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Nonstationarity and Kalman Filter

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Abstract

We illustrate that in time varying parameter models, model (especially transition equation) specification is very important in the presence of integrated variables. Allowing for high flexibility in the transition equation with a high transition coefficient (near random walk) decreases the probability of not finding a cointegrating relation to almost zero. We propose a i) state contingent transition equation with the help of conditional to unconditional variance ratio and ii) modification in the Kalman filter to prevent the risk of spurious regression. We find that time varying transition coefficient is more effective in distinguishing between a spurious and a cointegrating relation.

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Spurious regression robust Kalman Filter

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Introduction

In time series models, econometricians inevitably encounter non-stationary series as dependent and independent variables. The presence of such series in regression models require that econometricians adjust the estimation techniques or their statistics to avoid potential mistakes in inference (mainly due to false standard errors). The obvious adjustment that comes to one's mind is testing for the existence of spurious regression. Dating all the way back to the spurious correlation of Yule (1926), the concept of spurious regression has been formalized by quite a few studies (Granger and Newbold, 1974; Phillips, 1986; Durlauf and Phillips, 1988; Entorf, 1997; Granger, 2001). While measures to protect the empirical results against potential spuriousness is standard practice in classical econometric analysis with constant coefficients, no adjustments have been suggested for models when these parameters are allowed to vary with time. Our study attempts to illustrate the potential pitfalls of not adjusting for possible spuriousness and then suggests techniques to make time varying parameter estimation techniques (especially the Kalman Filter) robust when using integrated variables.

An early study by Canarella et al. (1990) shows that using time varying coefficients in the depiction of the relation between the exchange rate and relative prices (PPP) shows that the PPP holds relatively well between the currencies of five developed economies. A later comment by Honohan (1993) shows that such a depiction of time varying parameters (TVPs) in the presence of integrated variables can be problematic, and more specifically the lack of cointegration may go unnoticed by utilizing time varying specification for the cointegrating vector. In particular, in a system such as $Y_t = \beta_{1t} + \beta_2 Z_t + u_t$ with two unrelated integrated variables *Z* and *Y*, when β_{1t} is allowed to vary in time following a random walk, a spurious fit will be generated between the two unrelated series. In other words, he shows that when $Z_t = Z_{t-1} + w_t$ and $Y_t = Y_{t-1} + v_t$, one can easily define a series $\beta_{1t} = Y_{t-1} - Z_{t-1}$ such that the linear combination $Y_t - Z_t - \beta_{1t} = v_t - w_t$ will be stationary and there will be a resultant impression that *Z* and *Y* are cointegrated. This observation motivates our study and draws attention to the specification of the transition equation in state space models when analyzing potentially integrated variables.

Our contribution hinges on penalizing the movement in the TVP in a way that distinguishes spurious regression from cointegration. We utilize two methods in achieving our goal. The first one is modifying the Kalman Filter itself to prevent I(1) fluctuations in the TVP, which compensate for the lack of a relation between the two integrated variables. To do so, we adopt a particular time varying specification for the measurement equation error term such that the adjustment in the TVP is reduced with sample size (assuming quick initial convergence to correct coefficient value). The second correction we propose is on the state equation itself. In this method, we deflate the transition coefficient in the state equation if the (recursive) OLS estimation of the measurement equation shows signs of spuriousness. In other words, the time varying nature of the transition coefficient is tied to the time growth rate of the variance of OLS estimate of the measurement equation coefficient. In both of the proposed methods, we try to avoid specifying the model in a way that removes the evidence of any existing cointegration. Finite sample simulations show that the second method of penalizing the transition coefficient is much more powerful than in distinguishing between cointegrated series and unrelated ones.

Next section introduces the model and illustrates the problem of time varying parameter specification and Kalman Filter estimation in the presence of integrated variables. The following section proposes methods to separate cointegrated series from unrelated ones, discusses the results. The last section concludes.

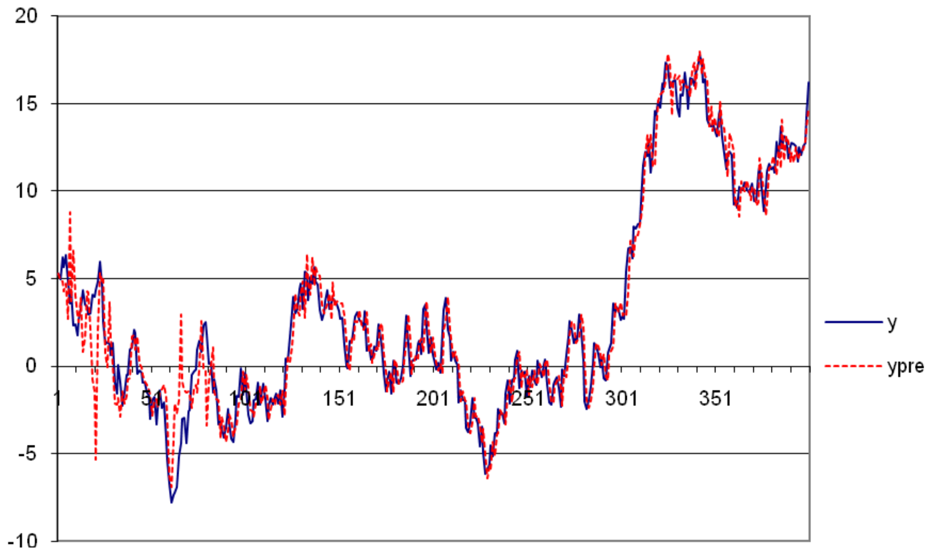
Model

The setup of model (using notation in Harvey, 1989) is

$$Y_t = Z_t a_t + d_t + \varepsilon_t$$
$$a_t = T_t a_{t-1} + c_t + R_t \eta_t$$

where the first equation is the measurement equation with a_t as the time varying coefficient displaying the transition dynamics in the second equation. Other assumptions are $E(\varepsilon_t) = 0$, $\text{var}(\varepsilon_t) = H_t$, $E(\eta_t) = 0$, $\text{var}(\eta_t) = Q_t$, and $E(\varepsilon_t \eta_s) = 0$ for all *s* and *t*. In such a setup, our conjecture is that depicting a_t as having a random walk ($T_t = 1$ for all *t*) will lead to a potential spurious regression relation (in a constant parameter sense) between two uncorrelated and integrated series. Figure 1 displays the nice fit between the actual and predicted Y_t series where the prediction is generated from an unrelated Z_t series and a random walk specification for a_t.[1]

Figure 1: Identification of the problem: Noncointegrated series with drift T=1

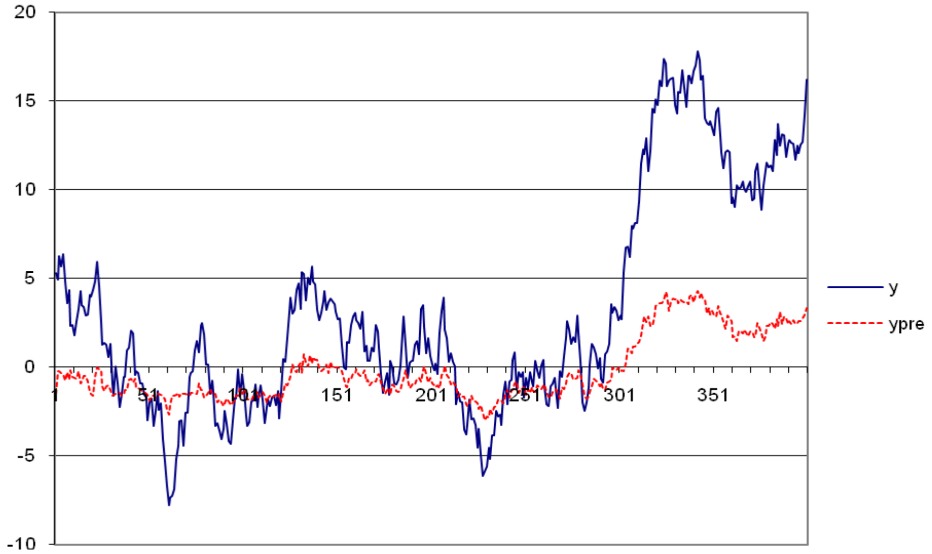


The cause of the problem is the high adaptability of the TVP when it is allowed to move with a random walk specification, adjusting fully to the forecast error caused by the unrelated

[1] Z_t and Y_t are two generated random walk series with drifts 0.02 and 0.04, respectively.

nature of the two independent and integrated series. Naturally, when one tests for a unit root, the residuals turn out to be stationary (pointing toward spurious cointegration) instead of being I(1) and having time dependent variance. However, the fit of the model worsens as the T_t coefficient is reduced. Figure 2 shows the fit between the actual and predicted series when the transition coefficient is set at 0.3 for all *t*. To save space, we do not report the graphs of other values for the transition coefficient. In both of these simulated series the d_t is taken as time invariant and the OLS estimate is used in generating the predicted *Y*. [2]

Figure 2: Identification of the problem: Noncointegrated series with drift T=0.3



Hence, the solution to potential spuriousness of the time varying coefficients seems to be an easy-fix with just a reduction in the T_t parameter. A brief inspection of two cointegrated series and a time varying parameter specification shows that it is not so. Figures 3 and 4 show two cointegrated series (with a cointegrating coefficient of 0.47) with two different (time invariant) specifications for the transition equation, $T_t = 1$ and $T_t = 0.3$ respectively.

[2] Using a proper prior for the constant term and a diffuse prior for the initial value of a_t does not affect the results significantly.

Figure 3: Identification of the problem: Cointegrated series with drift T=1

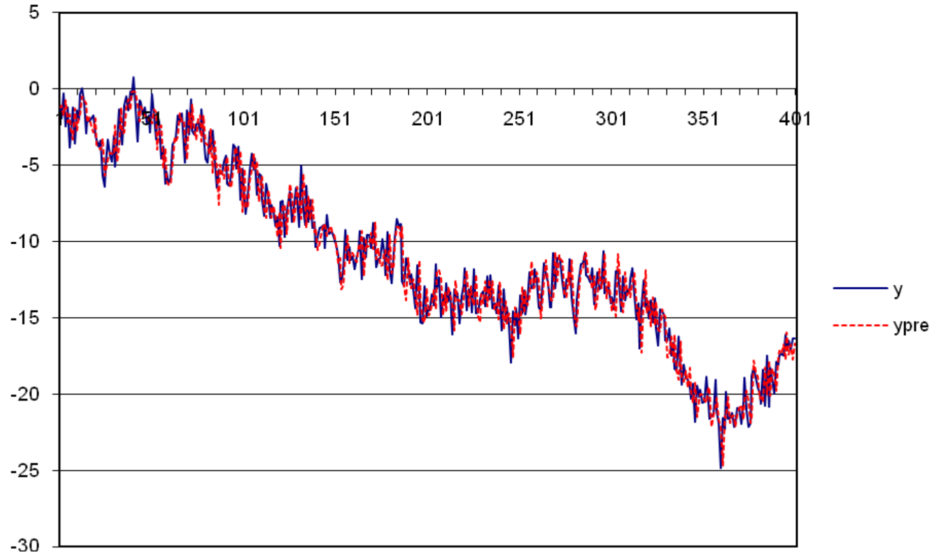
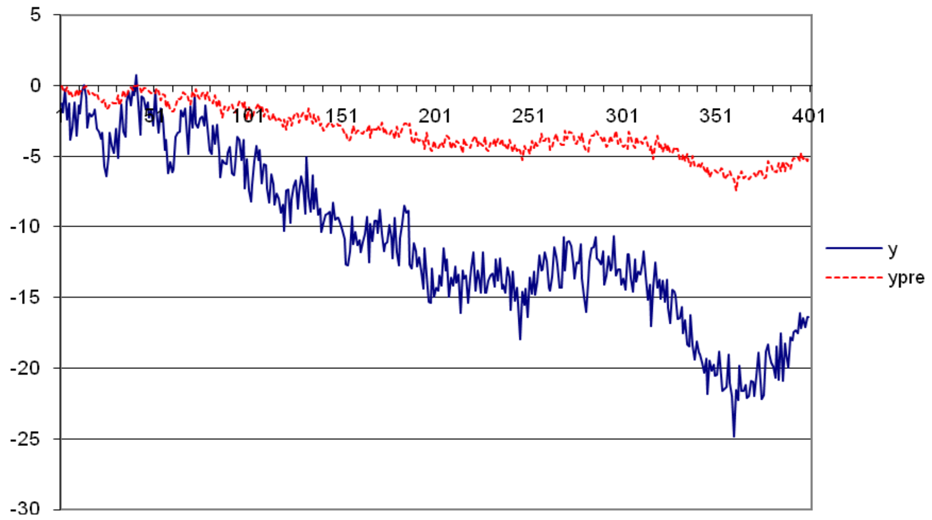


Figure 4: Identification of the problem: Cointegrated series with drift T=0.3



These figures show that when a cointegrating relation exists between the two series, specifying a larger T_t increases the fit of the model. Such a result is likely due to the lower adaptability of the TVP when the transition parameter is small and the predictions of the dependent variable consequently deviating away from the realized values. Hence, while higher transition (T_t) parameters provide a better fit in analyzing cointegrated series, they increase the risk of being too flexible and spuriously fit two unrelated yet integrated series to each other. The motivation for our study is to modify the state space estimation in such a way so that it will be robust in analyzing the relation between related and unrelated integrated series.

Table 1 confirms the graphical stories in the above figures. The simulations of 3000 iterations are done with a sample size of 400. The variances of ε_t (H_t) and η_t (Q_t) are taken to be constant and equal to 1 and 0.2, respectively. The initial values for the TVP and its variance are determined using diffuse priors. Variables *Z* and *Y* are generated as independent random walks with drifts, drift values 0.02 and 0.04 respectively, for the unrelated case. [3] For the case of cointegration, *Z* is again of the same specification, but this time Y_t is generated as $Y_t = 0.04 + 0.47Z_t + \varepsilon_t$, ε_t being standard normal. The regression model used in the estimations is $Y_t = d + a_t Z_t + \upsilon_t$; hence, without loss of generality we focus on a time invariant constant term, equaling the OLS estimate. [4] We also take $c_t = 0$ and $R_t = 1$.

In the tables below, we will illustrate the problem by reporting some statistics on the variability of the TVP, its relation with the data, and the overall fit of the model. The first column of the tables reports the levels of time invariant transition coefficient, *T*, while the second to fifth columns display the mean and standard deviation in the predicted and updated TVP, $a_{t|t-1}$ and $a_{t|t}$. The next columns report the correlation of $a_{t|t-1}$ with *Y*/*Z* and the Mean Squared Error (MSE).

One can observe a few interesting facts from the results. First, the MSE increases for the cointegrated series (and cointegration is not found) as *T* gets smaller while for the unrelated series, the probability of a spurious fit correctly diminishes with the same movement in *T*. As can also be seen from the figures above, as the flexibility of *T* increases the fit in the model of *Z* to *Y* improves whether it is a legitimate fit or not. One could tie this result to our second

[3] Smaller drift values or pure random walk DGPs do not radically alter the findings.
[4] Further extensions will include time varying MLE estimates for both the constant and slope parameters.

point, which notes that the variation in $a_{t|t-1}$, which seems to be much larger in the spurious case than in the cointegrated one. One can also observe that this variation in $a_{t|t-1}$ seems to be correlated with the ratio of the two integrated variables, again indicating that nonstationarity in the error term (the spurious relation) is compensated/reduced by excessive variation in the TVP. The third point regards the significant differences in the means of the TVP values. One notes that the $a_{t|t-1}$ used in the prediction of the measurement equation shows a movement toward the correct value zero in the spurious regression case as the transition coefficient T is reduced; however, it moves in the wrong direction with the same change in T in the cointegrated case. The updated values of the TVP, $a_{t|t}$, also shows that the problem is not solely in the prediction stage since the TVP is fixed at a wrong value of 0.1 in the spurious case despite being equal to the correct 0.47 value in the cointegrated scenario.

Results

We propose two modifications to make the state space estimation of integrated series spuriousness robust. The first modification regards the updating equation in the Kalman filter.

$$a_{t|t} = a_{t|t-1} + K_t \left(Y_t - Z_t a_t - d_t \right)$$

where

$$K_t = P_{t|t-1} Z_t{}' F_t^{-1}$$

$$P_{t|t-1} = T_t P_{t-1} T_t' + R_t Q_t R_t'$$

$$F_t = Z_t P_{t|t-1} Z_t{}' + H_t$$

In spurious regression since the error term will also be integrated and its variance will increase with *t*, we try defining H_t as a increasing function in *t*. Such a modification would reduce the Kalman filter by increasing F_t and lead to the time varying coefficient to be updated less with increasing *t*. Although the error variance will not increase in a cointegrating relation, the reduction in the updating process should not affect the fit between the cointegrated series due to superconsistency. This modification is aimed at eliminating the discrepancies between the prediction stage TVP, $a_{t|t-1}$, and updating stage TVP, $a_{t|t}$, that one sees in Table 1. These results are reported in Table 2 and illustrated in Figures 5 and 6.

Without loss of generality, the time variation added to the measurement error term is directly proportional with time, $H_t = t$, and the constant H example assumes $H = 1$. T is taken to be 1 for both of these examples. Figures 5 and 6 again show the fit in the predicted and actual Y_t. Comparisons of Figure 5 with Figure 1 and Figure 6 with Figure 3 show that there is a slight reduction (though not obvious) in the fit of the case with the independent series, but the cointegration relation does not seem to be affected. Such a time varying specification of the variance of the measurement equation error term should expectedly fix some of the errors in the spurious scenario (due to the correct specification of the time movement in H_t). However, such a specification should distort the results in the cointegrated case since its measurement error variance will be time independent.

Figure 5: Penalty applied to H(t): Noncointegrated series with drift H(t)=t, T=1

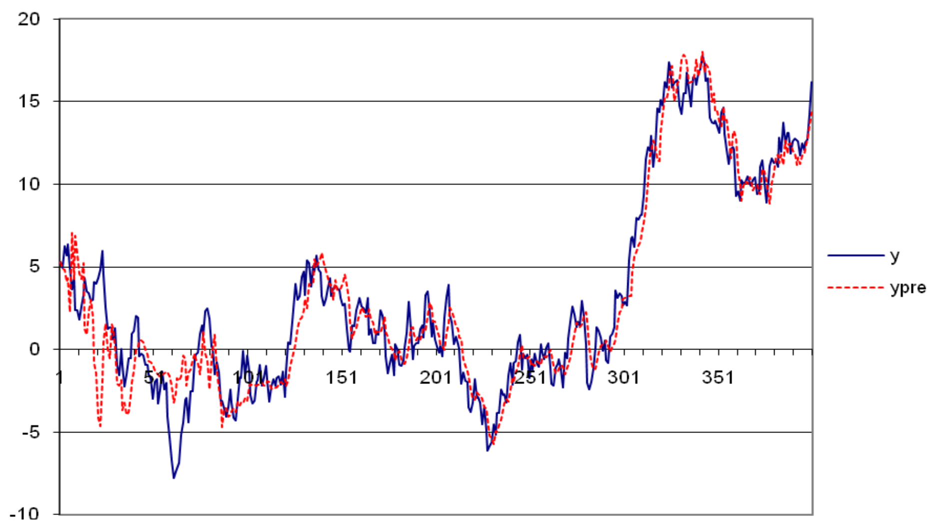
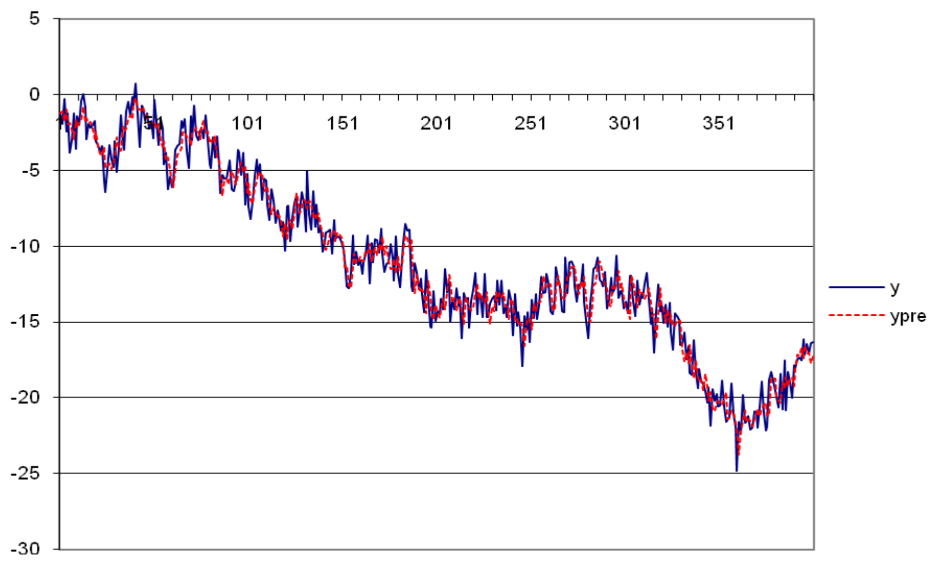


Table 2 confirms our graphical findings with changes in MSE in the desired direction for both scenarios. In the independent random walk case, the MSE increases slightly, a movement in the right direction. In the cointegrated case, the result is quite surprising since the reduction in the Kalman filter seems to have reduced the MSE (making the fit better) despite having increased the measurement error variance, H_t, by *t*. In short, we notice small movements in the MSE when we decrease the Kalman gain using a time increasing measurement error variance. The other values in Table 2 seems to be quite in line with the results in Table 1. One should also note that unreported results show larger penalty on the Kalman filter create larger improvements in MSE, but they can't be substantiated using a theoretical reason.

Figure 6: Penalty applied to H(t): Cointegrated series H(t)=t T=1

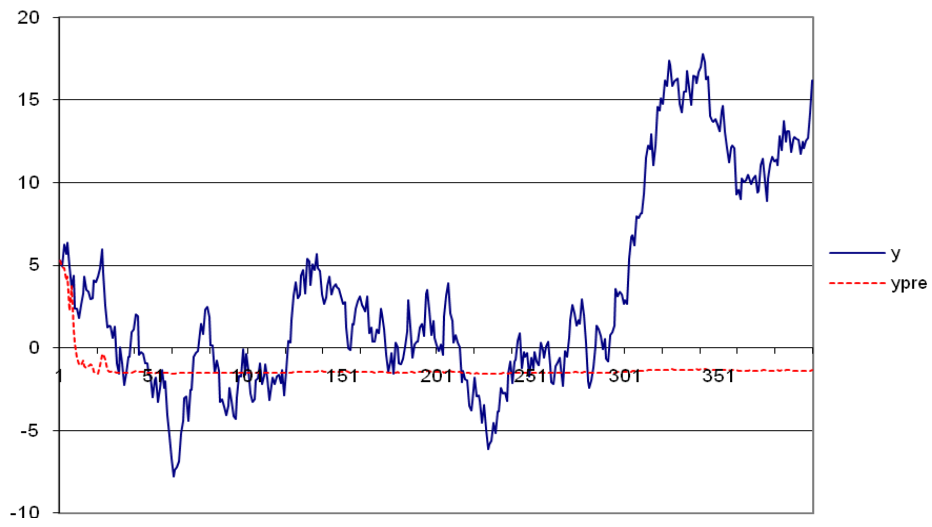


The second penalty enters the prediction equation in the form of a time variant transition parameter, T_t, where this variation is determined by the spuriousness of the cointegration relation. We should remind the reader at this point that larger values of the transition parameter is ideal for the analysis of a cointegrated pair of series while lower values of that parameter seems to avoid a spurious regression risk. Hence, we ideally want the time varying coefficient to have a random walk (T_t converging to 1 from below) if we have a cointegrating relation and to have stationary dynamics (converge to zero) when we have two independent

I(1) series. This way, we also don't allow the time varying coefficient to get stuck at the wrong parameter value due to a diminishing Kalman gain.

One possibility for the generation of an endogenous time varying T_t is inspired by the idea in the variance ratio test (Cochrane, 1988). Using the ratio of conditional variance to unconditional variance of the measurement equation residual (divols), we expect to have a ratio that decreases in a spurious regression due to increasing (with time) unconditional variance in the denominator. Since both the conditional and unconditional variances are time independent in the cointegrated case, this ratio should stay constant for a stationary measurement equation error term. We proxy for the conditional variance by averaging the pairwise variances of the residuals from an OLS regression of the measurement equation, and the unconditional variance comes from the recursive residual variance of the same error term. The effects of these corrections can be seen in Table 3 and Figures 7 and 8.

Figure 7: Penalty applied to prediction equation of at Noncointegrated series with drift, H(t)=t and T=1



Figures 7 and 8 show that the correction in the prediction equation seems to have driven down the predicted series to the estimated OLS constant term in the noncointegrated case while it had little to no effect on the fit of the cointegrated relation. These results show that there is a fast convergence to the correct value of TVP, namely zero or 0.47 depending on the scenario, once the transition coefficient is set to a self adjusting form. The unconditional variance increases as expected in the spurious regression case, pushing the TVP to zero, while it stays quite constant (and equal to the conditional variance) for the cointegrated case. These results are also confirmed with the findings in Table 3 as well.

Figure 8: Penalty applied to prediction equation of at Cointegrated series H(t)=t,T=1

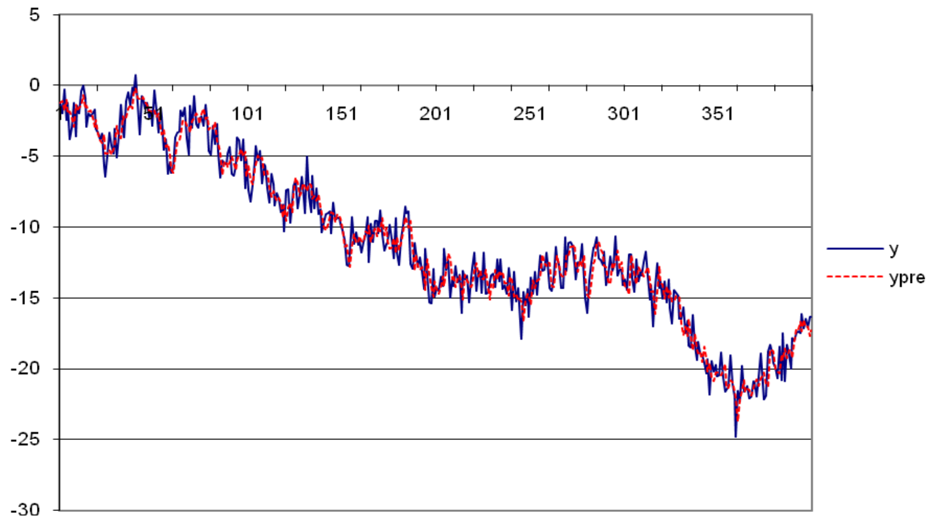


Table 3 reports the similar statistics as the previous tables. The first row of each subsection shows when there is no penalty term and both *T* and *H* are time invariant and set to 1. The second row shows the statistics only when a time varying *T* is used, and the last row shows the combined effect of time varying *T* and *H*. One can observe that the MSE in the spurious scenario moves in the desired direction while there is minimal distortion in the fit of the cointegrated model. One also observes the declines in the mean and the variability of the TVP with the penalty term on the transition coefficient. While these reductions are only in the TVP used in the prediction equation $\left(a_{t|t-1}\right)$, the correction of time varying *H* carries the decline in the mean and variability to the TVP in the updating equation as well $\left(a_{t|t}\right)$.

Conclusion

We illustrate that in time varying parameter models, model (especially transition equation) specification is very important in the presence of integrated variables. Allowing for high flexibility in the transition equation with a high transition coefficient (near random walk) decreases the probability of not finding a cointegrating relation to almost zero. We propose a i) state contingent transition equation with the help of conditional to unconditional variance ratio and ii) modification in the Kalman filter to prevent the risk of spurious regression. We find that time varying transition coefficient is more effective in distinguishing between a spurious and a cointegrating relation. Further research will include maximum likelihood estimation of the transition equation parameters and an application to extended Kalman filter.

Table 1: Statistics on the TVP and the fit of the model after the Kalman Filter estimation of two independent *I*(1) series

T	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
1.0	0.102	1.043	0.097	1.268	0.362	2078.46
0.6	0.102	0.750	0.061	0.450	0.362	25410.23
0.3	0.101	0.661	0.030	0.198	0.362	74180.31

Statistics on the TVP and the fit of the model after the Kalman Filter estimation of two cointegrated series

T	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
1.0	0.469	0.135	0.470	0.141	0.003	746.84
0.6	0.437	0.116	0.262	0.071	-0.002	7495.40
0.3	0.423	0.118	0.126	0.036	-0.001	21662.88

Note: The simulations

Table 2: Statistics on the TVP and the fit of the model after the penalty term (time varying *H*) is applied to the Kalman Filter estimation of two independent *I*(1) series

Penalty	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
$H = 1$	0.102	1.043	0.097	1.268	0.362	2078.467
$H_t = t$	0.098	0.736	0.093	0.975	0.351	3043.232

Statistics on the TVP and the fit of the model after the penalty term (time varying *H*) is applied to the Kalman Filter estimation of two cointegrated series

Penalty	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
$H = 1$	0.469	0.135	0.470	0.141	0.003	746.845
$H_t = t$	0.469	0.055	0.470	0.061	0.004	514.697

Table 3: Statistics on the TVP and the fit of the model after the penalty term (time varying *T*) is applied to the Kalman Filter prediction of two independent *I*(1) series

Penalty	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
No penalty	0.102	1.043	0.097	1.268	0.362	2078.46
$H = 1; T_t = divols$	0.101	0.763	-0.002	0.489	0.076	133650.58
$H = t; T_t = divols$	0.055	0.324	-0.003	0.473	0.057	137500.85

Statistics on the TVP and the fit of the model after the penalty term (time varying *T*) is applied to the Kalman Filter prediction of two cointegrated series

Penalty	mean($a_{t\|t}$)	stdc($a_{t\|t}$)	mean($a_{t\|t-1}$)	stdc($a_{t\|t-1}$)	corr $\left(\frac{Y}{Z}, a_{t\|t-1}\right)$	MSE
No penalty	0.469	0.135	0.470	0.141	0.003	746.84
$H = 1; T_t = divols$	0.465	0.133	0.448	0.137	0.004	948.55
$H = t; T_t = divols$	0.435	0.066	0.421	0.077	0.004	951.23

References

Canarella, G. , S. K. Pollard and K.S. Lai, 1990. Cointegration between Exchange rates and Relative Prices: Another View. *European Economic Review*, 34, 1303-1322.

Durlauf, S. N. and P. C. B. Phillips, 1988. "Trends versus Random Walks in Time Series Analysis", *Econometrica*, 56(6), 1333-1354.

Entorf, H. , 1997. "Random Walks With Drifts: Nonsense Regression and Spurious Fixed-effect Estimation", *Journal of Econometrics*, 80, 287-296.

Granger, C.W.J., 2001. "Spurious Regressions in Econometrics". Chapter 26 in *A Companion to Econometric Theory*, ed. B. Baltagi, Oxford, Blackwell Publishers, 557-61.

Granger, C.W.J. and P. Newbold, 1974. "Spurious Regression in Econometrics", *Journal of Econometrics*, 2, 111-120.

Honohan, P., 1993. "Cointegration and time-varying parameters: A comment" *European Economic Review*, 37, 1279-1281.

Philips, P.C.B, 1986. "Understanding the Spurious Regression in Econometrics", *Journal of Econometrics*, 33, 311-340.

Yule , G.U., 1926. "Why do we Sometimes get Nonsense-Correlations between Time-Series?: A Study in and the Nature of Time-Series", *Journal of the Royal Statistical Society*, 89(1), 1-63.