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Submission Number: ASSET2009-09-00274

Dynamic analysis of kidney exchange problems

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*Abstract*

The demand of kidney transplants is rapidly increasing and one possibility to increase the number of available kidneys is the organization of kidney exchanges between incompatible pairs. When kidney exchanges begin to be performed, one important point is the study of the organization of such exchanges in a dynamically evolving patients' pool. Despite many models have been proposed in order to study the static situation, less attention has been devoted to dynamics, though dynamic matching mechanisms should be expected to have advantages over static matching mechanisms. We consider the case where a centralized authority decides how to organize the exchanges, having the possibility of observing the evolution of the pool over time. In particular we focus on the problem of finding an optimal matching mechanism for such sequential decision process. We first present a model, which shows that the dynamic kidney exchange problem can be stated as a Markov decision process. Then we focus on two specific classes of dynamic matching policies, consisting in a sequence of maximum weight matchings. The novelty of our approach relies in the fact that the quality of the exchanges is taken into account. We will discuss the results of numerical simulations we developed using this model.

## Dynamic analysis of kidney exchange problems

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Kidney transplantation is the elective treatment for many kidney diseases and it is nowadays a routine therapy, so that the demand of kidney transplants is rapidly increasing. One possibility to increase the number of available kidneys is the use of living donors, who are typically friends, spouses or relatives of the patients. However, many potential donors are not donating because of incompatibilities of various kinds with their intended recipient. To overcome this problem, the possibility of organizing kidney exchanges has been suggested by the medical community [3]. When kidney exchanges begin to be performed, one important point is the study of the organization of such exchanges in a dynamically evolving patients' pool. Despite many models have been proposed in order to study the static situation (see [1, 4, 5, 6, 7]), less attention has been devoted to dynamics, with the exception of [8, 9], though dynamic matching mechanisms should be expected to have advantages over static matching mechanisms. We consider the case where a centralized authority decides how to organize the exchanges, having the possibility of observing the evolution of the pool over time. More precisely, on the basis of the available information, at each decision point the decision maker chooses a feasible (possibly empty) set of exchanges. This choice influences the composition of the pool for future exchanges, from which it derives an impact on the overall social welfare function employed by the institution in charge of the organization. Therefore, when taking a decision, the central authority must take into account not only the immediate costs and rewards, but also the future consequences of her decision. In this talk we consider the problem of finding an optimal matching mechanism for such sequential decision process. We first present a model, which shows that the dynamic kidney exchange problem can be stated as a Markov decision process. Then we focus on two specific classes of dynamic matching policies, consisting in a sequence of maximum weight matchings. The policies in the first class prescribe to organize maximum weight matchings at regular intervals of time. Policies in the second class prescribe to organize maximum weight matchings only when the number of patients in the pool is above a certain threshold. The idea is that letting the pool to become more populated should give the possibility of making exchanges of better quality.

The novelty with respect to [8] is the fact that the quality of the exchanges is taken into account. Moreover, the proposed policies are worth to

be studied since they resemble the policies adopted in the real implementations of kidney exchanges, as in The Netherlands for instance [2]. We will discuss the results of numerical simulations we developed using this model. In particular we will show that in some situations, to wait for a certain time in order to let the pool becoming more populated has some advantages over the policy that performs the exchanges as soon as they become available. A comparison between the proposed classes of policies enlightens that the two approaches give similar results. More precisely we observed empirically that in correspondence to each time interval there exists a suitable threshold giving approximately the same total expected discounted reward.

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