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Leveling and Evaluation Precision in Asymmetric Tournaments.

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Abstract

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Abstract

Asymmetric competitions with a relative performance evaluation of candidates usually presents the problem of preemption effect. It consists in that both advantaged and disadvantaged competitors anticipate that there is a clear favorite to win, so everybody performs worse in order to save in competing cost. The usual way of addressing this problem consists in making the competition symmetric through the application of leveling policies (affirmative action). We introduce an additional device to address preemption effect, which is precision in evaluation. By reducing precision in evaluation, there is more uncertainty about the final winner, therefore preemption effect is less likely to occur. We study the combination of these two procedures in a rank-order tournament model. We show how the impossibility of applying leveling policies (color blind affirmative action) can induce to design competitions with a higher influence of randomness. Moreover making the competition symmetric guarantees better performance induce lower levels of randomness.

1 Introduction.

Job allocation, award of a project, promotion in firms or assignment of students grants are good examples of a competition for a non-divisible and non-exchangeable object. In all these cases, in order to win, agents invest in a costly and unobservable variable (job market candidates preparation, quality of a project, effort of a worker or students level of knowledge). The principal's payoff is positively affected by these unobservable investments.

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Though, since he cannot know its true value he uses relative evaluation with respect a related observable variable. His final objective is to induce more investement on the agents' part.

One of the main obstacles to the principal's objective, when using relative performance evaluation, is the existence of asymmetry among competitors. When players are evaluated relatively to each other the weaker the opponent, the lower is the required level in order to win. Hence investment levels are reduced. This is known as preemption effect (see Che and Gale, 1998, for some discussion). When advantaged (disadvantaged) competitors anticipate their higher (lower) chances of winning they relax (become discouraged) and perform worse, trying to save in the cost of competing. A usual way to mitigate this problem consists in applying different treatments to different types in order to have a more balanced competition that prevents the occurence of this preemption effect. This is known as leveling policies (or Affirmative Action when it has some fairness concern). Here we also analyze a less direct way to address the preemption effect: precision in evaluation. Reducing accuracy in evaluation generates more uncertainty on the final result of a competition, and therefore reduces preemption.

As we mentioned above, using relative performance evaluation makes sense when the principal lacks information. This imperfect information consists in the impossibility of observing agents' investments. Imperfect observability means that although he can have some notion of agents' investments he cannot know its true value. Think for an instance in a university interested in motivating its students to learn a lot by giving a grant to the best alumni. The university cannot exactly know how much their students have learnt but can have some idea about it through their scores on a test. In other words, there is a difference between the real knowledge that each student has and on their performance (test score). They are related but different variables. The former is not observable and the latter is. Our idea is that it seems reasonable to consider that the principal can modify or can influence the level of observability through changing the accuracy in the evaluation of agents. Following the previous example we can consider that, in order to choose the winning student, the university can simply use a test scores or ,on the other hand, it can carry on a candidate's interview, some specific tests and ask for a dissertation. As more exhaustive the selection process, the more probabilities of observing the real level of knowledge of each student. A more precise evaluation increases investments observability. As we have explained before preemption effect can be mitigated by reducing agents' possibilities of anticipation, that is by reducing precision in evaluation. If there is more uncertainty about their chances of winning or losing preemption is reduced. Obviously this does not necessarily mean that we improve agents performance. Actually introducing more randomness in the final allocation also has negative effects on competitors investments, since it reduces its returns. In the extreme case of an absolutely random allocation agents will not invest. In the opposite case of no randomness (all-pay auction as Hillman and Riley, 1989) the asymmetry has much more influence on final allocation. There is a trade-off between reducing preemption or returns to investment when deciding precision in evaluation. This fact is reflected in Nti (2004), where he analyzes the optimal level of the random component in a contest framework instead of in a rank-order tournament one. Krkel and Sliwka (2001) and Krkel (2008) analyze the optimal decision of noise that agents want to take in an asymmetric tournament. In our case this decision is taken by the competition designer and analyzed together with leveling policies.

Both leveling policies (reduction in asymmetry) and precision in evaluation (reduction in anticipation with less precision) can prevent the previously mentioned preemption effect and therefore can stimulate better performance of competitors. Our objective is to analyze the principal's optimal choice regarding these devices. We study how the interaction of both policies can lead to some results which are different from when they are considered separately. When precision in evaluation is exogenous, and the principal can only apply leveling policies he is not always interested in completely compensating the asymmetry, since for small levels of observability this could generate too much randomness in the competition [1]. When the principal cannot apply leveling policies but can control precision in evaluation he may be interested in not reducing the unobservability at the minimum level in order to dissipate the previously mentioned preemption effect, by introducing some noise. Finally we find that when the principal can control both instruments it is optimal to maximize the accuracy in evaluation and completely equalize the competition, that is make it symmetric. In Chan and Eyster (2003) they also consider randomness as an alternative to affirmative action when principals are interested in ethnic diversity. They consider the possibility of making the selection process absolutely random for some candidates but not the optimal level of it. Moreover they describe the process as an adverse selection one instead of a moral hazard.

To model the competition we use rank-order tournaments first intro-

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[1] This result differs from previous literature on Affirmative Action. This is because in our case it is not possible to compensate unobservable investments but only observable performance. We discuss later the convenience of this modelization.

duced by Lazear and Rosen (1981). We present the basic framework in Section 2. A rank-order tournament is a relative evaluation mechanism consisting in allocating the object to the first ranked player according to candidates order with respect to an observable variable. This observable variable is composed by the sum of the unobservable and costly investment of candidates and the influence of some unsystematic elements (luck, errors...) represented by the realization of a random variable. The objective of the principal or rank-order tournament designer is only related with the unobservable investment. A good realization of the random element helps candidates to win but does not increase the payoff of the principal. For instance when the designer uses a multiple answer test-score evaluation he can not distinguish if correct answers correspond to a true knowledge of the topic or to a fortunate random decision. If it is the first case the test is informative about candidates knowledge, otherwise it is not. In the second case it does not reflect the level of the unobservable variable. The specific form of the principal's payoff depends on the kind of situation we want to represent. On one hand when a tournament designer can capture everybody's investments, he is interested in maximizing all individuals performance independently of if they win or not. This fits well with competition for an internal bonus in firms or prizes to best students. On the other hand sometimes the principal is not interested in making everybody to perform well but just in maximizing winner's performance, think for an instance in a job market, where employer does not care about the effort made by non-hired individuals. In this preliminary version we will focus in the first case.

Leveling policies make the competition more symmetric and reduce preemption effect. When some individuals are disadvantages, these policies will compensate them in order to have a more balanced competition. Equality of opportunities and affirmative action correspond to these kind of policies (applied when the disadvantage is considered unfair). When the level of observability of agents' investments is exogenous, we have some theoretical evidence that participants' performance is maximized when there is a symmetric environment. Nti (1998) and in Stein (2001) provides some evidence of this in a Tullock rent-seeking contest. We find also some literature focused on affirmative action policies that also agree with these results: Schotter and Weigelt(1992), Fu(2008) or Franke (2008). However we treat leveling policies in a different way than this previous research. In particular our model is similar to the one in Schotter and Weigelt(1992), but we considering that leveling polices cannot be applied directly to the unobservable agents' investmentsbut just to the observable part. Under imperfect observability of investments it is difficult to accept that tournament designer can directly compensate individuals'investement. That is why in our work, differently than the previous ones, leveling policies affects both the unobservable investement and the noise term. To illustrate this consider the previous example of a university that evaluates the knowledge of its students through a test. The university allocate a grant to the best student with the objective of motivating students to learn a lot. Imagine that we have two groups of students. The teacher of the second group gets sick and the second group have to study the last lesson by themselves. When doing relative evaluation university may want to compensate the disadvantaged group by valuating higher their scores. By doing this university is not only compensating disadvantaged students for their higher cost, due to learning last lesson by themselves, but is also influencing some other random circumstances that also affects final score as luck or nerves. University can only compensate the observable variable that uses to evaluate, not he unobservable one. This new and more reasonable (under our opinion) interpretation of compensation gives some results different from the ones obtained by directly compensating cost of effort or valuations with an exogenous level of observability. We study the optimal leveling policy with an exogenous level of observability in section 3.

Many times it is physically impossible or legally prohibited to apply differential treatments, consider for an instance the non possibility of distinguish individuals valuations or the legal prohibition of applying certain differential policies (imposition of color blind affirmative action). When this happens the contest designer may be interested in looking for more subtitle ways of strengthening the competiton. One of them could be the modification of random components in the competition by reducing precision in evaluation, since players are different, varying it may not affect all of them in the same way. In particular advantaged competitors see how their advantage is dissipated because of a higher level of uncertainty. We explore the impossibility of applying leveling policies in section 4.

Finally in section 5 we study the application of leveling policies accounting for the fact that evaluation precision is something endogenous for the principal.

2 The Model.

We will use a rank-order tournament in order to represent the competition for a non-divisible and non-exchangeable object. The winner selection proceeds as follows. First the designer specifies the details of relative performance evaluation: accuracy in the evaluation of candidates and/or the application or not of leveling policies. Once the candidates observe competition design, they decide their investments, we will denote them by e_i, and will call it effort from now on. Since the principal cannot directly observe the candidates investments he evaluates them accordingly to some related variable b_i composed by the unobservable investments (e_i) and the realization of a random variable, ϵ_i. Finally the auctioneer allocates the prize to the highest ranked player accordingly to observable performance (b_i).

2.1 Candidates.

Let $N = \{1, 2\}$ be the set of players. They are involved in a competition for a prize, they value V. In rank-order tournaments (see Lazear and Rosen 1981, for a detailed explanation), the prize is allocated to the highest bidder and candidates are not refunded for their cost of effort, they pay the cost independently of winning or not. To simplify computations cost of effort of individual $i \in N$ is $C(e_i) = \frac{1}{2c_i} e_i^2$ where c_i accounts for players asymmetry. The greater is c_i, the higher is the ability of individual i to compete, his cost of effort is lower. Agents' effort (investment levels) are not observable but bids are. Bids respond to the following function:

$$b_i = e_i + \epsilon_i$$

where e_i is individual i investment in a non observable variable and ϵ_i represents a random noise uniformly, identically and independently distributed, $\epsilon_i \tilde{U}(-k, k)$.

2.2 Tournament Designer.

Since tournament designer cannot observe efforts, he elects the winner by relative evaluations assigning the prize to the individual with a higher bid (i.e. individual i wins if $b_i > b_j$, $i \neq j$, $i, j = 1, 2$.) The designer is interested in maximizing total effort of competitors. To do so he can control for two different devices: leveling policies and evaluation precision.

Definition 1. *A leveling policy consist in modifying the value of competitors bids. In a leveled competition individual i wins if* $\alpha_i b_i > \alpha_j b_j$

Notice that applying leveling policies does not only modifies the value of unobservable part but also those of the random element ($\alpha_i b_i = \alpha_i e_i + \alpha_i \epsilon_i$). This is not a trivial difference with respect to previous models of leveling policies.

To modify precision in evaluation means that the designer can control for the variance (or equivalently for the bounds, k) of the distribution of ϵ_i for $i = 1, 2$. We impose a lowe bound on $k \geq \frac{\sqrt{V_1} V_2}{}$. This gurantees the existance of an equilibrium in pure strategies [2].

2.3 The competition.

Individuals are risk neutral so their maximization problem is the following:

$$max_{e_i} Prob(\alpha_i b_i > \alpha_j b_j)V - C_i(e_i)$$

$$max_{e_i} Prob(\alpha_i e_i - \alpha_j e_j > \alpha_j \epsilon_j - \alpha_i \epsilon_i)V - \frac{1}{2c_i} e_i^2$$

We denote by $\xi = \alpha_j \epsilon_j - \alpha_i \epsilon_i$. Given the previous distributional assumptions on (ϵ_i, ϵ_j) we can find the distribution of ξ, $F(\ )$.

Then we can rewrite previous expression as:

$$max_{e_i} F(\alpha_i e_i - \alpha_j e_j)V - \frac{1}{2c_i} e_i^2$$

First order condition has the following form:

$$e_i = f(\alpha_i e_i - \alpha_j e_j)\alpha_i V_i \qquad with \qquad V_i = c_i V$$

Where $f(\ )$ is the density function of ξ. The symmetry of distribution functions together with first order conditions guarantees that:

$$\alpha_i e_i = \frac{V_i}{V_j} \alpha_j e_j \qquad (1)$$

for $i \neq j$ and $i, j = 1, 2$.

We will us without loss of generality the following normalization:

$$\alpha_1 = \alpha \quad \alpha_2 = 1 \quad c_1 = 1 \quad c_2 = c > 1$$

Since 1 is the disadvantaged individual, leveling the competition implies $\alpha \geq 1$.

We need to use $\alpha \geq 1$ in order to construct different intervals for the distribution function of ξ. Using previous normalization and the fact that $\alpha \geq 1$ the distribution function of ξ has the following form:

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[2] Fixing lower levels of k induce a mixed strategy equilibria that converges to the one of an all-pay auction. However all-pay auction mixed strategy equilibria coincides with the rank-order tournament pure strategy equilibria with $k = \frac{\sqrt{V_1}}{2}$

$$F(\xi) = \begin{cases} \frac{(k+k\alpha+\xi)^2}{8k^2\alpha} & \text{if } -k-\alpha k < \xi < k-\alpha k \\ 1-\frac{1}{2\alpha} & \text{if } k-\alpha k < \xi < \alpha k - k \\ 1-\frac{1}{2\alpha}+\frac{(k(\xi-\alpha+1))(k(3+\alpha)-\xi)}{8k^2\alpha} & \text{if } \alpha k - k < \xi < \alpha k + k \end{cases}$$

Some other works find that total equalization maximize agents effort. According with them we define total equalization as the level of α that guarantees that when individuals incur into the same cost of effort they are equally likely to win the prize.

Definition 2. *Total equalization consist in a leveling policy such that; when* $C_1(e_1) = C_2(e_2)$ *then* $P(\alpha e1 \leq e2) = 1 - P(\alpha e1 \leq e2)$.

Lemma 1. *Total equalization is guaranteed if and only if:* $\alpha = \sqrt{c}$.

Proof. $C_1(e_1) = C_2(e_2)$ implies that $\frac{1}{2}e_1^2 = \frac{1}{2c}e_2^2$. Hence $\sqrt{c} = \frac{e2}{e1}$.

$P(\alpha e1 \leq e2) = P(\alpha e1 \geq e2)$ implies that both probabilities are equal to $\frac{1}{2}$. To guarantee this necessarily $\alpha = \frac{e2}{e1}$.

Using both conditions together immediately arises that $\alpha = \sqrt{c}$.

□

3 Leveling policies with exogenous noise.

First we consider the case of a tournament designer that can apply leveling policies for a given level of noise. This is very similar to the work in Schotter and Weigelt (1992) with the difference that we consider that it is not possible to compensate effort directly since it is not observable. Then the design problem just consists in deciding the optimal level of α.

$$max_\alpha e_1 + e_2$$

The solution to this maximization problem leads to the following proposition which is very different from other literature conclusions.

Proposition 1. *When evaluation level is exogenous it is optimal to apply total equalization only if* $k = \frac{\sqrt{V_1}}{2}$.

Proof. We have four different intervals for the distribution function (F( )): Let us consider the second interval(i.e. $k - \alpha k \leq \alpha e_1 - e_2 \leq 0$) In this case first order conditions are: $e_1 = \frac{V_1}{2k}$ $e_2 = \frac{V_2}{2\alpha k}$

Which implies that total effort decrease with α. Then effort is maximized when α is as small as possible.

Since we are considering the interval $k - \alpha k \leq \alpha e_1 - e_2 \leq 0$, it implies that:

$$\frac{k^2 + \sqrt{k^4 + 2k^2 V_2 + V_1 V_2}}{2k^2 + V_1} \leq \alpha \leq \sqrt{\frac{V_2}{V_1}} = \sqrt{c}$$

Hence when fixing $\alpha = \frac{k^2+\sqrt{k^4+2k^2V_2+V_1V_2}}{2k^2+V_1}$ is possible, it will be better than total equalization. However it is necessary to satisfy the participation constraint (to have an equilibrium in pure strategies). Having this into account leads to the following solution (in this interval):

$$\alpha^*| \leq \sqrt{c} = \begin{cases} \frac{k^2+\sqrt{k^4+2k^2V_2+V_1V_2}}{2k^2+V_1} & \text{if } V_2 < \frac{32k^6+8k^4V_1}{V_1^2} \\ \sqrt{\frac{2V_2}{4k^2+V_1}} & \text{otherwise} \end{cases}$$

Using this solution we can see that $\alpha = \sqrt{c}$ is only possible when $k = \frac{\sqrt{V_1}}{2}$ by substituting k in $\sqrt{\frac{2V_2}{4k^2+V_1}}$.

□

In our framework leveling policies also affects the noise term. In this case total equalization implies high levels of uncertainty that induce lower levels of effort. The density function is flat shaped at the center. For any $\alpha > 1$ zero is located on the right hand side of the beginning of the flat part. Increasing α in order to make $\alpha e_1 = e_2$ increases the variance of the distribution function and reduces the density function values in the flat shaped interval. Hence in case of having $\alpha e_1 = e_2$ we can increase the density function by moving to the value of α that allocates $\alpha e_1 - e_2$ in the left sided part of the flat shaped interval. This is only avoided when the noise level is equal to its lower bound.

4 Endogenous noise without leveling policies.

Now we want to study the case in which contest designer can control the level of noise that exist in the competition. By fixing different requirements the designer is able to obtain more or less information about individuals effort hence its reasonable to assume that he can affect the size of the random component. We assume that he can increase the observability of effort without no cost, which is not realistic but simplifies our analysis. As we will show if we remove this assumption results will still hold. In a symmetric competition and without any cost of implementing a more exhaustive evaluation the optimal level of noise will be zero transforming the tournament in an all-pay auction. On the other hand when there is some asymmetry and it is not possible to apply leveling policies the designer will be interested in fixing a higher level of noise in order to maximize total effort. The prove of the following proposition shows it.

Proposition 2. *When equalization policies are not available the following k's maximizes the effort level in an asymmetric tournament:*

$$k^* = \frac{\sqrt{V_2 - V_1}}{2} \ \textit{when } V_1 \leq \tfrac{1}{2}V_2 \qquad k^* = \frac{V_1}{2} \quad \textit{when } V_1 > \tfrac{1}{2}V_2$$

Proof. Since now $\alpha = 1$ the distribution function of the aggregate noise term follows a triangular distribution centered at zero, with bounds $-2k$ and $2k$.

Contest designer has to decide k in order to maximize everybody's effort, that is:

$$max_k \frac{2kV_1}{4k^2 + V_2 - V_1} + \frac{2kV_2}{4k^2 + V_2 - V_1} \quad s.t. \quad U_1, U_2 \geq 0$$

Using first order condition, we find the interior solution:

$$\frac{2(4k^2 + V_2 - V_1)(V_2 + V_1)}{4k^2 + V_2 - V_1} = 0$$

Isolating k:

$$k^* = \frac{\sqrt{V_2 - V_1}}{2}$$

Concavity of contest designer objective function evaluated at k^* guarantees that this is a local maximum. In order to be an equilibrium we have to check if participation constraint is satisfied. Since $U_1 < U_2$ we need to substitute the equilibrium effort levels and the optimal bounds k^* in player 1 utility function and check $U_1 \geq 0$. That is:

$$\frac{2k^2(4k^2 - V_1)V_1}{(4k^2 + V_2 - V_1)^2} \geq 0$$

Which is satisfied if and only if: $k \geq \frac{\sqrt{V_1}}{2}$. Then the interior solution is only feasible when

$$\frac{\sqrt{V_2 - V_1}}{2} \geq \frac{\sqrt{V_1}}{2}$$

, or equivalently when $V_1 < \frac{1}{2}V_2$. This means that previous expression for k^* is only an equilibrium in pure strategies when there is enough level of asymmetry between players. When this does not happen we need that $k \geq \frac{\sqrt{V_1}}{2} = \bar{k}$. It is easy to show that the derivative of pure strategies equilibrium efforts strictly decreases with k when $V_1 < \frac{1}{2}V_2$ and $k \geq \frac{\sqrt{V_1}}{2}$ [3] so the optimal level of noise distribution bounds that permits an equilibrium in pure strategies when $V_1 > \frac{1}{2}V_2$ is $k^* = \frac{\sqrt{V_1}}{2}$.

Finally comparing total effort in equilibrium we see that interior solution is superior than the corner one:

$$\sum e_i(k^* = \tfrac{\sqrt{V_2 - V_1}}{2}) = \tfrac{V_1\sqrt{V_2 - V_1}}{2V_2 - 2V_1} + \tfrac{V_2\sqrt{V_2 - V_1}}{2V_2 - 2V_1}$$

$$>$$

$$\sum e_i(k^* = \bar{k}) = \tfrac{V_1}{V_2}\sqrt{V_1}$$

Which concludes the proof.

$\square$

For low levels of asymmetry ($V_1 > \frac{V_2}{2}$) it is better to reduce the noise to the minimum in order to maximize total effort. For small levels of asymmetry preemption effect is not severe. In these cases it is better to maintain the lowest level of uncertainty in order to have higher returns to effort. On the other hand when asymmetry increases it accentuates preemption effect and its negative influence on effort. For enough levels of asymmetry it is optimal to fix levels of noise greater than the minimum one. Finally, as we see in the next corollary, once the asymmetry is big enough as to be interested in fixing a level of noise bigger than the minimum one, it is optimal to reduce precision in evaluation as long as asymmetry increases.

Proposition 3. *The optimal level of evaluation decreases with asymmetry.*

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[3] $\frac{\partial U_1}{\partial k} = -\frac{2(4k^2+V_1-V_2)(V_1+V_2)}{(4a^2-V1+V2)^2} \leq 0$, taking $V_1 < \frac{1}{2}V_2$ and $k \geq \frac{\sqrt{V_1}}{2}$ it is immediate to see that both numerator and denominator are positive, which guarantees that the whole expression is negative.

proof For small levels of asymmetry ($V_1 < \frac{V_2}{2}$) it is optimal to fix k as small as possible. However, it is immediate to see that when asymmetry increases and $k^* = \frac{\sqrt{V_2 - V_1}}{2}$, this strictly increases when the difference between V_1 and V_2 becomes greater.

This proposition implies that banning leveling policies can suppose a greater level of randomness in competitions.

5 Endogenous noise and leveling policies.

The most realistic and naturally most interesting case is when contest designer can both modify the level of evaluation of candidates and can also apply leveling policies. If we think in a firm that wants to promote one of his workers it can use both policies at the same time.

Proposition 4. *The effort maximizing policy is* $\alpha = \sqrt{c}$ *and* $k = \frac{\sqrt{V1}}{2}$.

Proof. Let us first check the middle intervals in the aggregate noise distribution function:

In these intervals:$e_1 = \frac{V_1}{2k}$ $e_2 = \frac{V_2}{2\alpha k}$ Hence in order to maximize effort we need to make k and α as small as possible subject to participation and interval constraints.

When $k - \alpha k \geq \alpha e_1 - e2 < 0$, the following constraints has to be satisfied:

$$U_1, U_2 >= 0 \qquad (a)$$

$$\frac{k^2 + \sqrt{k^4 + 2k^2 V_2 + V_1 V_2}}{2k^2 + V_1} \leq \alpha \leq \sqrt{V_2} V_1 \qquad (b)$$

$$k \geq \frac{\sqrt{V_1}}{2} \quad (c)$$

Notice that by lemma 1, $\alpha \leq \sqrt{V_2} V_1$ guarantees that $U_1 \leq U_2$ so when $U_1 \geq 0$ this holds automatically for U_2. $U_1 \geq 0$ implies that:

$$\frac{V_1(-2V_2 + (4k^2 + V_1)\alpha^2)}{8k^2\alpha^2} \geq 0$$

which can be written as:$\alpha \geq \sqrt{2}\sqrt{\frac{V_2}{4k^2 + V_1}} \qquad (a')$

We have two possible lower bounds for α determined by conditions (a') and (b). Depending on which of them is binding we have two candidates to be the solution:

If we take the lower bound for α from (b), that is $\alpha = \frac{k^2+\sqrt{k^4+2k^2V_2+V_1V_2}}{2k^2+V_1}$, we need the following condition to satisfy (a'):

$$V_2 \leq \frac{32k^6+8k^4V1}{V1^2}$$

. Since efforts strictly decrease with k this has to hold with equality, which determines a lower bound for k such that inequality in (c) strictly holds.

When on the other hand when taking the lower bound for α from (a') we need to satisfy $\alpha = \sqrt{2}\sqrt{\frac{V_2}{4k^2+V_1}}$, when this happens satisfying condition (b) only is possible when $V_2 > \frac{32k^6+8k^4V_1}{V_1^2}$, which only determines an upper bound for k, then condition (c) is binding, so $k = \frac{\sqrt{V_1}}{2}$. Using this in the expression for α we find the second candidate to the optimum.

By comparing both candidates we see that total effort is always greater when $k = \frac{\sqrt{V_1}}{2}$ and $\alpha = \sqrt{2}\sqrt{\frac{V_2}{4k^2+V_1}} = \sqrt{\frac{V_2}{V_1}}\sqrt{c}$.

When $0 \geq \alpha e_1 - e2 \geq \alpha k - k$ we have the same conditions (a) and (c) as before but now condition (b) consists in $\alpha \geq \sqrt{\frac{V_2}{V_1}}$. Now condition (a) always holds when (b) is binding and the unique candidate to equilibrium coincides with the one previously found, that is $k = \frac{\sqrt{V_1}}{2}$ and $\alpha = \sqrt{c}$.

Notice that payoffs in the previous equilibria implies full rent-dissipation of candidates, $U_1, U_2 = 0$. When $\alpha = \sqrt{c}$ the tournament becomes symmetric. In this case, preemption effect has been avoided, hence the only effect of introducing some noise is to decrease effort returns. This result coincides with that of an all-pay auction as the one in Hillman and Riley. In this case $P_1(e^*) = P_2(e^*)$

If we are in the first interval $-\alpha k - k \geq \alpha e_1 - e_2 \geq -\alpha k + k$, necessarily $\alpha < \sqrt{c}$. With a lower α the probabilities of winning of individual 1 are lower, so $P_1(\alpha < \sqrt{c}) < \frac{1}{2}$ (and can only become equal to $\frac{1}{2}$ if $k = \infty$, which generates no effort). Since there were full rent dissipation the effort level of individual 1 has to decrease, otherwise he obtains a negative utility. Since $e_2 = \frac{V_2}{\alpha V_1}e_1$, effort of second individual also reduces, so contest designer payoff decreases. Hence $-\alpha k - k \geq \alpha e_1 - e_2 \geq k - \alpha k$ never contains an optimal leveling policy.

Finally, by symmetry of the distribution function, we can argue that if there is no optimum in the first interval it will neither be any in the fourth one $\alpha k - k \geq \alpha e_1 - e_2 \geq \alpha k + k$.

So we conclude that effort maximizing policy consists in: $k = \frac{\sqrt{V_1}}{2}$ and $\alpha = \sqrt{c}$.

□

Finally we can see that when tournament designer can control both the application of leveling policies and precision in evaluation he will be interested in making the competition symmetric (total equalization) and making the competition as less uncertain as possible. That is Affirmative Action works better in order to prevent preemption effect. However as we see in the following corollary it will be only optimal to apply total equalization in the case of being able of fix enough levels of precision in evaluation. Otherwise it is optimal to leveling less than total equalization.

Corollary 1. *When there is a lower bound for k such that* $\frac{32k^6+8k^4V_1}{V_1^2} \geq V2$ *then it is not optimal to apply total equalization.*

Proof. To see this consider the payoffs induced by previous leveling candidates to equilibrium as a function of k.

$e_1 = \frac{V_1}{2k}$ for any α. On the other hand, $e_2 = \frac{V_1}{2\alpha k}$ changes according to the value of α. In order to maximize total effort contest designer should maximize the value of α with respect to k.

Considering both candidates when $\frac{32k^6+8k^4V_1}{V_1^2} \geq V2$ we can see that:

$$\frac{k^2+\sqrt{k^4+2k^2V_2+V_1V_2}}{2k^2+V_1} < \frac{V_2}{V_1}$$

for any $k > 0$.

Hence the level of α is lower when $\alpha = \frac{k^2+\sqrt{k^4+2k^2V_2+V_1V_2}}{2k^2+V_1}$, so when the level of noise is the same it is not optimal to apply total equalization.

□

When for some reason it is not possible to apply enough accuracy in evaluations to reduce the noise term to a sufficiently small level ($\frac{32k^6+8k^4V_1}{V_1^2} \leq V2$ then it is better not to total equalize. This can account for those cases in which the evaluation cost is higher than its benefits at some level above this critical point or when technology does not allow for sufficient precision in evaluation. In those cases we can talk about constrained evaluation. When there is no restriction in accuracy, total equalization allows to apply higher levels of precision without violating participation constraint. When precision in evaluation is determined by some other circumstance (as evaluation cost or physical constraints) then it is optimal to not completely balance the competition.

6 Conclusions and Further Research.

This paper analyze the application of leveling policies (Affirmative Action) with two important differences with respect to previous literature. On one hand it endogenizes the level of noise in the competition. Previous literature on Affirmative Action has considered it exogenous. This permits to analyze the interaction effects of precision in evaluation and leveling policies on the candidates' effort. On the other hand it asses that a leveling policy cannot compensate competitors for their unobservable investments but only for their observable performance.

The application of leveling policies without considering the impossibility of compensating unobservable investments produce the usual result that total equalization is the total effort maximizing policy. Once we change this assumption we obtain that total equalization is never optimal (except for $k = \frac{\sqrt{V_1}}{2}$) when there is an exogenous noise term.

By considering the modification in evaluation precision we can observe that the impossibility of applying leveling policies generates higher levels of randomness in the final allocation. This contradicts the classical critique to Affirmative Action that argues taht this policies reduce meritocray

The application of total equalization maximizes effort when precision in evaluation is unconstrained, that is the principal can reduce it to its lower bound. However when this is not possible the level of equalization is lower than the one that provides an absolutely balanced competition.

Here we have analyzed the competition design when the objective of the principal is to maximize everybody's effort. This fits well with competitions in which the designer can capture all candidates investments. Think for instance in the allocation of a bonus in a firm. However in many cases the principal is only interested in maximizing the winner's investment. This fits well with job allocation where the employer is only interested in maximizing the preparation of the hired employee. In some other works the same equalization policy maximizes both kind of objective function. Our objective is to check if this also happens here.

References

[1] Chan, Jimmy and Eyster Erik. "Does Banning Affirmative Action Lower College Student Quality?". *American Economic Review.*, Vol. 93(3), (June, 2003) ,pp. 858-872.

[2] Che, Yeon-Koo and Gale, Ian. “Difference -Form Contests and the Robustness of All-Pay Auctions.” *Games and Economic Behavior*, Vol.30 (1), (Jan., 2000), pp. 22-43.

[3] Fu, Quiang. A Theory of Affirmative Action in College Admissions, *Economic Inquiry,* Vol. 44, (2006), pp. 420428.

[4] Franke, Jorg.Does Affirmative Action Reduce Effort Incentives? A Contest Game Analysis. UAB and IAE Working Paper Series 711.07. Barcelona. 2007

[5] Hillman, Arye L. and Riley, John G. “Politically Contestable Rents and Transfers,” *UCLA Economics Working Papers* 452, UCLA Department of Economics. 1987.

[6] Krkel, Matthias. “Optimal Risk Taking in an Uneven Tournament Game with Risk Averse Players,” *Journal of Mathematical Economics*,Vol. 44(11),(Dec., 2008) pp. 1219-1231.

[7] Krkel, Matthias and Sliwka, Dirk. “Risk Taking in Asymmetric Tournaments,” *German Economic Review,*, Vol. 5 (1), (Feb, 2004), pp.103-116.

[8] Lazear, Edward P. and Rosen, Sherwin. “Rank-Order Tournaments as Optimum Labor Contracts,” *The Journal of Political Economy,* Vol. 89 (5) (Oct., 1981), pp. 841-864.

[9] Nti, Kofi O. Rent-Seeking with Asymmetric Valuations, *Public Choice*,Vol. 98(3-4),(March.,1999), pp. 415430.

[10] Nti, Kofi O. Maximum Efforts in Contests with Asymmetric Valuations, *European Journal of Political Economy.*, Vol. 20(4), (Nov., 2004), pp. 10591066.

[11] Schotter, Andrew and Weigelt, Keith. “Asymmetric Tournaments, Equal Opportunity Laws And Affirmative Action: Some Experimental Result” *C.V. Starr Center for Applied Economics, New York University. Working Papers.* 90-14. (1990)

[12] Stein, William E. Asymmetric rent-seeking with more than two contestants, *Public Choice*, Vol. 113(3-4),(Dec., 2002), pp. 325336.

[13] Tullock, Gordon. “Efficient rent seeking.” in (J. M. Buchanan, R. D. Tollison, and G. Tullock, eds.), Toward a Theory of Rent Seeking Society, College Station: Texas A&M University Press, pp. 97-112.