Abstract

The reaction of hours worked to technology shocks has represented one of the key controversies between RBC and NK explanations of the business cycle. It sparked a large empirical literature with contrasting results. Here we introduce imperfect factor substitutability in the supply side of both a flexi-price and a sticky-price DSGE model. In both models, we show that when the production function is not Cobb-Douglas – as emphasized by recent evidence – technology shocks can have either positive or negative impacts on hours worked. The effect depends on the value of the elasticity of substitution between capital and labor and whether technology shocks are capital- or labor-augmenting. Price rigidities do matter for the sign of the reaction of hours, but not necessarily in the standard way of generating negative responses. The impact of technology shocks on hours worked, hence, can hardly be taken as evidence in support of a particular class of business cycle model on its own.

**JEL Classification:** E32, E23, E25.

**Keywords:** Technology Shocks, Hours Worked, Capital-Labor Substitution, Factor-Augmenting Technical Change, DSGE.
1 Introduction

The reaction of hours worked to a technology shock has been the focus of important controversies in macroeconomics in the last decade. According to the benchmark real business cycle (RBC) model, hours should raise after a productivity shock. However, in an important paper, Gali (1999), using a structural VAR (SVAR) with long-run restrictions, found that this impact is negative. This evidence has long since been interpreted as favoring the New-Keynesian sticky-price model (NKM) of fluctuations. This is because NKMs, by introducing nominal frictions, can generate negative correlations between technology and factor inputs in the short run.

The results of Gali (1999) spurred a large literature on the effects of technology shocks that was mostly supportive of a negative correlation between technology and hours-worked [see, for instance, Francis et al. (2003) and Francis and Ramey (2005)]. Christiano et al. (2003) challenged these results arguing that these are driven by the way researchers treat hours worked, whether in levels or first differences. Econometric identification, hence, took center stage in the debate. Fernald (2007) emphasizes the importance of low-frequency trend-breaks in productivity and finds a negative impact of technology on hours. Dedola and Neri (2007) use sign restrictions for VAR identification and found that hours worked are likely to increase. Uhlig (2004), using a medium-run identification scheme, also finds support for a mildly positive impact. Pesavento and Rossi (2005) use an agnostic method that does not require choosing between a specification in levels or in first differences. They find that hours worked fall after a technology shock, but the effect is short lived. Chari et al. (2008), however, argue that SVAR models are not capable of distinguishing between different explanations of the business cycle (i.e. RBC vs sticky prices).1 They argue that, if non-technology shocks account for an important part of business cycle fluctuations, SVAR models can incorrectly conclude in favor of a NKM when the data has been generated using an RBC model. Basu et al. (2006) take a different approach and construct a measure of aggregate technology change from sectoral-level data. This method is free from the shortcomings of the SVAR identification restrictions. They find that technological change and factor inputs are negatively correlated. They conclude that technology

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1Indeed, the role of technology shocks in explaining the variation of hours worked is also at the core of many methodological debates in macroeconomics.
improvements are contractionary on impact, which would constitute an important stylized fact of business cycle fluctuations.

In this paper we take a different route. While remaining agnostic about empirical identification methods, we present a theoretical argument that needs to be considered when interpreting the results of the empirical literature. Following Francis and Ramey (2005), we argue that relaxing some standard assumptions in business cycle models, we can obtain dynamic responses of hours worked that are compatible with both positive and negative impact effects of technology shocks. Basu et al. (2006) also argue that short-run correlations between technology and hours need not be interpreted as a “test” of a particular business cycle model, since there can be more than one alternative hypothesis. Our main concern here is the introduction of the issue of factor substitution in business cycle models. Standard models assume that the production function governing output is Cobb-Douglas, implying net substitutability between capital and labor. However, evidence for the US economy stands in stark contrast with this assumption. Recent evidence in Klump et al. (2007), for instance, finds support for an elasticity of substitution between 0.5 and 0.7 for the US. Antrs (2004) also rejects the Cobb-Douglas specification (i.e. unitary substitution elasticity) although his estimates are somewhat higher between 0.65-0.9. Chirinko et al. (2004), using a panel of U.S. firms, find support for an elasticity of 0.4. Similar low values are also found by León-Ledesma et al. (2010a) using quarterly time series aggregate data for the U.S. economy.\textsuperscript{2} León-Ledesma et al. (2010b) show that, contrary to previous belief, the joint identification of biased technical progress and the elasticity of substitution is achievable within a normalized supply-side system. They review previous empirical studies showing that, with some exceptions, estimates of the elasticity of substitution below unity for the US economy are most frequent. Despite most evidence rejecting unitary factor substitution elasticities, business cycle models, perhaps because of tractability, continue to focus on simple Cobb-Douglas production functions. Theoretically, Jones (2003) has argued that the evolution of capital shares in developed countries is not consistent with a Cobb-Douglas. Capital income shares show large variations and even trends for European countries.

\textsuperscript{2}See Chirinko (2008) for a review of empirical evidence on the elasticity of substitution. Although estimates differ depending on the sample period, estimation technique, level of aggregation and type of data, most evidence favors low elasticities.
He then proposes a nested production function. This function will exhibit a less than unitary substitution elasticity in the short-run (the relevant time frame for business cycle models) and be Cobb-Douglas in the long run.

The effect of technical change on employment is, in fact, a long-standing debate in economics. The traditional Ricardian ‘machinery effect’ defended by Hicks (1969) supported the idea that the introduction of machinery reduces employment in the short run, but increases it in the long run. The kind of mechanism behind this effect, however, did not rest on the introduction of nominal rigidities that characterizes modern macroeconomics, but on the existence of complementarities in production between factors. These mechanisms, hence, can play an important role in understanding the short run impact of technology shocks.

The introduction of imperfect substitutability in a business cycle model can have several potential impacts. First, the reaction of labor (and capital) to a technology shock may be dampened (or exacerbated) if factors of production are gross complements (or gross substitutes). Second, it allows us to introduce factor-biased technical change, as opposed to the standard Hicks neutral specification. This permits the analysis of the impact of technology shocks on hours worked considering their factor bias and for alternative constellations of the elasticity of substitution.

We hence introduce a flexible specification of the production function, Constant Elasticity of Substitution (CES), with biased (non-neutral) technical change in both a standard RBC model, and a sticky price NK model. We then compare the results of different technology shocks on hours worked for a standard Cobb-Douglas and a CES with different values of the elasticity of substitution. As a technical point, we introduce the concept of production function “normalization” in calibrated DSGE models, which is important when the production function is not Cobb-Douglas. Our results show that, both RBC and NK models can yield

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3This is in contrast to the Marxian theories that supported the existence of permanent effects on the level of employment. See Beach (1971) for a discussion.

4Greenwood et al. (1997) analyze the effect of investment-specific technical change. This is introduced by Fisher (2006) in a short-run model of fluctuations for business cycle accounting but he still assumes net factor substitutability. For a robust analysis of empirical effects of neutral and investment-specific shocks, see also Canova et al. (2009). Ríos Rull and Santeulália-Llopis (2007) also analyze the impact of redistributive shocks affecting the labor share but also consider the case of a Cobb-Douglas production function. These works, nevertheless, emphasize the potential role of biased technical change for economic fluctuations and income distribution.
either positive or negative responses to technology shocks in hours worked if the production function is CES. The response will depend on whether the shock is labor- or capital-augmenting, and on the value of the elasticity of substitution. In the RBC model, capital-augmenting shocks always yield positive hours responses, whilst labor-augmenting shocks can lead to either response sign. We derive an approximate rule for this response, which will depend on the relative magnitudes of the elasticity of substitution between capital and labor and the elasticity of output with respect to capital. The rule is still relevant in the NKM. However, in this model labor-augmenting shocks are most likely to yield negative responses (although positive responses are not ruled out). Capital-augmenting shocks can lead to positive or negative responses. In the NKM, however, these responses depend heavily on the monetary policy rule. We explain the intuition behind these results and also carry out a comprehensive sensitivity analysis.

We do not interpret our results in terms of favoring a particular model of business cycles. Our results indicate the possibility of alternative interpretations of empirical evidence on technology and hours. This interpretation is based on previous empirical evidence and theoretical arguments and is hence reasonable. It is possible, of course, that new results uncover regularities that support or reject particular theories of fluctuations. Our results simply call into question the sole interpretation of technology-hours correlations to distinguish between business cycle models.

The paper is organized as follows. Next section presents some basic concepts about the CES production function and biased technical change. Section 3 discusses the concept of normalization of CES functions. Sections 4 and 5 present the RBC and NKM respectively. Section 6 presents the equilibrium conditions and parameter calibration. Section 7 presents and discusses the results, and Section 8 concludes.

Of course, negative responses of hours in a standard RBC could also be generated by altering the parameters of the utility function relating labor dis-utility. However, these would require empirically implausible parameter values.
2 Factor Substitutability and the CES Function

The degree of factor substitution and the related concept of biased technical change feature an important role in many areas of economics such as income distribution, growth, dynamic stability, fiscal policy and business cycles (see La Grandville (2008)). There has also been an increasing interest in the relationship between the elasticity of substitution and growth arising from the concept of normalized Constant Elasticity of Substitution (CES) production functions (La Grandville (1989), Klump and de La Grandville (2000), La Grandville and Solow (2008)).

The nature of technical change, on the other hand, is important to explain labor-market inequality and skills premia (Acemoglu (2002b)); the evolution of factor income shares (Kennedy (1964), Acemoglu (2003)) etc. If the production function driving output was Cobb-Douglas (i.e., unitary substitution), then technological progress degenerates to the Hicks-Neutral representation, and factor shares in total value added are constant by definition.\(^6\) However, modern business cycle models have generally ignored these important supply-side considerations when analyzing dynamic responses to supply and demand shocks.

The CES production function is represented as follows:

\[
F(\Gamma^k_t k_t, \Gamma^h_t h_t) = u \left[ \alpha (\Gamma^k_t k_t)^{\frac{1}{1-\sigma}} + (1 - \alpha) (\Gamma^h_t h_t)^{\frac{1}{1-\sigma}} \right]^\frac{1}{\frac{1}{1-\sigma}} \tag{1}
\]

where distribution parameter \(\alpha \in (0, 1)\) reflects capital intensity in production; \(u\) is an neutral efficiency parameter and the elasticity of substitution \(\sigma\) between capital \(k_t\) and labor \(h_t\) (hours worked) is given by the percentage change in factor proportions due to a change in the marginal products (or factor price ratio):

\[
\sigma \in (0, \infty) = \frac{d \log (k/h)}{d \log (F_h/F_k)} \tag{2}
\]

Equation (1) nests Cobb-Douglas when \(\sigma = 1\); the Leontief function (i.e., fixed factor proportions) when \(\sigma = 0\); and a linear production function (i.e., perfect factor substitutes) when \(\sigma \to \infty\). Finally, when \(\sigma < 1\), factors are gross complements in production and gross substitutes when \(\sigma > 1\) (Acemoglu (2002a)).

The terms \(\Gamma^k_t\) and \(\Gamma^h_t\) capture capital and labor-augmenting efficiency. To

\(^6\) An observation that is violated if we look at the cyclical movements of factor shares in the US and even long swings in many European countries.
circumvent problems related to Diamond et al. (1978)’s impossibility theorem, researchers usually assume specific functional forms for these functions, e.g., $\Gamma_k^t = \Gamma_0^k e^{z_{kt}}$ and $\Gamma_h^t = \Gamma_0^h e^{z_{ht}}$ where $z_{it}$ can be a stochastic or deterministic technical progress function for factor $i$. The case where $z_{kt} = z_{ht} > 0$ denotes Hicks-Neutral technology; $z_{kt} > 0$, $z_{ht} = 0$ yields Solow-Neutrality; $z_{kt} = 0$, $z_{ht} > 0$ represents Harrod-Neutrality; and $z_{kt} > 0$, $z_{ht} > 0$ indicates general factor-augmentation.

La Grandville (2008) stresses that the prime motive for introducing the concept of factor substitution was to account for the evolution of income distribution. This can be easily seen by assuming that factors are paid their marginal products. Then, assuming competitive markets and profit maximization, relative factor income shares ($sh^{k/h}$) and relative marginal products are (dropping time subscripts) are:

$$\frac{r_k}{w_h} = sh^{k/h} = \frac{\alpha}{1 - \alpha} \left( \frac{\Gamma_k^k}{\Gamma_h^h} \right)^{\frac{\sigma - 1}{\sigma}}$$

$$\frac{F_k}{F_h} = \frac{\alpha}{1 - \alpha} \left[ \left( \frac{k}{h} \right)^{-\frac{1}{\sigma}} \left( \frac{\Gamma_k^k}{\Gamma_h^h} \right)^{\frac{\sigma - 1}{\sigma}} \right]$$

where $r_t$ and $w_t$ represent the real interest rate and the real wage, respectively.

Thus, capital deepening, ceteris paribus, assuming gross complements (gross substitutes) reduces (increases) capital’s income share. Likewise, a relative increase in, say, capital-augmentation assuming gross complements (gross substitutes) decreases (increases) its relative marginal product and factor share. Accordingly, it is only in the gross-substitutes case that, for instance, a capital augmenting change in technology is capital-biased (i.e., raises its relative marginal product and share for given factor proportions). Naturally, the relations between the substitution elasticity, technical bias and factor shares evaporates under Cobb-Douglas.

This illustrates the potential importance of factor substitutability and the bias in technical change for the transmission of technology shocks. For instance, the impact of technology on factor payments depends on $\sigma$ and factor augmentation, hence potentially influencing the dynamic response of wages (and hence hours) to technology shocks. For instance, in a standard model without frictions, a capital-augmenting technology shock would contribute to an increase the relative wage if $\sigma < 1$ (ceteris paribus). This would trigger a different impact on capital dynamics.
and hours worked than in the standard CD case. The dynamics in a model with sticky prices, however, will also depend on the impact on the marginal cost and the reaction of the monetary policy authority. The picture, of course, becomes more complex with the introduction of other frictions such as capital adjustment costs and sticky wages. What is important to note, however, is that the responses to technology shocks will crucially depend on the value of \( \sigma \) and the direction of technical change (or rather, the interaction between them). Which kinds of technology shocks dominate at different points in time can also alter the dynamic responses, that may become time-varying. It is then relevant to introduce these aspects into a dynamic general equilibrium model. First we need to explain the important concept of normalization.

### 3 Normalization of the CES production function

In this section we discuss the concept of normalization of CES production functions introduced by La Grandville (1989) and further analyzed in Klump and de La Grandville (2000), Klump and Preissler (2000), and León-Ledesma et al. (2010b). Normalization, i.e. expressing the production function in terms of index numbers, is important so that the parameters of the CES are deep and not a mixture of production parameters. This has important consequences for growth theory (Klump and de La Grandville (2000)) and the estimation of production functions (León-Ledesma et al. (2010b)). In our context, it is also necessary so that we choose a normalization point corresponding to a steady state where factor shares directly map to certain CES parameters. Using non-normalized production functions not only obscures calibration results, but could affect dynamic responses to shocks as the elasticity of output with respect to production inputs can change at different steady states.\(^7\) We will also compare dynamic responses for different values of the elasticity of substitution. A meaningful and consistent comparison requires analyzing the models at the same normalization point. This is not necessary when working with Cobb-Douglas functions, since factor shares do not change, and hence it is not common practice in calibration of business cycle models.

We first present some general features necessary to understand normalization.

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\(^7\)Klump and Saam (2008) discuss aspects of normalization for calibration purposes.
We start with the definition of the elasticity of substitution in a linear homogenous production function $y_t = F(\Gamma^k_t k_t, \Gamma^h_t h_t) = \Gamma^h_t h_t f(\kappa_t)$ where $\kappa_t = (\Gamma^k_t k_t) / (\Gamma^h_t h_t)$ is the capital-labor ratio in efficiency units. Likewise $\varphi_t = y_t / (\Gamma^h_t h_t)$ represents per-capita production in efficiency units. The elasticity of substitution can be expressed as (Arrow et al. (1961):

$$\sigma = \frac{f(\kappa)}{\kappa f'(\kappa)} \left[ f'(\kappa) - \kappa f''(\kappa) \right].$$

This definition can then be viewed as a second-order differential equation in $\kappa$ having the following general CES production function as its solution:

$$\varphi_t = a \left[ \kappa_t^{\frac{r}{\sigma}} + b \right]^{\frac{1}{\sigma}} \Rightarrow y_t = a \left[ (\Gamma^k_t k_t)^{\frac{r}{\sigma}} + b (\Gamma^h_t h_t)^{\frac{r}{\sigma}} \right]^{\frac{1}{\sigma}}$$

where $a$ and $b$ are two arbitrary constants of integration with the following correspondence with the non-normalized form: $C = a (1 + b)^{\frac{1}{\sigma-1}}$ and $\alpha = 1 / (1 + b)$.

A meaningful identification of these two constants is given by the fact that $\sigma$ is a point elasticity relying on three baseline values: a given capital intensity $\kappa_0 = \Gamma^k_0 k_0 / (\Gamma^h_0 h_0)$, a given marginal rate of substitution $[F_k / F_h]_0 = w_0 / r_0$ and a given level of per-capita production $\varphi_0 = y_0 / (\Gamma^h_0 h_0)$. For simplicity, and without loss of generality, we scale the components of technical progress such that $\Gamma^k_0 = \Gamma^h_0 = 1$. Thus,

$$y_t = C \left[ \alpha (\Gamma^k_t k_t)^{\frac{r}{\sigma}} \left( 1 - \alpha \right) (\Gamma^h_t h_t)^{\frac{r}{\sigma}} \right]^{\frac{1}{\sigma-1}} \Rightarrow$$

$$y_t = y_0 \left[ \alpha_0 \left( \frac{\Gamma^k_t k_t}{k_0} \right)^{\frac{r}{\sigma}} \left( 1 - \alpha_0 \right) \left( \frac{\Gamma^h_t h_t}{h_0} \right)^{\frac{r}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

where $\alpha_0 = r_0 k_0 / (r_0 k_0 + w_0 h_0)$ is the capital income share evaluated at the point of normalization.

The problem with the parameters of non-normalized functions is that they are not “deep”, i.e. they depend on the point of normalization and also $\sigma$. Looking at equation (7) we can see that:
C (σ, ⋄) = y_0 \left[ \frac{r_0k_0^{1/\sigma} + w_0h_0^{1/\sigma}}{r_0k_0 + w_0h_0} \right]^\sigma \quad (8)

α (σ, ⋄) = \frac{r_0k_0^{1/\sigma}}{r_0k_0^{1/\sigma} + w_0h_0^{1/\sigma}}. \quad (9)

Hence, maintaining C and α as constants, each non-normalized function (1), corresponding to different values of σ, goes through a different point of normalization belonging to different families. In the non-normalized formulation the parameters C and α have no clear theoretical or empirical meaning. Instead, they are composite parameters conditional on, besides the selected fixed points, the elasticity of substitution. Hence, varying σ, whilst keeping C and α constant, is inconsistent for comparative-static purposes. Each of the resulting CES functions goes through different fixed points and we can say that each resulting CES function belongs to different families.

Since, in a non-normalized case, parameter α depends on the point of normalization and σ, it is obvious that the dynamic responses to shocks can change as we change σ, since the elasticity of output w.r.t. capital and labor will change. In our dynamic general equilibrium setting, we are interested on the dynamic responses of variables in a stationary model. Hence, we need to ensure that factor shares in steady state (the initial and end point of our stochastic simulations) are constant and equal to α and 1 − α. Also, output, capital, labor, consumption and factor payments are common at this point for different σ’s. We hence choose to make the steady state our normalization point.

A logical way is then to choose a steady state and then calibrate the model using this normalization point. We can, for instance, chose y^* and h^* and set them arbitrarily to 1. In a simple RBC model, for instance, since the interest rate is pinned down by preferences and capital depreciation, we can then choose the steady state capital stock as \( k^* = \alpha_0 / r \), where \( \alpha_0 \) is our chosen steady state capital income share and r is the steady state interest rate. Since \( y = rk + wh \) this automatically implies that \( w^* = 1 - \alpha_0 \) (as h = 1). ***In order to ensure that the model yields this result, we modify the scaling term of leisure in the utility function so that \( w^* \) is attained***. This simply ensures that the model is consistent, so that factor

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shares sum up to one (as does the CES production function) and consumption plus investment also equal output. Hence, we calibrate the model’s parameters and enter all the supply side (CES and first order conditions for $k$ and $h$) in normalized form. This implies also that changing $\sigma$ does not change our steady state or factor shares, and hence IRFs are directly comparable. In the explanations that follow, we will use any variable with subscript 0 as the normalization point of the variable which, in our case, actually corresponds to the steady state.

4 RBC model

In this section we present the RBC model where we introduce a CES production technology in the supply side. Given that the model is otherwise standard, we describe it here only briefly. The equilibrium conditions are given by:

\[ c_t^{-\sigma_c} = \beta c_{t+1}^{-\sigma_c} [1 + r_{t+1} - \delta], \]  \hspace{1cm} (10)

\[ c_t + k_t - (1 - \delta)k_{t-1} \leq y_t, \]  \hspace{1cm} (11)

\[ w_t = vh_t^\sigma c_t^{\sigma_c}, \]  \hspace{1cm} (12)

\[ y_t = CES_t = y_0 e^{z_N} \left[ \alpha_0 \left( \frac{e^{z_k} k_{t-1}}{k_0} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_0) \left( \frac{e^{z_h} h_t}{h_0} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \]  \hspace{1cm} (13)

\[ w_t = CES_{ht} = (1 - \alpha_0) \left( \frac{y_0}{k_0} e^{z_h} \right)^{(\sigma-1)/\sigma} (y_t/h_t)^{1/\sigma} (e^{z_N})^{(\sigma-1)/\sigma}, \]  \hspace{1cm} (14)

\[ r_t = CES_{kt} = \alpha_0 \left( \frac{y_0}{k_0} e^{z_k} \right)^{(\sigma-1)/\sigma} (y_t/k_t)^{1/\sigma} (e^{z_N})^{(\sigma-1)/\sigma}, \]  \hspace{1cm} (15)

\[ z_i^t = \rho^i z_{i-1}^t + \varepsilon_i^t \quad \{i = k, h, N\}, \]  \hspace{1cm} (16)

\( ^8 \text{Details on the normalization for both the RBC and NKM are available together with the necessary Matlab codes. To save space, we do not present here the whole derivation of normalization, but in all cases the results are internally and accounting consistent.} \)
where \( c_t \) is consumption and \( z^i_t \) are technology shocks for \( i = k, h, N \) meaning capital-augmenting, labor-augmenting, and Hicks-neutral shocks respectively. Equation (10) and (12) are the usual Euler and labor supply equations associated with the household problem using a CRRA utility function of the form:

\[
U_t = \left\{ \frac{c_t^{1-\sigma_c}}{1-\sigma_c} - v \frac{h_t^{1+\gamma}}{1+\gamma} \right\}.
\]

Equation (11) is the resource constraint of this economy while equations (13), (14) and (15) are the CES production function and its derivatives with respect to labor and capital in their normalized form. Given the way we model the supply side of the economy we have three source of technology shocks: Hicks-neutral shock, Solow-neutral shock (capital-augmenting) and Harrod-neutral shock (labor-augmenting). The three shocks follow an AR(1) process.

Clearly the only difference between this model and the textbook RBC model is in the supply side where we use a more general (normalized) CES production function and, consequently, also the first order conditions associated with the representative-firm problem are affected. Note that the model nests the Cobb-Douglas as a special case when \( \sigma = 1 \).

5 The New Keynesian Model

The model economy we describe below follows a standard DSGE model with sticky prices and capital adjustment costs.\(^9\) The model described in this section is a simplified version of Schmitt-Grohe and Uribe (2007) in which we do not consider the role of government, money, and cash in advance constraints to firms. Again, the main difference arises in the supply side where we use the more general (normalized) CES production function for the monopolistically competitive firm. We chose a model without other sorts of rigidities, sectors or distortions because we want to keep the structure as simple as possible in order to make the role of factor substitution and factor-biased technology shocks more transparent. As in any DSGE model, conclusions could potentially change as we change the number of distortions in the model, but these would not necessarily invalidate the relevance of our

\(^9\)Adjustment costs to capital turn out to be essential to obtain a negative response of hours in the baseline model with a Cobb-Douglas.
core argument. Since we want to show the potential effect of factor substitution, it is sufficient to settle for a relatively simple model.

The model essentially augments a standard neoclassical growth framework with nominal rigidities à la Calvo (1983) so that only some monopolistically competitive firms are allowed to reset optimally their price each period. The model economy is populated by two types of agents (households and firms) and by a monetary authority. The only sources of shocks in the economy are given by hicks-neutral, capital- and labor-augmenting technology shocks, which will drive our main results, and a monetary policy shock. We describe here the problem faced by households, firms, and the monetary authority. The complete set of equilibrium conditions will be presented in the Appendix, and an in-depth discussion can be found in Schmitt-Grohe and Uribe (2007).

5.1 Households

As in the RBC model, the representative household maximizes the following stream of utility

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \left\{ \frac{c_{t+l}^{1-\sigma_c}}{1-\sigma_c} - \frac{h_{t+l}^{1+\gamma}}{1+\gamma} \right\}, \quad (17)$$

where the consumption good is assumed to be a composite good produced with a continuum of differentiated goods, \(c_{i,t}, i \in [0, 1]\), via the usual aggregator function:

$$c_t = \left[ \int_0^1 c_{i,t} \frac{1}{2} dt \right]^{\frac{1}{1-\eta}}, \quad (18)$$

where \(\eta\) represents the intratemporal elasticity of substitution across different varieties of consumption goods. To find total consumption demand for each variety \(i\), one can solve the standard problem of minimizing total cost subject to (18), which yields the downward sloped demand function:

\(10^9\)For simplicity we do not introduce the government to abstract from any other potential sources of shocks.

\(11^9\)At this stage of the work we only focus on technology shocks, the latter will become relevant once we start considering the effect of factor substitutability on monetary policy.
\[ c_t = \left( \frac{P_{it}}{P_t} \right)^{-\eta} c_t, \]  \hspace{1cm} (19)  

where \( P_t \) is the nominal price index given by:

\[ P_t = \left[ \int_0^1 P_{it}^{-\eta} dP_{it} \right]^{\frac{1}{1-\eta}}, \]  \hspace{1cm} (20)  

The household faces a budget constraint each period given by:

\[ E_t d_{t, t+1} + \frac{x_{t+1}}{P_t} c_t + i_t = \frac{x_t}{P_t} + w_t h_t + r^k_t k_t + \Pi_t, \]  \hspace{1cm} (21)  

where \( d_{t, s} \) is a stochastic discount factor, such that \( E_t d_{t, s} x_s \) is the nominal value in period \( t \) of a random nominal payment \( x_s \) in period \( s \geq t \). The variable \( w \) stands for the real wage, \( r^k \) for the real rental rate of capital, \( P_t \) the nominal price level and \( \Pi \) denotes profits received from the ownership of firms.

The resource constraint is

\[ c_t + i_t \leq y_t, \]  \hspace{1cm} (22)  

where \( i_t \) denotes investment, and the capital accumulation equation

\[ k_t = (1 - \delta) k_{t-1} + i_t \Psi \left( \frac{i_t}{i_{t-1}} \right), \]  \hspace{1cm} (23)  

where changes in the capital stock are assumed to be subject to a convex adjustment cost, \( \Psi \left( \frac{i_t}{i_{t-1}} \right) = 1 - \frac{\psi}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \), with \( \psi \) being a positive constant.

We assume the investment good \( i_t \) to be a composite made of the aggregator function (18). Hence, investment demand for each variety has the same form as the consumption function: \( i_{it} = (P_{it}/P_t)^{-\eta} i_t \). The maximization problem provides the equilibrium conditions which are listed in the Appendix (9).

5.2 Firms

We assume that single firms operating in a monopolistically competitive environment produce one good variety \( i \). It does so by using capital and labor following a production technology:
\[ e^{z_N^i} F(e^{z_k^i} k_{it}, e^{z_h^i} h_{it}) - \chi \]

where the \( F \) is the same normalized CES production function presented in previous section and \( \chi \) represents fixed costs in production. Given the consumption and investment demand functions, aggregate demand for good \( i \), \( y_t = c_t + i_t \), will then be given by:

\[ y_{it} = \frac{P_{it}^{\eta}}{P_t} y_t \]

Real profits for firm \( i \) at time \( t \) expressed in terms of the composite good are given by:

\[ \Pi_{it} = \frac{P_{it}}{P_t} y_{it} - r_t^k k_{it} - w_t h_{it}, \]

assuming, as usual in this literature, that firms rent capital services from a centralized market and that capital input can be readily reallocated across industries.

Assuming that the firm satisfies demand at the posted price, we then have the constraint that \( e^{z_N^i} F(e^{z_k^i} k_{it}, e^{z_h^i} h_{it}) - \chi \geq y_{it} \). Profit maximization subject to this constraint leads to the following first order conditions with respect to capital and labor:

\[ mc_{it} e^{z_N^i} F_h(e^{z_k^i} k_{it}, e^{z_h^i} h_{it}) = w_t \]

(25)

\[ mc_{it} e^{z_N^i} F_k(e^{z_k^i} k_{it}, e^{z_h^i} h_{it}) = r_t^k \]

(26)

where \( mc_{it} \) stands for marginal costs, associated with the budget constraint of the firm.

Following Calvo (1983) we assume price stickiness. Each period a random fraction \( \theta \) of firms is not allowed to change the nominal price of the good it produces. The remaining \( 1 - \theta \) fraction of firms choose price optimally by maximizing the expected discounted value of profits:
\[ E_t \sum_{s=t}^{\infty} d_{is} \theta^{s-t} \left[ \left( \frac{\tilde{P}_d}{\tilde{P}_s} \right)^{1-\eta} y_s - r^k s k_{is} - w_s h_{is} \right] + m c_{is} \left( e^{z_k} f(e^{z_k} k_{is}, e^{z_h} h_{is}) - \chi - \frac{\tilde{P}_d^{-\eta}}{\tilde{P}_s} y_s \right) \]  

and we denote \( \tilde{P}_{it} \) the chosen price.

The associated first order condition with respect to \( \tilde{P}_{it} \) is

\[ E_t \sum_{s=t}^{\infty} d_{is} \theta^{s-t} \left[ \left( \frac{\tilde{P}_d}{\tilde{P}_s} \right)^{1-\eta} y_s - r^k s k_{is} - w_s h_{is} \right] + m c_{is} \left( e^{z_k} f(e^{z_k} k_{is}, e^{z_h} h_{is}) - \chi - \frac{\tilde{P}_d^{-\eta}}{\tilde{P}_s} y_s \right) \]  

We do not assume price indexation to keep the analysis as simple as possible.

5.3 Monetary authority

We assume that the monetary authority follows a simple Taylor rule. We denote \( R_t \) the nominal interest rate and \( \pi_t = P_t / P_{t-1} \) (inflation rate if in logs). Calling \( R, \pi \) and \( y \) without the time subscript the nominal interest rate, the inflation rate and output in steady state, the monetary authority exogenously sets the nominal rate of interest following:

\[ \log(\frac{R_t}{R}) = \alpha_r \log(\frac{R_{t-1}}{R}) + \alpha_\pi \log(\frac{\pi_t}{\pi}) + \alpha_y \log(\frac{y_t}{y}) + \epsilon_t. \]  

We do not assume price indexation to keep the analysis as simple as possible.

6 Equilibrium and Calibration

In the Appendix we list the non-linear equilibrium conditions of the sticky-price version of the model. Here we discuss briefly some aggregation issues as well as some of those equilibrium conditions. By looking at expression (28) we can see that all firms that are able to change their price in a given period will chose the same price so we can drop the subscript \( i \) from the equilibrium conditions. By taking
into account relative price dispersion across varieties\textsuperscript{12}, the resource constraint in this model is given by the following three expressions:

\begin{equation}
    y_t = \frac{1}{s_t} [e^{\varepsilon_N} F(e^{\varepsilon_N} k_{i,t}, e^{\varepsilon_N} h_{i,t}) - \chi]
\end{equation}

\begin{equation}
    y_t = c_t + i_t
\end{equation}

\begin{equation}
    s_t = (1 - \theta) \bar{p}_{t-1}^{-\eta} + \theta \pi^n s_{t-1}
\end{equation}

where \( s \) is a state variable that measures the resource costs induced by the inefficient price dispersion present in the Calvo problem in equilibrium\textsuperscript{13}. Both the RBC and NK models are calibrated with the same parameter values (when shared) and around the same normalization point. We choose the time unit to be a quarter by setting the discount factor \( \beta = 0.9902 \) which gives a nominal rate of interest around 4% per year. Utility function parameters are chosen in order to have an intertemporal elasticity of consumption \( (\sigma_c) \) equal to 1 and \( \gamma = 1 \),\textsuperscript{14} the parameter \( v \) is varied in order to ensure a steady state at the normalization point and its set equal to 0.8379. Capital share \( (\alpha_0) \) in the production function is chosen to be equal to 0.4 and the adjustment cost to investment parameter \( (\psi) \) is set equal to 2.48 as in Christiano et al. (2005). The price elasticity of demand \( (\eta) \) is calibrated in order to have a steady-state price mark-up of 25 percent over marginal costs as in Basu and Fernald (1997). The depreciation rate of capital is assumed to be 10 percent per year. The value of the Calvo parameter \( (\theta) \) is set to be equal to 0.8 which implies that on average firms can change prices every 5 quarters. The value of the elasticity of substitution between capital and labor,
which will be the key parameter for our results, will be varied for values equal to 1 (i.e. Cobb Douglas Case) and above and below 1 to consider cases where capital and labor are gross complements and gross substitutes. The autoregressive parameter of the three shocks is set to 0.8556. For the Taylor-rule we assume that the Monetary authority responds to inflation deviations from the steady state with a coefficient of $\alpha_\pi = 2$, to output gap with a coefficient of $\alpha_y = 0.2$ and with a smoothing parameter equal to $\alpha_r = 0.8$.\textsuperscript{15}

7 Simulations and Results

The equilibrium conditions listed in Sections 4 and 5 and in the Appendix were solved and simulated using second-order approximation methods around the non-stochastic steady state of the economy.\textsuperscript{16}

We present the workings of the model by analyzing the dynamic response of the endogenous variables to a one percent shock in $\varepsilon^n_t$ and $\varepsilon^n_k$. We also computed dynamic responses for $\varepsilon^n_N$, but do not report them here for reasons of space.\textsuperscript{17} We do this by computing a linear approximation, and present impulse-response figures expressed in percentage deviations of the endogenous variables from their steady state value.

Figures (1,2) show the impact of a capital augmenting and labor augmenting shock in the RBC model. Figures (3,4) show the impact of a capital augmenting and labor augmenting shock in the NKM model.

Focusing only on the reaction of hours in the RBC model, we can observe that the response in all cases for a capital-augmenting shock are positive and similar to the CD case. This is regardless of the value of $\sigma$. Of course, $\sigma$ affects the reaction of output, consumption and factor payments.\textsuperscript{18} It also affects the movement of factor

\textsuperscript{15}We also vary policy parameters in our sensitivity analysis and consider the case of a Ramsey policy, which we will make available in future versions of the paper.

\textsuperscript{16}For the second order approximation we follow the pure stochastic perturbation approach of Schmitt-Grohe and Uribe (2004) and Collard and Juillard (2001).

\textsuperscript{17}The IRFs for $\varepsilon^n_N$ are available in a technical appendix to the paper.

\textsuperscript{18}For $\sigma > 1$, it is possible to obtain perpetual growth even without technical progress. The critical threshold level for the substitution elasticity (to generate such perpetual growth) can be shown to be increasing in the growth of labor force and decreasing in the saving rate, see La Grangdville (1989) and Klump and Preissler (2000). For this reason, we used a value for $\sigma$ only reasonably above unity.
shares: in the CD case they remain constant, in the CES case, a k-augmenting shock favors k if $\sigma > 1$ and it favors labor if $\sigma < 1$. The responses of factor shares are obviously perfectly symmetric. In the case of labor-augmenting shocks, we can observe that hours actually fall, albeit slightly, for small values of $\sigma$ (in the region of those obtained in empirical estimates as reviewed by Chirinko (2008)). That is, it appears that sufficient factor complementarity can lead to negative hours responses even in the case of the canonical RBC model.\textsuperscript{19} For $\sigma = 1.4$, hours increase after a labor-augmenting shock. Correspondingly, wages fall for $\sigma = 0.4$ and increase for $\sigma = 1.4$ for labor-augmenting shocks. Further analysis of these results will follow below. At this point, we just emphasize the fact that hours can potentially fall in the RBC model after a positive technology shock for sufficiently low values of the ES and labor-augmenting shocks.

We now move onto the basic NKM results. For labor-augmenting shocks we can observe that hours always fall, regardless of the value of $\sigma$. Wages fall slightly for low values of the ES and increase slightly for higher values of the ES. When we turn to capital-augmenting shocks, we can see that hours fall for high values of $\sigma$, but they increase slightly for $\sigma = 0.4$. Wages increase in all cases, but they do so more for the low ES values. Hence, we can see that in the NKM, we can obtain both negative and positive hours responses. In this case the positive response may happen for for capital-augmenting shocks when $\sigma$ is small. That is, it appears that the results from the RBC model somehow revert, but we are still able to obtain both positive and negative hours responses in this model.

In fact, the negative response of hours for the RBC model with labor-augmenting shocks and the positive one for the NKM in the case of capital-augmenting shocks are even larger the smaller the elasticity of substitution and as we approach the Leontief corner. This is a case considered by Francis and Ramey (2005), which we are thus generalizing here. Further inspection of the reaction of hours to technology shocks shows that:

- In the RBC model, the positive response of hours to capital-augmenting shocks always occurs. However, the negative response for labor-augmenting shocks appears to depend on the relative magnitude of $\sigma$ and $\alpha_0$. When $\sigma$

\textsuperscript{19}One could, of course, obtain negative responses by changing parameters of the utility function, but these changes would require empirically unfeasible values.
is in the vicinity of $\alpha_0$ the response is negative. The $\sigma$ threshold appears to be $\alpha_0$ plus small number. This is shown in Figure 5. There we show the impact response of hours for different values of $\sigma$ for values of the capital share of $\alpha_0 = 0.1$ and 0.4. We can observe that, regardless of $\sigma$ and the capital share, the response to capital-augmenting shocks is always positive. For labor-augmenting shocks, though, the response is negative for values of $\sigma$ in the vicinity of 0.1 when $\alpha_0 = 0.1$ and 0.4 when $\alpha_0 = 0.4$. That is, the sign of the response depends on the relative value of the ES and the capital share. We provide below a rule and intuition for this result.

- In the NKM, the negative response for labor-augmenting shocks seems to be more robust to changes in $\sigma$. However, as we will show in our sensitivity analysis, for sufficiently large $\sigma$ and a strong reaction of monetary policy to inflation, one can actually obtain positive effects on hours. A similar result is obtained, not surprisingly, for an optimal Ramsey-policy setting.

- In the NMK, the response to capital-augmenting shocks is positive for sufficiently small values of $\sigma$, but there appears to be no simple threshold as in the RBC model. We also give below an intuitive explanation for this and the previous point.

The first obvious question arising is what explains these results intuitively? The explanation we give is an approximation that simplifies the intuition behind these dynamic responses, but other factors will only have a small effect on impact. Looking at the household problem, what determines labor supply $h_t$ is the $w_t/c_t$ ratio, see equation (12) with $\gamma = \sigma_c = 1$. However, because of consumption smoothing, $c_t$ only changes very slightly following (10). The reaction of hours relative to the steady state, hence, depends on the reaction of wages plus a small effect of the change in consumption. So we can focus on what happens to $w_t$. If wages increase, as can be seen in the RBC figure, hours increase, and vice versa if they fall. Hence, given that the technology shocks affect the demand for labor then inducing a change in the marginal rate of substitution between leisure and work, it is sufficient to focus on labor demand movements to analyze the impact effect of technology.

\footnote{The explanation with other values for $\gamma$ and $\sigma_c$ follows straightforwardly.}
The elasticity of output with respect to \( z_t^h \) (ceteris paribus) is exactly \( (1 - \alpha_0) \). That is, \( (1 - \alpha_0) \) is the elasticity of output with respect to labor, and hence also labor-augmenting shocks. Hence, the level increase in output that we obtain after the shock is \( (1 - \alpha_0)z_t^h \). If we now take the FOC for labor:

\[
 w_t = (1 - \alpha_0)\left[\left(\frac{y_t}{h_0}\right)^{(\sigma-1)/\sigma}\right](y_t/h_t)^{1/\sigma}.
\]

(33)

Then take its log:

\[
 \log(w_t) = \log(1 - \alpha_0) + \sigma - 1 \frac{y_0}{h_0} + \sigma - 1 \frac{z_t^h}{\sigma} + \frac{1}{\sigma} \log(y_t/h_t).
\]

(34)

We can then calculate the percent deviation of wages from steady state as:

\[
 \log(w_t) - \log(w_{SS}) = -\frac{1}{\sigma} \log(y_0/h_0) + \sigma - 1 \frac{z_t^h}{\sigma} + \frac{1}{\sigma} \log(y_t/h_t),
\]

(35)

But we know that \( y_t \) has now increased by \( (1 - \alpha_0)z_t^h \)%, since \( (1 - \alpha_0) \) is the elasticity of output with respect to labor and labor-augmenting shocks. Now, since we are interested on labor demand movements we can fix \( h_t \) (at its steady state value). With this in mind, after some manipulation, we arrive at:

\[
 \log(w_t) - \log(w_{SS}) = \frac{\sigma - 1}{\sigma}z_t^h + \left(\frac{1}{\sigma}\right)(1 - \alpha_0)z_t^h = \left[\frac{\sigma - \alpha_0}{\sigma}\right] z_t^h
\]

(36)

Hence, the reaction of wages depends on the difference between \( \sigma \) and \( \alpha_0 \). In fact this gives a very simple view of how wages react to labor shocks: for \( \sigma < 1 \), a labor augmenting shock reduces wages, but the increase in output induced by the shock increases labor demand. Both effects cancel out when \( \sigma = \alpha_0 \). That is, there is a pure substitution effect and an output effect of the technology shock going in opposite directions.

The explanation for why we get always positive responses for hours for k-augmenting shocks in RBC follows the same logic. Now the elasticity of output with respect to capital is \( \alpha_0 \), so equation (??) becomes:

\[
 \log(w_t) - \log(w_{SS}) = \frac{\sigma - 1}{\sigma}z_t^h + \left(\frac{1}{\sigma}\right)\alpha_0 z_t^k,
\]

(37)

and since \( z_t^h = 0 \), we can simplify to get:
\[ \log(w_t) - \log(w_{SS}) = \left( \frac{1}{\sigma} \alpha_0 z_t^k \right) = \frac{\alpha_0}{\sigma} z_t^k, \] (38)

which is always positive.

What happens in the NKM? Here, it is obvious that the FOCs change as they include the marginal cost multiplying the marginal product as in equations (25) and (26). Since the marginal cost will always fall as output increases with the technology shock, this will change the mechanisms at work. The marginal cost is the shadow price of the firm’s budget constraint, which is relaxed by a positive technology shock hence reducing its marginal value. In this case we have:

\[ w_t = mc_t(1 - \alpha_0) \left[ \left( \frac{y_0}{h_0} e^{z_t^h} \right)^{\sigma - 1/\sigma} \right] \left( y_t / h_t \right)^{1/\sigma}, \] (39)

\[ \log(w_t) = \log(mc_t) + \log(1 - \alpha_0) + \frac{\sigma - 1}{\sigma} \frac{y_0}{h_0} + \frac{\sigma - 1}{\sigma} z_t^h + \frac{1}{\sigma} \log(y_t / h_t), \] (40)

Following the same logic as above, we would end up with the following two expressions for changes in wages for the two shocks:

\[ \log(w_t) - \log(w_{SS}) = \left[ \frac{\sigma - \alpha_0}{\sigma} + \Delta mc_t \right] z_t^h, \] (41)

\[ \log(w_t) - \log(w_{SS}) = \left[ \frac{\alpha_0}{\sigma} + \Delta mc_t \right] z_t^k, \] (42)

where \( \Delta mc_t \) is the percentage deviation of \( mc_t \) from steady state. This is always negative. In the case of labor shocks, it is clear that it is going to be difficult to get a positive reaction as there are two negative forces. Nevertheless, \( \Delta mc_t \) depends on how aggressive monetary policy is, since it will determine how much output deviates from equilibrium. If one sets a very aggressive policy rule one might get positive reaction of wages for high \( \sigma \)’s, as we effectively get in the simulations. But in this case, the most likely outcome is a fall. In the capital augmenting case, it should be obvious that a positive wage reaction can occur more easily, especially for small \( \sigma \). But the reaction always depends on the policy rule, since it affects \( \Delta mc_t \), so there is no simple “rule of thumb” for this case as in the RBC model.

How plausible are these responses? It is not unusual to find small estimates of
σ in the region of 0.1–0.2, especially when using disaggregate data (see Chirinko (2008) and León-Ledesma et al. (2010b)). Also, the shorter the time horizon, the less likely it is that firms can easily substitute capital and labor, so one may think of a Leontief corner as a possible approximation for the time-frames used in business-cycle analysis. We are not, in essence, forcing the results towards implausible corners. In all, the introduction of factor substitutability allows us to distinguish between different sorts of factor augmenting shocks, which have very different effects on the dynamics of hours worked (and other variables in the model). What is clear from the results is that it is not only conceivable, but also likely, that technology shocks will have different impacts on hours depending on the nature of the shocks and the production technology. It is difficult, hence, to argue that these differences can shed light on alternative theoretical explanations of the business cycle (flexible vs sticky prices).

Note also that the results presented can be relevant to interpret some recent empirical evidence. The results in Fisher (2006), Fernald (2007), and Gambetti (2006), amongst others, report relevant changes in the impact of technology shocks since the mid 1960s. Technology shocks appear to have a strong negative effect before the 1980s which then becomes almost insignificant (with another possible change in the mid-1990s).\footnote{Results differ by study but, in general, changes on the impact response can be observed around 1973, 1982 and also the first half of the 1990s} The dynamics of the the labor share in the US show a sharp decrease after the early 1980s and then a steady but slow increase until the mid 1990s. It is then likely that shocks that reduce the labor share were more prominent in the 1980s as suggested by theories of directed technical change (Acemoglu (2002a) and Acemoglu (2003)) since firms introduce technologies that reduce the use of the factors with a larger cost share. One would then also expect that the reaction of hours to technology shocks changes through time.

### 7.1 Sensitivity Analysis

In this section we present sensitivity analysis for hours when we vary the calibration value of various parameters in both models. For the RBC figure (6) shows sensitivity for the autoregressive parameter of the shocks and the elasticity of substitution between capital and labor. Figure(7) shows sensitivity analysis for the parameter
of the utility function.

For the NKM we also present sensitivity analysis for \( \rho \) and \( \sigma \) (figure 8) and for the utility function parameters (figures 9 and 10) but we also conduct sensitivity analysis for the parameters of the monetary policy rule (figures 11 and 12).

Comments in progress.

8 Conclusions

The reaction of hours worked to technology shocks has represented one of the key controversies between RCB and NK explanations of the business cycle. It sparked a large empirical literature with contrasting results, and it remains a critical issue in business cycle theory. The usual interpretation is that in an RBC model hours increase after a technology shock whereas, in a sticky price New-Keynesian model, they fall. In this paper we challenge this simple view: while remaining agnostic about empirical identification of shocks we provide theoretical arguments that show that hours worked can increase or decrease after a technology shock regardless of the price-setting behavior of the economy.

We do so by introducing the important issue of factor substitutability in the supply side of both a flexi-price and a sticky-price DSGE model. Standard models assume that capital and labor are net substitutes as implied by the use of Cobb-Douglas production functions. However, recent evidence is in sharp contrast with this net substitutability assumption. We hence introduce a Constant Elasticity of Substitution (CES) which is appropriately normalized so that the results are not the artifact of differences in steady states. We show that when the production function is not Cobb-Douglas technology shocks can have either positive or negative impacts on hours worked. The effect depends on the value of the elasticity of substitution and whether technology shocks are capital- or labor-augmenting. In the RBC model, capital-augmenting shocks always yield positive hours responses, whilst labor-augmenting shocks can lead to either response sign. We derive an approximate rule for this response, which will depend on the relative magnitudes of the elasticity of substitution between capital and labor and the elasticity of output with respect to capital. That is, there is a factor substitution and an output effect that affects wages and hence labor supply, and these can show opposite signs. The rule is still relevant in the NKM, however, in this model labor-augmenting shocks
are most likely to yield negative responses (although positive responses are not ruled out). Capital-augmenting shocks can lead to positive or negative responses. In the NKM, however, these responses depend heavily on the monetary policy rule. We explain the intuition behind these results and also carry out a comprehensive sensitivity analysis, where we show that these results are reasonably robust and, importantly, always affect the sign of the reaction of hours to technology shocks.

We conclude that the impact of technology shocks on hours worked can hardly be taken as evidence in support of a particular class of business cycle model. This is not to say that empirical evidence cannot discriminate amongst these models, but that the mere focus on this particular variable may not be a promising research avenue.

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References


9 Appendix: Equilibrium Conditions

In what follows, we describe in details the equilibrium conditions of the model and the calibration used. Here, variables without time subscript reflects steady state values.
\[ k_t = (1 - \delta) k_{t-1} + \Psi \left( \frac{i_t}{i_{t-1}} \right) i_t \quad (43) \]
\[ c_t = \lambda_t \quad (44) \]
\[ v \lambda_t c_t = w_t \quad (45) \]
\[ \lambda_t = \lambda_t q_t \left[ 1 - \psi \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \psi \left( \frac{i_t}{i_{t-1}} - 1 \right) \right] + + \beta \psi E_t \left\{ \lambda_{t+1} q_{t+1} \left( \frac{i_{t+1}}{i_t} \right)^2 \left( \frac{i_{t+1}}{i_t} - 1 \right) \right\} \]
\[ \lambda_t q_t = \beta E_t \lambda_{t+1} \left( u_{t+1} + q_{t+1} (1 - \delta) \right) \quad (47) \]
\[ \ln(r_t/\bar{r}_t) = \alpha_r \ln(r_{t-1}/\bar{r}_{t-1}) + \alpha_x \ln(\pi_t/\bar{\pi}_t) + \alpha_y \ln(y_t/\bar{y}_t) + e_m \quad (49) \]
\[ w_t = mc_t \quad (50) \]
\[ u_t = mc_t \quad (51) \]
\[ x_1 = \tilde{p}_t^{-1-\eta} (c_t + i_t) mc_t + \theta \frac{\lambda_{t+1}}{\pi_{t+1}} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-1-\eta} x_{1t+1} \quad (53) \]
\[ x_2 = \tilde{p}_t^{-\eta} (c_t + i_t) + \theta \frac{\lambda_{t+1}}{\pi_{t+1}} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta} x_{2t+1} \quad (54) \]
\[ x_2 = \frac{1}{\eta} x_1 \quad (55) \]
\[ y_t = \frac{1}{s_t} [ces_t - \chi] \quad (56) \]
\[ y_t = c_t + i_t \quad (57) \]
\[ s_t = (1 - \theta) \tilde{p}_t^{-\eta} + \theta \pi_t^{\eta} s_{t-1} \quad (58) \]
\[ ces_t = ces_0 e^{z_{Hicks}^t} \left( \alpha \left( e^{z_K^t k_{t-1}} \right) \frac{\eta}{\eta - 1} + (1 - \alpha) \left( e^{z_L^t h_{t-1}} \right) \frac{\eta}{\eta - 1} \right) \quad (59) \]
\[ ces_h^t = (1 - \alpha) \left( ces_0 \left( e^{z_{Hicks}^t e^{z_{Hicks}^t}} \right) \frac{\eta}{\eta - 1} \left( ces_0 \right) \frac{1}{\eta} \right) \quad (60) \]
\[ ces_k^t = \alpha \left( ces_0 \left( e^{z_{Hicks}^t e^{z_{Hicks}^t}} \right) \frac{\eta}{\eta - 1} \left( ces_0 \right) \frac{1}{\eta} \right) \quad (61) \]
\[ z_t^{Hicks} = \rho_{z_t}^{Hicks} + \epsilon_t^{Hicks} \quad (62) \]
\[ z_t^K = \rho_{z_t}^{K} + \epsilon_t^K \quad (63) \]
\[ z_t^L = \rho_{z_t}^{L} + \epsilon_t^L \quad (64) \]
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Table 1: Parameters calibration
Figure 1: RBC model - Capital Augmenting Shock
Figure 2: RBC model - Labor Augmenting Shock
Figure 3: NKM model - Capital Augmenting Shock
Figure 4: NKM model - Labor Augmenting Shock
Figure 5: RBC model - Impact response of hours for different $\sigma$ values. For $\alpha_0 = 0.1$ and $\alpha_0 = 0.4$
Figure 6: Sensitivity analysis in the RBC model for $\rho$ and $\sigma$
Figure 7: Sensitivity analysis in the RBC model for $\sigma_c$ and $\gamma$
(a) K-aug $\sigma$

(b) L-aug $\sigma$

(c) K-aug $\rho$ with $\sigma = 0.4$

(d) K-aug $\rho$ with $\sigma = 1.4$

(e) L-aug $\rho$ with $\sigma = 0.4$

(f) L-aug $\rho$ with $\sigma = 1.4$

Figure 8: Sensitivity analysis in the NKM model for $\rho$ and $\sigma$
Figure 9: Sensitivity analysis in the NKM model for $\sigma_c$ and $\gamma$ when $\sigma = 0.4$
Figure 10: Sensitivity analysis in the NKM model for $\sigma_c$ and $\gamma$ when $\sigma = 1.4$
Figure 11: Sensitivity analysis in the NKM model for monetary policy parameters when $\sigma = 0.4$
Figure 12: Sensitivity analysis in the NKM model for monetary policy parameters when $\sigma = 1.4$. 