with probability } (1-\psi)dt \end{array} \right\}.$$

To focus on operational risk capital charge, we assume that the book value of bank's illiquid asset minus its debt is equal to the credit risk capital requirements. Further, we assume that equity issue is prohibitively costly[8] Therefore, absent operational risk capital requirements, the bank would be closed the first time its level of cash reserves falls below zero. In presence of operational risk charge (ORC), the bank is closed if it's capital falls below zero or if an audit reveals that it's capital is below the required operational risk capital. We also assume limited liability for the bank. This will be crucial for the solution of our problem.

The manager has also control over the dividend payments and decides the level of efforts. A dividend policy L_t represents the cumulated dividend payed up to time t. L is an *adapted* right-continuous non decreasing process with $0 \leq \Delta L_t \leq m_t$ for all $t \geq 0$ $P - a.s.$ The later condition states that the manager cannot pay an amount of dividends larger than cash reserves. We denote Π the set of all admissible strategies.

Under the manager's control the reserves evolve according to

$$dm_t^{L,\,e} = (\mu - e_t.c)dt + \sigma dw_t - \lambda(e_t)dN_tY_{N_t} - dL_t, \tag{3.2}$$

The manager chooses a dividend policy and the effort level to maximize the expected value

[7] *Biais et al(2009)*

[8] Milne and Whalley (2001) assume fixed costs for equity issue

of future dividend payments. We denote by v the optimal value function,

$$v(m) = \sup_{(L,\, e)\in\Pi} \mathbb{E} \int_0^{\tau_{(L,e)}} e^{-\rho t} dL_t / m_0 = m, \tag{3.3}$$

where ρ is the discount factor and $\tau_L = \inf\{t/m(t) \leq 1_{\{A_t=1\}} ORC\}$ is the bankruptcy time. The bank is liquidated in two cases: first, if an audit reveals that its level of capital is less than the required operational risk capital or if the level of cash reserves falls below zero.

The problem defined in (3.3) is a singular stochastic control problem. We define the following operator

$$A(e)v(m) = \frac{1}{2}\sigma^2 v''(m) + (\mu - ec)v'(m) - (\rho + \psi 1\!\!1_{\{m \leq ORC\}})v(m) + \lambda(e)\delta \int_0^{+\infty} (v(m-y) - v(m)) \exp(-\delta y) dy. \tag{3.4}$$

Let we introduce the following function:

$$DV(m) = A(1)V(m) - A(0)V(m) = \Delta\lambda\delta \int_0^{+\infty} (V(m) - V(m-y)) \exp(-\delta y) dy - cV'(m). \tag{3.5}$$

The function DV measures the benefits from efforts. It is equal to the gain associated with risk exposure reduction minus the cost of effort multiplied by the marginal value of cash.

Using stochastic optimal control techniques, we can show that if the optimal value function v is C^2, then it satisfies the following HJB equation:

$$Max(A(0)v, A(1)v, 1 - v') = 0, \tag{3.6}$$

with $v(0) = 0$.

However finding a solution to (3.4) does not guarantee that it is the value function. The following lemma gives necessary conditions for optimality. The proof relies on standard verification techniques and is omitted.

Lemma 3.1 *Let V a twice continuously differentiable concave solution to (3.4) with bounded first derivative then V is the optimal value function.*

The following results[9] will be useful in our analysis. Consider the following integro-differential equation

$$\frac{1}{2}\sigma^2 V''(m) + \mu V'(m) - (\rho+\lambda)V(m) + \lambda\delta\int_0^{+\infty}(V(m-y))\exp(-\delta y)dy = 0. \tag{3.7}$$

Differentiating this equation yields

$$\frac{1}{2}\sigma^2 V'''(m) + \mu V'' - (\rho+\lambda)V'(m) - \lambda\delta^2\int_0^{+\infty}(V(m-y))\exp(-\delta y)dy + V(m)) = 0. \tag{3.8}$$

Combining these equalities yields

$$\frac{1}{2}\sigma^2 V'''(m) + (\mu + \frac{1}{2}\delta\sigma^2)V''(m) - (\delta\mu - (\rho+\lambda)V'(m) - \delta\rho V(m) = 0. \tag{3.9}$$

We define the polynomial

$$P_{(\mu,\rho,\lambda)}(d) = \frac{1}{2}\sigma^2 d^3 + (\mu + \frac{1}{2}\delta\sigma^2)d^2 + (\mu\delta - (\rho+\lambda))d - \delta\rho. \tag{3.10}$$

This polynomial satisfies

$$\lim_{\theta\to-\infty} P(\theta) = -\infty,\ P(-\delta) = \delta\lambda > 0,\ P(0) = -\delta\rho < 0, \text{ and } \lim_{\theta\to+\infty} P(\theta) = +\infty.$$

Thus, it has three real zeroes has three real zeros d_1, d_2, and d_3 such that $d_1 < -\delta < d_2 < 0 < d_3$. Then solutions of eq (3.9) can be written as linear combinations of $e^{d_1 m}$, $e^{d_2 m}$ and $e^{d_3 m}$.

We define

$$f_\psi(m) = a_1 e^{\theta_1 m} + a_2 e^{\theta_2 m} + a_3 e^{\theta_3 m}, \tag{3.11}$$

where θ_1, θ_2, and θ_3 are the three reals zeros of $P_{(\mu,\rho+\psi,\bar{\lambda})}$, $a_1 = \frac{1}{2}\sigma^2(\theta_2^2 - \theta_3^2) + \mu(\theta_2 - \theta_3)$, $a_2 = \frac{1}{2}\sigma^2(\theta_1^2 - \theta_3^2) + \mu(\theta_1 - \theta_3)$, and $a_3 = \frac{1}{2}\sigma^2(\theta_2^2 - \theta_1^2) + \mu(\theta_2 - \theta_1)$. The function f_ψ is solution of $A(0)V(m) = 0$, with $f(0) = 0$. We will see later on that for small cash reserves, the value function is proportional to f_ψ.

[9]See Belhaj 2009

4 No Operational Risk Charge

In this section, we will analyze the optimal policies of the bank when there is no operational risk capital requirements. The bank is liquidated only when the level of cash reserves falls below zero. This problem has been studied by Belhaj (2009) when $c = \frac{\bar{\lambda}}{\delta}$ and $\underline{\lambda} = 0$, i.e, effort fully eliminates operational risk. Here we extend his results when $\underline{\lambda} \geq 0$ and for $c \leq \frac{\Delta\lambda}{\delta}$. We recall the HJB equation,

$$Max(A(0)v, A(1)v, 1 - v') = 0, \tag{4.1}$$

To get some insights into the solution, first note that it is not optimal to exert effort near liquidation since $Dv(0) = -\frac{\lambda}{\delta}v'(0) < 0$.

Lemma 4.1 *There exists m_1 such that $Df_0(m_1) = 0$.*

Proof. Let $\bar{m}$ =inf$\{m \geq 0, f''(m) = 0\}$ and define $V_0(m) = \frac{f_0(m)}{f_0'(\bar{m})}$ for $m \leq \bar{m}$. The function V_0 is concave[10] and satisfies the following properties $V_0''(\bar{m}) = 0$ and $V_0'(\bar{m}) = 1$. We have $\mu - \rho V_0(\bar{m}) + \lambda\delta \int_0^{+\infty}(V_0(m-y) - V(\bar{m}))\exp(-\delta y)dy = 0$, then $DV_0(\bar{m}) = \Delta\lambda\frac{-\rho V_0(\bar{m})+\mu}{\bar{\lambda}} - c$. Since $c \leq \frac{\Delta\lambda}{\delta}$, we obtain $DV_0(\bar{m}) \geq \Delta\lambda\frac{-\rho V_0(\bar{m})+\mu-\frac{\bar{\lambda}}{\delta}}{\bar{\lambda}} > 0$. As V_0 is concave and $V'(m) < 1$ for $m < \bar{m}$, we can easily show that $\rho V_0(\bar{m}) < \mu - \frac{\bar{\lambda}}{\delta}$. Therefore we obtain $DV_0(\bar{m}) > 0$. We also have $DV_0(0) = -\frac{\lambda}{\delta}V_0'(0) < 0$. As Df is continuous, there is $m_1 \in [0,\ \bar{m}]$ such that $DV_0(m_1) = 0$. ∎

We have that f_0 is solution $A(0)V = 0$ by construction. Now we construct a solution g of $A(1)V(m) = 0$,

$$g(m) = b_1e^{\alpha_1 m} + b_2e^{\alpha_2 m} + b_3e^{\alpha_3 m},$$

where α_1, α_2, α_3 are the three real zeros of $P_{(\mu-c,\rho,\underline{\lambda})}$.

The parameters b_1,b_2, and b_3 are chosen in order to insure that $g_0(m_1) = f_0(m_1)$, $g_0'(m_1) = f_0'(m_1)$, and $g_0''(m_1) = f''(m_1)$.

[10] V_0 is the optimal value function when the bank has no control over its operational risk

Let $B = \begin{pmatrix} e^{\alpha_1 m_1} & e^{\alpha_2 m_1} & e^{\alpha_3 m_1} \\ \alpha_1 e^{\alpha_1 m_1} & \alpha_2 e^{\alpha_2 m_1} & \alpha_3 e^{\alpha_3 m_1} \\ \alpha_1^2 e^{\alpha_1 m_1} & \alpha_2^2 e^{\alpha_2 m_1} & \alpha_3^2 e^{\alpha_3 m_1} \end{pmatrix}$ and $M = \begin{pmatrix} e^{\theta_1 m_1} & e^{\theta_2 m_1} & e^{\theta_3 m_1} \\ \theta_1 e^{\theta_1 m_1} & \theta_2 e^{\theta_2 m_1} & \theta_3 e^{\theta_3 m_1} \\ \theta_1^2 e^{\theta_1 m_1} & \theta_2^2 e^{\theta_2 m_1} & \theta_3^2 e^{\theta_3 m_1} \end{pmatrix}$

We obtain $(b_1, b_2, b_3) = B^{-1} M\ (a_1, a_2, a_3)$

Let $\bar{m}$ =inf($m > m_1$ such that $g''(m) = 0$). Now, we give the solution of eq (4.1) such that the bank pays dividend in excess of a level m^* and exerts effort only for cash reserves larger than m_1.

V is solution of

$$\left\{ \begin{array}{c} A(0)V(m) = 0 \ for\ m \leq m_1 \\ A(1)V(m) = 0 \ for\ m \in [m_1, m^*] \\ V'(m) = 1 \ for\ m \geq m^* \end{array} \right\}, \tag{4.3}$$

At the threshold m_1, the cost of efforts equal its benefits ($DV = 0$).

The solution of (4.3) is given by

$$V(m) = \left\{ \begin{array}{c} \frac{f_0(m)}{g'(m^*)} \ for\ m \in [0,\ m_1] \\ \frac{g_0(m)}{g'(m^*)} \ for\ m \in [m_1, m^*] \\ \frac{g(m^*)}{g'(m^*)} + m - m^* \ for\ m \geq m^* \end{array} \right\}, \tag{4.4}$$

Optimizing V with respect to m^* yields $g''(m^*) = 0$.

Lemma 4.2 *The equation $g''(m) = 0$ has at least one solution.*

Proof. Consider the barrier policy at $\tilde{m} = \frac{\mu - c}{\rho}$, and assume that for all $m < \tilde{m}$, $g''(m) < 0$. Then the corresponding value function $V_{\tilde{m}}$ is concave. Since $A(1)V_{\tilde{m}}(\tilde{m}) = 0$, we have that $V_{\tilde{m}}(\tilde{m}) < \tilde{m}$. This contradicts $V'_{\tilde{m}}(m) > 1$ for $m < \tilde{m}$. ■

Let $m^* = \inf\{m, g''(m) = 0\}$ and let V the corresponding value function. We obtain the following result.

Proposition 4.1 *The barrier policy at m^* and the function V given in eq (4.4) is the optimal value function*

Proof. Note that V is twice continuously differentiable and concave by construction. First, we show that it is not optimal to exert effort for cash reserves less than m_1. We have $(DV)'(m) = -\delta DV(m) + (\Delta\lambda - \delta c)V'(m) - cV''(m)$. Therefore if $DV(m) = 0$ then $(DV)'(m) > 0$, this means that the equation $DV(m) = 0$ has only one solution given by m_1. It follows that $A(0)V(m) > A(1)V(m)$ for $m < m_1$, and $A(0)V(m) < A(1)V(m)$ for $m > m_1$.

Second, we show that the barrier policy at m^* is optimal.

For $m > m^*$, we have $V'(m) = 1$. Then, we need to show that the function V satisfies $A(1)V(m) \leq 0$ for $m > m^*$.

Let $m > m^*$, we have

$A(1)V(m) = \mu - c - (\rho + \underline{\lambda})V(m) + \underline{\lambda}\delta \int_0^{+\infty}(V(m-y)\exp(-\delta y)dy$ and

$V(m) = V(m^*) + m - m^*$.

We differentiate the equation above, we obtain

$A(1)V'(m) = -(\rho + \underline{\lambda}) + \underline{\lambda}\delta \int_0^{m}(V'(m-y)\exp(-\delta y)dy$.

We use integration by part, we find

$A(1)V'(m) = -(\rho + \underline{\lambda}) + \underline{\lambda}\delta(V(m) - \delta \int_0^{m} V(m-y)\exp(-\delta y)dy)$.

Combining these two equations we obtain

$A(1)V'(m) + \delta A(1)V(m) = \delta(\mu - c) - (\rho + \underline{\lambda}) - \delta\rho V(m)$

Since $A(1)V(m^*) = 0$, we obtain $A(1)V'(m) + \delta A(1)V(m) = A(1)V'(m^*) - \delta\rho(m - m^*)$.

Using equation (3.9) (we replace μ by $\mu - c$ in eq (3.9)), we obtain $A(1)V'(m^*) = \frac{1}{2}\sigma^2 V'''((m^*)^-) < 0$. Therefore we have $A(1)V'(m) + \delta A(1)V(m) < 0$ and $A(1)V'(m^*) < 0$. It can be easily shown (by contradiction) that $A(1)V(m) < 0$ for all $m > m^*$ ■

The optimal dividend policy is a barrier strategy. The bank keeps cush inside for small levels of capital and distributes everything in excess of m^* as dividends. The bank mitigates its operational risk by holding capital to absorb losses and also by reducing its operational risk exposure. However, only well capitalized banks ($m > m_1$) reduce their operational risk exposure. For undercapitalized banks, the marginal value of cash is too high and the bank is more likely to go bankrupt because of credit risk rather than operational risk. Therefore the bank does not exert costly effort and chooses to avoid these costs in order to insure itself against downside credit risk.

5 Operational Risk Charge

In the previous section, we have found that, absent operational risk capital charge, banks have no incentive to reduce it exposure to operational risk for small levels of capital. This gives a rationale for the use of operational risk capital requirements. We also find that banks optimally hold capital in excess of the required credit capital to absorb future operational and credit losses. Therefore, imposing a capital charge would be counter productive if it does not encourage banks to reduce their operational risk exposure. Here, we adopt the approach that operational risk capital charge should be computed in a way to ensure that banks complying with the regulatory requirements (well capitalized banks) choose to reduce their exposure to operational risk[11].

5.1 Risk Insensitive Operational Risk Capital

First, we start by analyzing the case where operational risk capital is set independently from banks operational risk exposure. This means that monioring operational risk capital does not affect the required capital. This is consistent with the BIA.

Let x such that $Df_{\psi}(x) = 0$. When $ORC = x$, the banks exerts effort only when cash reserves is larger than x. The optimal value function is given by

$$V(m) = \left\{ \begin{array}{c} \frac{f_{\psi}(m)}{g'_{\psi}(\bar{m})} \; for \; m \in [0, \; x] \\ \frac{g_{\psi}(m)}{g'_{\psi}(\bar{m})} \; for \; m \in [x, m^*] \\ \frac{g_{\psi}(\bar{m})}{g'_{\psi}(\bar{m})} + m - \bar{m} \; for \; m \geq \bar{m} \end{array} \right\}, \tag{5.1}$$

with

$$g_{\psi}(m) = c_1 e^{\alpha_1 m} + c_2 e^{\alpha_2 m} + c_3 e^{\alpha_3 m}, \tag{5.2}$$

where c_1,c_2, and c_3 are chosen in order to insure that $g_{\psi}(x) = f_{\psi}(x)$, $g'_{\psi}(x) = f'_{\psi}(x)$, and $g''_{\psi}(x) = f''_{\psi}(x)$. Technically, compared to the previous section, the discount rate will be

[11] The same approach has been used in Bhattacharya *et al* (2002) to determine a closure rule that eliminates risk shifting for banks with asset values larger than the required level.

increased by the amount ψ in the region $[0\ x]$. We obtain the following:

Proposition 5.1 *The level x is the minimal capital requirement such that well capitalized banks exert effort.*

Proof. Let z the smallest level at which the bank exert effort. We have that the optimal value function is proportional to the function f_ψ in the interval $[0, min(z, ORC)]$.

First case : $ORC > x$, since we have $Df_\psi(x) = 0$, then we obtain $z = x$.

Second case: $ORC < x$, we have $Df_\psi(ORC) < 0$ or equivalently $Dv(ORC) < 0$. Therefore the bank does not exert effort in the neighborhoods of ORC. ■

Note that setting operational risk capital requirements will not induce undercapitalized banks to reduce their operational risk exposure since near when the bank capital falls the marginal value of internal funds are too high and larger than the benefits of operational risk reduction[12]. Operational risk capital charge increases the probability of failure for undercapitalized banks. It artificially increases the discount rate in the region $[0\ x]$ by the amount ψ. This means that the bank becomes more impatient and it discount future cash at a higher rate. This decreases future franchise value and the marginal value of internal cash. But at the same time it decreases the benefits from insurance. Proposition allows to analyze the effect of increase of operational risk capital requirements on bank's behavior. If the capital requirements is less an x, increasing the required level to x would make the banking system more stable. However increasing capital requirements when they are larger than x, will increase the probability of failure and will have no effect on bank's operational risk exposure. Therefore, the level x is the best candidate for operational risk capital charge from an incentive point of view. However, an important question is raised. Does operational risk capital charge really create incentive to exert effort under BIA?, i.e, is x less than m_1? Numerical simulations shows that x is larger than m_1. This means that an unregulated bank does better than a regulated one. Under BIA, banks will consider operational risk capital

[12]Dangl and Lehar (2004) find that regulatory capital requirements do not prevent distressed banks from increasing their asset risk.

charge as an extra cost. Moreover the level x is increasing with audit frequency. Therefore, BIA is counterproductive, it does not really create incentive to exert effort.

5.2 Risk Sensitive Operational Risk Capital

So far, we have considered that operational risk capital requirements is independent from bank's operational risk exposure. Under AMA, regulator can align operational risk capital charge with bank's operational risk exposure. She rewards goods banks by imposing less capital charge then for bad banks. Let ORC(1) be the operational risk charge for good banks (e=1) and ORC(0) be the operational risk capital charge for bad banks. By exerting effort in the region $[ORC(1),\ ORC(0)]$, good banks obtain an extra gain measured by ψV. The gain from effort is now given by

$$\Delta V(m) = DV(m) + 1_{\{m\in[ORC(1),\ ORC(0)] \text{ and } e=1\}}\psi V(m) \tag{5.3}$$

Let y be the solution of $\Delta f_\psi(m) = 0$, and assume that ORC(0) is sufficiently large (note that $y < x$). $Let ORC(1) = y$, the bank exerts effort only for m larger than y and chooses a barrier policy at a level m^*. The value function is given by

$$V(m) = \left\{ \begin{array}{c} \frac{f_\psi(m)}{h'_\psi(m^*)}\ for\ m \in [0,\ y] \\ \frac{h_\psi(m)}{h'_\psi(m^*)}\ for\ m \in [y, m^*] \\ \frac{h_\psi(m^*)}{h'_\psi(m^*)} + m - m^*\ for\ m \geq m^* \end{array} \right\}, \tag{5.4}$$

with

$$h_\psi(m) = b_1 e^{\alpha_1 m} + b_2 e^{\alpha_2 m} + b_3 e^{\alpha_3 m}. \tag{5.5}$$

The parameters b_1,b_2, and b_3 are chosen in order to insure that $h_\psi(x) = f_\psi(y)$, $h'_\psi(y) = f'_\psi(y)$, and $h''_\psi(y) = f''_\psi(y)$.

We have the following:

Proposition 5.2 *The capital charge y is the minimal operational risk charge for good banks (ORC(1)) such that well capitalized banks always exert efforts.*

Proof. The bank exerts effort if $\Delta f_\psi \geq 0$.

Let z the smallest level at which the bank exerts effort.

We have that the optimal value function is proportional to the function f_ψ in the interval [0, min(z,$ORC(1)$)]. We distinguish between two cases

First case $ORC(1) \leq y$: we have $\Delta v(z) = 0 \Leftrightarrow \Delta f_\psi(z) = 0$ then $z = y$.

Second case $ORC(1) > y$: let $m \leq y$, we have $\Delta f_\psi(m) = Df_\psi(m) < 0$ then $z > y$

■

The level of capital requirements for good behaving banks is the minimum level such that banks complying with capital regulation reduce their exposure to operational risk. Since bank's effort is not observable, capital requirements cannot give full incentives for banks to always reduce their exposure to operational risk,i.e, undercapitalized banks has no incentives to monitor their operational risk. Now we discuss the choice of the level $ORC(0)$. This level has to be chosen in order to avoid that banks may choose to shirk for levels of capital larger than $ORC(0)$, i.e, the difference between $ORC(0)$ and $ORC(1)$ should be sufficiently large in order to insure that the gain from effort at $ORC(0)$ is larger than ψv, since for $m > ORC(0)$, the extra gain ψV vanishes. We obtain the following straightforward result. Let z such that satisfies $Dh_\psi(z)) = 0$

Proposition 5.3 *The capital charge z is the minimal operational risk charge for bad banks ($ORC(0)$) such that well capitalized banks always exert effort .*

To summarize the best candidates for operational risk capital requirements $ORC(1)$ and $ORC(0)$ satisfy $\Delta f_\psi(ORC(1)) = 0$ and $Dh_\psi(ORC(0)) = 0$. By aligning operational risk capital charge with bank's risk, the regulator reward good banks. She gives real incentives to banks to monitor their operational risk and she reduces at the same time the required capital. Note that the levels of operational risk required capital are independent from the dividend policy (as soon as the barrier is larger than operational risk required capital).

The following example illustrates that AMA creates incentives to reduce risk exposure and that BIA is counterproductive. The level of capital requirements x, computed under BIA, insures that well capitalized banks always exerts effort. The level x does not depend on bank's risk. The level of capital requirements y for good banks, computed under AMA, insures that well capitalized banks always exerts effort (with z chosen as in previous proposition). We remark that x is slowly increasing with the frequency of audit, but y is decreasing with the level of audit.

Example: $\mu = 0.01$, $\delta = 1000$, $c = 0.001$, $\overline{\lambda} = 2$, $\underline{\lambda} = 0$, $\sigma = 0.0035$, $\rho = 0.05$.

ψ	0	0.5	1	2	5	6
x	0.0445	0.0447	0.0450	0.0457	0.0478	0.0486
y	0.0445	0.0351	0.0293	0.0223	0.0132	0.0116

One of the objective of Basel committee is to push banks to move to the AMA. However, if capital requirements for good banks is not well designed, it will not necessarily encourage banks to adopt AMA given that it needs non negligible investment in operational risk management.

6 Conclusion

This paper develops a framework to compute operational risk capital requirements. Undercapitalized banks have no incentives to monitor their operational risk. Operational risk charge can be computed in order to enhance banks' incentives to manage operational risk. Under BIA, operational risk capital is counterproductive. Aligning the required capital with bank's risk is more efficient in reducing the moral hazard problem. This work can straightforwardly be extended to allow for a richer setting. We can allow for remuneration of cash reserves, other distribution of jumps and the introduction of two types of operational risk; one that can be controlled the other not. We can also introduce fixed recapitalization costs. However, we can't obtain closed form solutions for the value function. Therefore we should rely on numerical solutions.

References

Barth J.R., 2004. Bank regulation and supervision: what works best?. *Journal of Financial Intermediation, vol 13(2), 205-248.*

Belhaj M., 2009. Optimal dividend policy when cash reserves follow a jump-diffusion process, *Forthcoming in Mathematical Finance*

Belhaj M., 2006. Excess Capital, Operational Risk, and Capital Requirements for Banks, Working Paper,

Bhattacharya, S. , Plank, M., Zechner, J., and G. Strobl, 2002. Bank capital regulation with random audits. *Journal of Economic Dynamics and Control (26) 1301-1321.*

Blum J., 1999. Do capital adequacy requirements reduce risks in banking?. *Journal of Banking and Finance, 755-771.*

Choulli, T., Taksar, M., and X.Y. Zhou (2000). A Diffusion Model for Optimal Dividend Distribution for a Company with Constraints on Risk Control. *SIAM Journal on Control and Optimization, 41 (6), 1946-1979.*

Dangl T. and A. Lehar, 2004. Basel accord vs building block regulation in banking. *Journal of Financial Intermediation, 13(2), 96-131.*

Decamps, J.P. , Rochet, J.C. and B. Roger , 2004. The three pillars of basel II: optimizing the mix. *Journal of Financial Intermediation, vol. 13(2), 132-155.*

De Fontnouvelle, P, De Jesus-Rueff, V., Jordan, J. S. and E. S. Rosengren, 2003. Capital and Risk: New Evidence on Implications of Large Operational Losses. *Journal of Money, Credit and Banking, Blackwell Publishing, vol. 38(7), 1819-1846.*

Diamond, D. and Dybwig, P. (1983), Bank runs, deposit insurance, and liquidity. *Journal of Political Economy, 91, 401-419.*

Fries, S., Mella-Barral, P., and, W. Perraudin, 1997. Optimal bank reorganization and the fair pricing of deposit guarantees. *Journal of Banking and Finance, 441-468.*

Froot K. A. and J. C. Stein, 1998. Risk management, capital budgeting and capital structure policy for financial institutions: an integrated approach. *Journal of Financial Economics, 47, 55-82.*

Jaffe, D.M. and T. Russell (1997). Catastrophe Insurance, Capital Markets and Uninsurable Risks. *Journal of Risk and Insurance, 64, 205 - 230.*

HØjgaard, B. and M. Taksar (2004). Optimal Dynamic Portfolio Selection for a Corporation with Controllable Risk and Dividend Distribution Policy. *Quantitative Finance, 3, 315-327.*

Hellmann, T. F. and K.C. Murdock, 2000. Liberalization, moral hazard in banking, and prudential regulation: are capital requirements enough?. *The Americain Economic Review, 147-164.*

Hovakimian A. and E J. Kane, 2000. Effectiveness of capital regulation at US commercial banks. *Journal of Finance, vol 55(1), 451-468.*

Jeanblanc, P. and A.N Shiryaev, 1995. Optimization of the flow of dividends. *Russian Mathematical Surveys, 50, 257-277.*

Merton, R., 1977. An analytic derivation of the cost of deposit insurance and loan guarantees: an application of modern option pricing theory. *Journal of Banking and Finance, 1, 3-11.*

Merton, R., 1978. On the cost of deposit insurance when there are surveillance costs. *Journal of Business, 51, 439-452.*

Milne, A. and D. Robertson, 1996. Firm behavior under the threat of liquidation. *Journal of Economic Dynamics and Control, 20(8), 1427-1449.*

Milne A. and Whalley A E , 2001. Bank capital regulation and incentives for risk-taking. *Bank of England working paper no 90.*

Opler T., Pinkowitz, L., Stulz, R., and R. Williamson (1999). The Determinants and Implications of Corporate Cash Holdings. *Journal of Financial Economics, 52, 3-46.*

Peura, S. and J. Keppo (2006). Optimal bank capital with costly recapitalization. *The Journal of Business,79, 2163-2201 .*

Prescot E. S., 2001. Regulating bank capital structure to control risk. *Economic Quarterly, 35-52.*

Radner, R. and L. Shepp (1996). Risk vs. profit potential: A model for corporate strategy. *Journal of Economic Dynamics and Control, 20, 1373-1393.*

Rochet J.C. and S. Villeneuve, 2004. Liquidity risk and corporate demand for hedging and insurance. *Toulouse University Working paper.*

Santos, J , 2001. Bank capital regulation in contemporary theory: a review of the literature. *Financial Markets, Institutions and Instruments, 41-84.*

Stolz, S, 2002, The relationship between bank capital, risk-taking, and capital regulation: a review of the literature, Kiel Working Paper No. 1105