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Equilibrium Rejection of a Mechanism

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Equilibrium Rejection of a Mechanism*

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Abstract

We study a mechanism design problem in which players can take part in a mechanism to coordinate their actions in a default game. By refusing to participate in the mechanism, a player can revert to playing the default game non-cooperatively. We demonstrate with an example that some allocation rules are implementable only with mechanisms which will be rejected on the equilibrium path. A refusal to participate conveys information about the types of the players. This information causes the default game to be played under different beliefs, and more importantly under different *higher order* beliefs, than the interim ones.

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1 Introduction

There are many mechanism design problems that involve agreements about how to play some *default game*. Cartel agreements govern how firms compete against one another; members of an auction bidding ring agree on how they should bid against each other; trade agreements limit governments' ability to

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use trade barriers to increase their share of trade; organizations govern the efforts of workers who might otherwise compete against one another.

In some of these situations, binding agreements require unanimous consent. Partnerships have this property by definition. Collusive agreements are often like this. Since collusion is illegal in most contexts, a non-cooperative partner can threaten any collusive agreement in which she is not included. These environments are special since any potential participant can veto an agreement and revert to playing the default game non-cooperatively. This idea underlines the concept of *ratifiability* (Cramton and Palfrey, 1995) which requires agreements to be acceptable by players no matter what their type. The concepts of *durability* (Myerson and Holmstrom, 1983), *attainability* (Crawford, 1985), and *resilience* (Lagunoff, 1995) attempt to characterize default games (and their equilibrium allocation rules) where all alternative agreements to the non-cooperative play of the default game will be rejected by at least one player. Similarly, *collusion proofness* (Laffont and Martimort, 1997, 2000, and more recently Che and Kim, 2006) requires contracts to have the property that one cannot find a collusive agreement which will be acceptable no matter what the participants' private information.

Our point here is simply to illustrate with an example that there are allocation rules that can only be supported with equilibria in which some types of some players accept the agreement but some others reject it. Since the participation decision is type dependent, a refusal to participate conveys information that causes the default game to be played differently than it would be if players used only their interim beliefs.

We base our example on a Cournot game played by two firms which we will call the *incumbent* and the *entrant*.[1] The entrant's production cost is either high or low, and its realization is private information. The mechanism designer suggests a cartel agreement which is accepted by the low cost type of the entrant, but rejected by its high cost type. When the incumbent learns that the entrant has rejected the agreement, it plays the default Cournot game as if it knows that the entrant has the high cost. When the entrant accepts the agreement, the incumbent simply does whatever the agreement tells it to. In the equilibrium we construct, the incumbent chooses to accept this agreement.

[1] Cartel agreements by asymmetrically informed firms are already studied as examples of the design setup with default game by Cramton and Palfrey (1990 and 1995). Similar examples can be constructed with auction and public good provision games, as we have done in earlier versions of this paper.

At first glance, it seems as if the designer could implement[2] the same allocation rule with full participation by suggesting a *direct revelation mechanism* which will be accepted by both types of the entrant. If the entrant reports low cost to this mechanism, then the mechanism does whatever the original cartel agreement does. If the entrant reports high cost, the mechanism instructs the firms to mimic their non-cooperative play of the Cournot game under the updated belief that the entrant has high cost. The reason for the failure of this direct mechanism is that if the entrant accepts this mechanism for sure, then the incumbent has an alternative that was not available under the original cartel agreement. That is, the incumbent can refuse to participate and then play the default game using its interim beliefs on the entrant's type. If the entrant realizes this, it will play the default game differently as well. In particular, the entrant will play the default game in a way that the incumbent prefers.[3] The cartel agreement we construct forces the incumbent to be informed of the entrant's type, and this cannot be accomplished with a direct mechanism in which all types of the entrant participate.

The idea that full participation is "without loss of generality" is widely used in mechanism design and conjectured to be an implication of the "revelation principle." It is important to understand what it is about default games that complicates this. Consider the direct revelation mechanism described in the previous paragraph. By refusing to participate in this mechanism, the incumbent ensures to play the default game under the interim higher order beliefs[4] of the entrant. The entrant's "sequentially rational" response to these beliefs turns out to be favorable to the incumbent. The revelation principle, on the other hand, is a statement about solution concepts, which do not put sequential rationality restrictions on the "off the equilibrium path" events.[5] If the incumbent's refusal is not on the path of the expected play, these solution concepts impose no restriction whatsoever on how the entrant

[2] The implementation concept we use is "weak" implementation. That is, a mechanism implements an allocation rule if the game induced by the mechanism has an equilibrium supporting the allocation rule (as opposed to all of its equilibria supporting the allocation rule).

[3] Kim (forthcoming) makes a similar observation in the context of common value first price auctions: If the value of the auctioned object is submodular in the bidders' signals, then a bidder prefers to be uninformed of her rival's signal.

[4] That is, the entrant's beliefs about the incumbent's beliefs on the entrant's type.

[5] One such solution concept is Bayesian equilibrium.

responds. Therefore the need for an equilibrium rejection does not arise. Of course, there is little point worrying about default games as outside options and then ignoring sequential rationality. Hence, our example should be considered as a contribution to the literature on default games and not as a comment about the revelation principle itself.

Here is the organization of the rest of the paper. We give the details of our default Cournot game, the cartel agreement the firms will consider, and other features of their interaction in the following section. In Section 3, in light of the earlier literature, we show how the updated beliefs affect the acceptability of a cartel agreement. In Section 4, we move to the main focus of the paper: the non-degenerate participation behavior of the firms. We show that there are allocation rules implementable only under non-degenerate participation. In Section 5, we check for the robustness of our arguments under alternative modeling choices. In the same section, we also consider equilibria where a type may randomize over its participation decision. We conclude with a discussion of our result in Section 6. We relegate the technical details of our analysis to the Appendix.

2 The Model

2.1 The Cournot Game with Private Cost

We consider an industry with two firms which are (potential) producers of an homogenous good. Both firms have linear cost functions. One of these firms, the incumbent (N), has the unit cost 0.7. The unit cost of the other firm, the entrant (E), is either high (h =1) or low (l =0.65) with equal probabilities. E observes the realization of its own unit cost. However N is only informed of the distribution. The inverse demand function for the good is given as $P = 1 - (q_N + q_E)$, where P is the price and q_N, q_E are the production levels for N and E respectively.

The firms make their production decisions simultaneously to maximize their expected profit levels. In the Appendix, we show that the resulting game of incomplete information has a unique *Bayesian equilibrium* (BE) under the interim beliefs, where N sets $q_N = \frac{17}{140}$ and receives the expected profit $\left(\frac{17}{140}\right)^2$, the low cost type of E sets $q_{E_l} = \frac{4}{35}$ and receives profit $\left(\frac{4}{35}\right)^2$, the high cost type of E sets $q_{E_h} = 0$ and receives profit 0. Even though the low cost type of E has a lower cost than that of N, the former firm has a lower market share and profit than does the latter firm when both these firms are active in the market. This is due to the fact that the *expectations* on the cost levels are as relevant as the realized cost levels for the firms' production decisions. N expects that its rival will produce zero output with probability $1/2$ and therefore chooses a higher production level than it would have chosen if it knew its rival was the low cost E. Believing that N produces a relatively high output level, the low cost E compensates by choosing a lower output level for itself. How the changing expectations would affect the firms' behavior and their profit in the Cournot game will be the key to the discussion to follow.

2.2 The Cartel Agreement

Suppose these two firms are able to sign a cartel agreement prior to making their production decisions. Following Cramton and Palfrey (1990), we model the cartel as a mechanism that is offered by a third party, which we will call the *designer*. The designer does not know the type of E, and does not posses any private information herself. Our aim in this paper is to discuss what this designer is capable of doing, rather than what she would choose to do. Therefore we will not be very specific on the designer's objective for now. She may be maximizing a weighted average of the firms' expected profits or any other function of the firms' production and profit levels.

The mechanism is a message game, where the messages of E are mapped into output levels for the firms and monetary side transfers between them.[6] (Each firm maximizes its expected profit level net of the side transfer.) When offered a mechanism, each firm has an inalienable right to reject it and play the Cournot game non-cooperatively. Following the literature, we assume that the firms make their ratification decisions simultaneously. If both firms accept the mechanism, then E sends its message to the mechanism, which in turn determines the output and side transfer levels. If either one of the firms rejects the mechanism, then they learn which firm(s) rejected it and play the Cournot game by choosing their production levels simultaneously. The design problem in this setup is nonstandard since the rejection payoffs are not exogenously specified but are determined by the subsequent actions of the firms.

[6] The messages are the means of collecting the information that is endowed by the firms. Since only E has private information in this setting, it suffices to have a singleton as the message set for N.

2.3 The Equilibrium

After the announcement of a mechanism, the interaction between the firms can be considered as a sequential game of imperfect information. The solution concept we consider here is *Perfect Bayesian equilibrium* (PBE). This solution concept is defined as a collection of *sequentially rational strategies* (which govern the ratification decisions of the firms, the message choice of E if the mechanism takes effect, and the production decisions of the firms if the mechanism is rejected) and *consistent beliefs* (of N on the types of E after observing E's ratification decision).[7] We relegate the formal definitions of the strategies, the beliefs, and the PBE to the Appendix.

An *allocation rule* in this environment is defined as a mapping from the two states of nature (recall that the cost level of E is either high or low) to the randomizations over the production and side transfer levels of the firms. A mechanism *implements* an allocation rule if there exists a PBE after the announcement of the mechanism, which supports the allocation rule in question. An allocation rule is called *implementable* if there exists a mechanism implementing it.

Before we proceed with the analysis of our model, a brief note is in order. What we outline above is certainly not the unique way to model an agreement between two players. Nevertheless our assumptions are in line with most of the earlier literature. Alternative modeling choices include pre-play communication in the default game, unobservable ratification behavior, and agreements offered by the players instead of a designer. After constructing our example (Section 4), we will discuss the relevance of our arguments under these alternative assumptions as well (Section 5).

3 Belief Update

Whether a firm will accept a cartel mechanism depends on the continuation profits it expects from accepting or rejecting it. Our discussion will be based on the rejection profits. One approach to this issue involves taking the BE payoffs under the interim beliefs as the outside option each firm expects from

[7] Our main contribution will be through considering equilibria where both ratification decisions are on the equilibrium path. Therefore utilizing an alternative solution concept which extends the consistency requirement for the off the equilibrium path beliefs (such as *sequential equilibrium*) would not enrich the discussion.

rejecting the mechanism. Suppose firms N and E indeed expect to play the Cournot game with the interim beliefs after a possible rejection by either one of the firms. Then the low cost type of E could guarantee its BE payoff of $\left(\frac{4}{35}\right)^2 \approx 0.013061$, by rejecting any mechanism.

The approach above is called the *passive beliefs* approach, since it does not allow for any change in the beliefs of a firm after observing the ratification decision of its rival (See Cramton and Palfrey, 1990 for an application). However, if the rejection of the mechanism is an off the equilibrium path behavior for E, then a Perfect Bayesian equilibrium does not put much restriction on the beliefs of N (regarding the type of E) after observing a rejection by E. This degree of freedom in the determination of the off the equilibrium path beliefs may make the rejection of a mechanism costlier than under the passive beliefs.

To illustrate, we consider a PBE such that both types of E accept the mechanism with probability one. In this case, rejection by E is an off the equilibrium path behavior. Suppose N attributes any such rejection to the high cost type of E. Given this belief, N expects E not to produce (since this is the dominant production decision for the high cost type of E in the Cournot game). Therefore, after observing E's rejection of the mechanism, N produces its monopoly output level $q_N = 0.15$. The best response of the low cost type of E is choosing $q_{E_l} = 0.1$, which yields a profit of 0.01. Notice that this profit is lower than the BE profit under the interim beliefs for this firm.

The designer may make use of such an off the equilibrium path belief update. To see this, consider mechanism $\mathcal{M}_1$, which has message set $\{L, H\}$. When E sends the message H, $\mathcal{M}_1$ instructs the firms to choose output levels $q_E = 0$, $q_N = 0.15$ and not to make any side transfers. When E sends the message L, $\mathcal{M}_1$ instructs the firms to choose output levels $q_E = 0.175$, $q_N = 0$ and E to pay $(0.175)^2 - (0.1)^2$ as a side transfer to N. If $\mathcal{M}_1$ is accepted by all types of all firms, the high type of E sends the message H and the low type sends the message L. The equilibrium outcome maximizes the total industry profits, since the lower cost firm produces the monopoly output level in either state of nature. The high and low cost types of E are left with profit levels 0 and 0.01 respectively. After the announcement of this mechanism, there exists a PBE where all types of all firms accept the mechanism and in the off the equilibrium path event that E rejects, N believes E is the high cost type. (The complete construction of the PBE is relegated to the Appendix.) In fact, the allocation rule that is supported by this PBE is the one that maximizes N's expected profit among all the implementable allocation rules.[8]

The idea that the outside option of a mechanism is affected by belief updates after an off the equilibrium path rejection is well known in the literature. A player may be punished (or rewarded) in the default game since it revealed some information by rejecting a mechanism. The construction of the PBE above relies on such belief updates. As we mentioned, a Perfect Bayesian equilibrium does not put much restriction on the off the equilibrium path beliefs. Cramton and Palfrey (1995) propose *ratifiability* as a refinement of these beliefs by suggesting that the support of the post rejection beliefs should consist only of the types that are not strictly worse off by rejecting the mechanism.[9] We do not employ any such refinement here. Instead, we take the opposite route and examine a larger class of equilibria, where rejection of the mechanism may be on the equilibrium path.

4 Equilibrium Path Rejection

With the above discussion, we have identified the optimal implementable allocation rule of a designer whose objective is maximizing the expected profit of N. Now suppose instead that the designer is concerned with the ex-ante expected payoff for E (That is, the designer's objective function is the sum of half the profit of the high cost type and half the profit of the low cost type of E). As before, maximizing the industry profits requires letting only the lowest cost firm produce its monopoly output in both states of nature. The question is the extent of the industry profits that should be left to N. Unlike the unit cost of E, the unit cost of N is common knowledge. Therefore there is no room for a belief update in case that N unexpectedly rejects the mechanism. In fact, any PBE, where both types of E accept the mechanism with probability one, has to leave N with at least its expected

[8] Since this last claim is not central to our arguments, we do not provide a proof.

[9] Cramton and Palfrey (1995) consider the joint profit maximizing cartel agreement for two competing firms with cost levels uniformly and independent distributed on $[0, 1]$. They argue that rejecting this agreement would signal a low production cost for these firms. In their setup, maximizing the joint profits is not compatible with ratifiability. More recently, Tan and Yilankaya (2007) show that the same refinement rules out efficient bidder collusion in a second price auction with participation costs as well.

profit from the unique BE of the Cournot game under the interim beliefs: $\left(\frac{17}{140}\right)^2 \approx 1.4745 \times 10^{-2}$.

We now consider another class of PBE, where E's ratification behavior reveals its type. Suppose the mechanism is accepted by the low cost type of E but rejected by its high cost type. Recall that N observes E's ratification decision. Therefore consistency requirement of PBE implies that N infers E's type after the ratification stage. Being informed on E's type changes the way N plays the Cournot game. When E has the high cost, N chooses the monopoly output level of $q_N = 0.15$, and receives the profit $(0.15)^2$. When E has the low cost, N's output level is much lower at $q_N = \frac{1}{12}$, yielding a profit of $\left(\frac{1}{12}\right)^2$ in the complete information Cournot game to follow. The expected profit level of N is the average of these two profit levels, ($\pi_N = \frac{1}{2}(0.15)^2 + \frac{1}{2}\left(\frac{1}{12}\right)^2 \approx 1.4722 \times 10^{-2}$), which is smaller than the unique BE payoff from the Cournot game played under the interim beliefs.

The remaining task is constructing a mechanism which will be accepted by the low cost type of E and rejected by its high cost type. Consider mechanism $\mathcal{M}_2$ which instructs the firms to set production levels $q_E = 0.175$, $q_N = 0$ and E to pay $\left(\frac{1}{12}\right)^2$ to N as a side payment (The message set is singleton in this mechanism). Once $\mathcal{M}_2$ is announced, there exists a PBE, where N and the low cost type of E accept the mechanism, but the high cost type of E rejects it. In case of a rejection by E, N learns that its rival has the high unit cost and therefore chooses the monopoly output level of $q_N = 0.15$. If both firms accept, then the mechanism instructs the lower cost firm to produce the monopoly output and to compensate the higher cost firm. (The complete formal construction of the equilibrium is in the Appendix.) This equilibrium supports an allocation rule which leaves N with the expected profit of $\pi_N \approx 1.4722 \times 10^{-2}$.

Construction of this equilibrium is based on the fact that N's expected profit is lower in the Cournot game whenever N infers its competitor's type. This is a consequence of the higher order beliefs of these firms. To see why, recall that E's low cost type produces a strictly positive level of output and its high cost type does not produce at all in the non-cooperative play of the Cournot game. When N is uninformed of E's type, it chooses an output level as a best response to the average of these two possible production levels by E. On the other hand, when N knows that E has a low (high) production cost, choosing a best response to the higher (lower) production level compels N to reduce (increase) its own output choice. At first glance, this adjustment in N's output level seems to have a positive effect on its profits. After all, more information allows N to better respond to E's production choice. However, since E also knows that N is informed of E's type, it adjusts its production level as well. In particular, E with the low production cost now chooses a higher production level which reduces the profits of N. Under the parameterization we use, this latter effect dominates and therefore N's expected profit is lower with symmetric information.[10]

The mechanism and the PBE we outline above show that the designer can reduce the expected profit of N below its BE level under the interim beliefs. However, this requires revealing E's type credibly to N. In the above construction, the designer uses E's ratification decision as the means of revealing its type to N. Notice that the allocation rule above cannot be replicated as the PBE allocation rule of another mechanism which is unanimously accepted on the equilibrium path. Under such a PBE, N would always have the option of rejecting the mechanism and playing the Cournot game under the interim beliefs. Therefore, if both types of E accept a mechanism, the expected payoff of N must be at least $\left(\frac{17}{140}\right)^2$ as we have noted above.

5 Remarks

5.1 Pre-play Communication in the Default Game

The Cournot game we introduce has a unique BE, which yields $\left(\frac{17}{140}\right)^2$ as the expected profit for N under the interim beliefs. However, as Matthews and Postlewaite (1989), Palfrey and Srivastava (1991), and Forges (1999) argue, pre-play communication between the players dramatically extends the set of equilibria. Indeed, our Cournot game also has a *communication equilibrium* where E sends a cheap talk message revealing its type prior to the production

[10] The mathematical intuition for this result comes from considering N's expected profit from the Cournot game as a function of its beliefs on E's type. Let v be the probability N assigns to the event that E has low cost. We show in the Appendix that N chooses the equilibrium output level $\frac{0.6-0.35v}{4-v}$, which yields the expected profit $\pi_N(v) = \left(\frac{0.6-0.35v}{4-v}\right)^2$. Function $\pi_N(\cdot)$ gives the monopoly profit at $v = 0$, and the expected BE profit under the interim beliefs at $v = 0.5$. As expected, this profit function is decreasing in v on the relevant interval $[0, 1]$. However it is not convex. The prior belief of $v = 0.5$ is a convex combination of the extreme beliefs of $v = 0$ and $v = 1$ (with equal weights). But the convex combination of $\pi_N(0)$ and $\pi_N(1)$ is smaller than $\pi_N(0.5)$.

decisions and which yields exactly the same expected payoff to N as does mechanism $\mathcal{M}_2$ and the PBE outlined above. Suppose we allow for pre-play communication between the firms playing the default game. Do we still need equilibrium rejection of a mechanism to implement certain allocation rules?

In order to answer this question, we modify our example by reducing the unit cost level for the high cost type of E to $h' = 0.9$ from $h = 1$. The Cournot game still has a unique BE where N produces $q_N = \frac{17}{140}$, E with cost 0.65 produces $\frac{4}{35}$, and E with cost 0.9 does not produce as in our original example. However, this time, there exists no communication equilibrium of the Cournot game, where E reveals its type truthfully with a cheap talk message. To see this last point, recall that when N believes that it is facing the low cost competitor with certainty, it reduces its production level to $\frac{1}{12}$. Both types of E would strictly benefit from this degenerate belief (even E with the high cost would produce a positive output level $\frac{1}{60}$ and make a positive profit $\left(\frac{1}{60}\right)^2$). Therefore, if there were a cheap talk message revealing in equilibrium that E has the low cost, both types of E would be willing to send this message, which in turn would eliminate the information content of the message in question.

There is no communication equilibrium of the modified Cournot game, where E's type is revealed. Yet, mechanism $\mathcal{M}_2$ still induces the same PBE as before, where E accepts $\mathcal{M}_2$ if it has low cost and rejects it otherwise. Even though the high cost type of E has the incentive to trick N into believing that it is facing a low cost competitor, accepting $\mathcal{M}_2$ would be a very costly means of achieving the objective.

5.2 Unobservable Ratification Decisions

We constructed our example under the assumption that the ratification decision of a player is observable to all the others. An alternative assumption would be that players observe whether a mechanism is accepted by all players or not instead of observing each individual ratification behavior. Under this alternative assumption, by rejecting a mechanism, each player could remain uninformed of the ratification behavior of the other players, and therefore maintain the interim beliefs while playing the default game. Is there still a case for the necessity of equilibrium rejection when the individual ratification decisions are unobservable?

To answer this question, we consider mechanism $\mathcal{M}_3$ which instructs the firms to set production levels $q_E = 0$ and $q_N = 0.15$, without any side transfer between them (As in $\mathcal{M}_2$, the message set is a singleton). In words, $\mathcal{M}_3$ asks the firms to replicate their BE output choices for the Cournot game played under the belief that E has the high cost. Under our initial assumption that individual ratification decisions are observable, there is a PBE of the game ensuing the announcement of $\mathcal{M}_3$, where N and the high cost type of E accept the mechanism, but the low cost type of E rejects it. In case of a rejection by E, N infers that its rival has the low cost. Therefore E and N play the unique BE of the induced complete information Cournot game by choosing output levels $q_E = \frac{2}{15}$ and $q_N = \frac{1}{12}$. (The complete construction of the equilibrium is relegated to the Appendix.) The resulting allocation rule does not maximize the joint profits but it is implementable. Under this allocation rule, the expected profit of N is equal to the same expected profit ($\approx 1.4722 \times 10^{-2}$) as under the allocation rule implemented by $\mathcal{M}_2$.

We now argue that the allocation rule outlined above is still implementable, with the same mechanism and the same equilibrium ratification and production decisions of the firms, even when the individual ratification decisions are not observable. The critical decision to check is the ratification decision of N. Suppose N accepts the mechanism as instructed by its equilibrium strategy. Under the new assumption, N does not directly observe E's ratification decision. However, it knows that the mechanism takes effect if and only if E accepts it. Therefore N infers E's type by only observing if the mechanism comes into effect or not. As we noted in the previous paragraph, after N's acceptance of the mechanism, the continuation equilibrium yields N an expected profit of approximately 1.4722×10^{-2}.

Now suppose N deviates from the equilibrium strategy and rejects the mechanism. By doing so, it exercises the option of remaining uninformed on E's type. Let us consider the default game ensuing after N's rejection of the mechanism. The high cost type of E infers that N had rejected the mechanism and it chooses its dominant output level $q_E = 0$ in the default game. The low cost type of E, who has rejected the mechanism, still believes that N accepted the mechanism and therefore N inferred that E's type is low. Consequently, the low cost type of E chooses the unique BE output level $q_E = \frac{2}{15}$ of the symmetric information Cournot game. Notice that, regardless of N accepts or rejects the mechanism, it faces the same type contingent output levels by its rival. However, N can learn the type of the rival and adjust its best response only by accepting the mechanism. This reveals that it is not optimal for N to deviate from the equilibrium ratification behavior.

Therefore the allocation rule detailed above is implementable even when individual ratification decisions are not observed.

Finally, notice that there exists no mechanism to implement this allocation rule with full participation. As under the initial assumption, if both types of E are accepting the mechanism, N can always force its rival to play the Cournot game under the common knowledge of the interim beliefs and receive a higher expected payoff than suggested by the allocation rule.

5.3 Informed Principal Problem Revisited

So far we have followed the Bayesian mechanism design tradition and assumed that the mechanism is offered by a designer who is uninformed of the types of the players. In this part of the paper, we open a parenthesis to consider the alternative assumption of allowing one of the players, E, to offer the mechanism to the other player. Under this alternative modeling, E is the *informed principal* offering a mechanism to the *uninformed agent* N. As before, a mechanism consists of a message set for E and a mapping from the messages to the output levels and the side transfers. After observing the offered mechanism, N decides whether to accept it or to play the Cournot game with E. This version of the model is reminiscent of the informed principal model studied by Myerson (1983) and Maskin and Tirole (1990, 1992). However, unlike in these seminal papers, the outside option of the agent is not exogenous here. Instead, this outside option is determined by a default game with both the principal and the agent as the game's players.

The informed principal variation of our model induces a sequential game of incomplete information between firms E and N as well. In this alternative formulation, the allocation rule implemented by mechanism $\mathcal{M}_2$ above can still be supported as a PBE allocation rule. However, construction of such a PBE requires E to reveal its type to N through the mechanism it offers. For instance, we can construct an equilibrium, where the low cost type of E offers mechanism $\mathcal{M}_2$ and the high cost type of E offers mechanism $\mathcal{M}_3$ (defined in Section 5.2). N infers its competitor's type from the offer and accepts either offer since the payoff from either mechanism is exactly the same as the payoff from playing the Cournot game under symmetric information. (A more detailed sketch of the equilibrium is relegated to the Appendix.)

In the equilibrium outlined above, different types of E offer different mechanisms. If we were to look for an equilibrium where both types of E offer the same mechanism, then N would guarantee the BE payoff of the

Cournot game under the interim beliefs by rejecting this mechanism. This observation implies that the allocation rule which leaves N with an expected profit of approximately 1.4722×10^{-2} is implementable only with equilibria where more than one single mechanism is offered on the equilibrium path. This is in contrast with the *inscrutability principle*,[11] which applies to the informed principal models with exogenous outside options.

5.4 Stochastic Ratification

In this part of the paper, we revert back to our initial modeling assumptions. With the analysis in Section 4, we showed that the designer can reduce N's expected profit below its Cournot game BE level by inducing E to reveal its type with its ratification behavior. We will now show that the designer can reduce N's expected profit even further by making E reveal only *partial* information on its type. To see this, consider mechanism $\mathcal{M}_4$ with the message set $\{L, H\}$. When E sends the message H, mechanism $\mathcal{M}_4$ instructs the firms to choose output levels $q_E = 0$, $q_N = 0.15$ and not to make any side transfers. When E sends the message L, mechanism $\mathcal{M}_4$ instructs the firms to choose output levels $q_E = 0.175$, $q_N = 0$ and E to pay 6.875×10^{-3} as a side transfer to N. There exists a PBE of the induced game where N and the low cost type of E accept the mechanism with probability one, however the high cost type of E accepts it with probability $1/4$ only. In case that E rejects this mechanism, N learns that E has high unit cost and therefore chooses the monopoly output level of $q_E = 0.15$ and receives the profit $(0.15)^2$. In case that E accepts, its cost level is revealed by the message it sends to the mechanism. If E sends the message H, then the rejection outcome is replicated. Otherwise E produces the monopoly output level and compensates N. (The complete characterization of the PBE is relegated to the Appendix.)

The mechanism and the PBE above yield a lower expected profit to N (at the amount 1.4688×10^{-2}) than mechanism $\mathcal{M}_2$ and the PBE that fully reveals E's type at the ratification stage. In fact, the resulting allocation rule here is the one that maximizes the industry profits and minimizes N's share of these profits among all the implementable allocation rules.[12]

[11] Formally, inscrutability principle states the following for the informed principal model with exogenous outside options. Consider an allocation rule that is supported by a PBE. There exists (perhaps another) PBE, which supports the same allocation rule and where all types of the principal offer the same mechanism.

[12] Since this last claim is not central to our arguments, we do not provide a proof.

6 Conclusion

As our example illustrates, mechanisms with full participation give the players an option to play the default game under the interim beliefs. The reason that this may be advantageous involves a higher order consideration: In the example we provide, the entrant (to be exact, one type of the entrant) chooses a lower output level in the non-cooperative play of the Cournot game if it believes the incumbent is using the interim beliefs. The equilibrium with partial participation takes this option away and supports a continuation equilibrium in which the incumbent plays as if it knows the type of the entrant.

This argument suggests why default games are more difficult to handle than (possibly type contingent) allocations as outside options. A sensible treatment of the default game demands some kind of sequential rationality restriction. This restriction limits the set of allocation rules that a mechanism designer can support. If we had used Bayesian equilibrium as a solution concept for the overall game instead of the Perfect Bayesian equilibrium, this distinction would not arise. To see why, return once again to the allocation rule we showed to be implementable with equilibrium rejection by the high cost type of the entrant. Consider the direct revelation mechanism suggested in the Introduction, which tried to implement the same allocation rule with full participation by having the designer instruct the firms to set the non-cooperative output levels whenever the entrant reports high cost. The problem arises because the incumbent can then refuse to participate and play the Cournot game without knowing the entrant's type. In this construction, such a rejection by the incumbent is an off the equilibrium path event. A Bayesian equilibrium does not impose any restriction on the behavior of the entrant in the continuation game following this off the equilibrium path rejection. If the incumbent expects the entrant to use the same type contingent behavior here as in the original equilibrium without full participation, then there would be no advantage to refusing to participate. The incumbent would do generally worse by rejecting the direct mechanism since it would be best replying to the same type contingent behavior, but would not know the entrant's type.

Bayesian equilibrium is also the solution concept underlying Myerson's model of *games with contracts* (1991, Chapter 6). In his model, a mechanism can still instruct the complying players how they should play the default game when there exists some other player(s) rejecting it. Myerson considers equilibria where all types of all players accept the mechanism. Since he uses Bayesian equilibrium as the solution concept, there is no sequential rationality restriction on the post-rejection instructions of the mechanisms. If our Cournot game is played with such *Myerson mechanisms*, in the event that one firm rejects the mechanism and the other one accepts, the mechanism may instruct the complying firm to flood the market by setting its production level to $q_i = 1$. Such an instruction eliminates the possibility of making a profit by rejecting a mechanism and reduces the outside option for each firm to zero profit. This amounts to setting the reservation utility of each player to zero.[13] Hence, the problem reduces to a standard design problem, eliminating the need for an equilibrium rejection.[14,15]

When the outside option is an exogenously specified allocation rule, demanding sequential rationality off the equilibrium path does not restrain the mechanism designer. Moreover, as Myerson illustrates, once we give up sequential rationality, a default game boils down to an allocation rule. If one is willing to use Bayesian equilibrium instead of Perfect Bayesian, there is no conceptual difference between default games and type contingent allocations as outside options. On the other hand, if the objective is to understand the restrictions that the default game imposes, then the relevant solution concept must be Perfect Bayesian equilibrium or some other refinement of Bayesian equilibrium based on sequential rationality.

[13] What is relevant here is not the magnitude of the rejection profit, but the fact that it is independent from the allocaton rule implemented by the mechanism. Under complete information, Myerson shows that any implementable allocation rule is implementable with unanimous ratification of a mechanism, which punishes a rejecting player by *minimaxing* her payoff in the default game.

[14] *Auctions with externalities*, as studied by Jehiel, Moldovanu, and Stacchetti (1996 and 1999) constitute another example of such mechanisms. In this model, a bidder who does not acquire the auctioned object may incur a negative externality if a *competitor* receives the object. Jehiel, Moldovanu, and Stacchetti show that the seller may extract surplus even from the bidders who do not acquire the object. The seller achieves this by threatening the bidders to give the object to their strongest competitor if they do not participate.

[15] Another way of modeling mechanisms which are not completely void after rejection is proposed by Dequiedt (2006). According to his model, once a mechanism is rejected, players choose sequentially rational actions in the default game. However, a rejected mechanism can still send messages to players, signaling relevant information on the types of the complying players. As in our example, what is crucial here is not the mechanism's communication with the non-participants, but that this communication is common knowledge.

7 Appendix

In the Appendix, we first derive the Bayesian equilibrium output and profit levels of the Cournot game under different beliefs on the cost of E. Then we discuss the cartel mechanism and the timing of the interaction between the firms involved. After that, we formally define the Perfect Bayesian equilibrium and we provide the details of the equilibria which are referred to in the main body of the paper. Finally we replicate the definition of Perfect Bayesian equilibrium for the informed principal variation of our model.

7.1 BE of the Cournot Game

When solving the Cournot game with private information, the first point to note is the dominant strategy of E with the high cost level. Whenever E has unit cost 1, its profit function is given as $-(q_N+q_{E_h})q_{E_h}$, which is maximized with the output choice $q_{E_h} = 0$. Therefore the equilibrium output level of the high cost type of E is determined as zero regardless of the beliefs. Zero output brings zero profit to this type of the entrant firm.

Now we move to the output levels of N and the low cost type of E. Since the latter firm has unit cost 0.65, its profit level is given by the function $(1 - q_N - q_{E_l} - 0.65)\, q_{E_l}$, which is maximized with the output choice $q_{E_l} = \frac{0.35-q_N}{2}$ for the relevant values of q_N. To derive a similar reaction function for N, we define v as the probability this firm attributes to the event that E has the low cost. Accordingly, the expected output level by E is vq_{E_l}. Since N has cost 0.7, its reaction function is written as $q_N = \frac{0.3-vq_{E_l}}{2}$ for the relevant values of q_{E_l}. When we solve for the two reaction functions simultaneously, we get the *unique* BE output levels as functions of parameter v:

$$\begin{aligned} q_N &= \frac{0.6 - 0.35v}{4 - v}, \\ q_{E_l} &= \frac{0.4}{4 - v}. \end{aligned}$$

After substituting these values in the (expected) profit functions, we see that the maximized levels for the profits are $(q_N)^2$ and $(q_{E_l})^2$ for these two firms. As expected, when we increase parameter v, the output and profit levels of N decrease whereas the output and profit levels of E with low cost increase. If v equals the interim belief 0.5, the output levels are $q_N = \frac{17}{140}$ and $q_{E_l} = \frac{4}{35}$. If v equals 0, q_N equals N's monopoly output level 0.15.

In the construction of the PBE for the sequential game that starts after the proposal of a cartel mechanism, we will refer to the belief specific output and profit levels we derived above.

7.2 The Cartel Mechanism

The cartel mechanism, if accepted, instructs the firms what output levels they should produce as well as how much side transfer they should make to each other. The mechanism can condition these choices on revelations by (the message of) E. Formally, a mechanism is a collection of a message set M_E for E, two output functions determining each firm's (randomization over) production level $\chi_i : M_E \to \Delta R_+$ for $i = E, N$, and a transfer function determining the (randomization over) side transfer from E to N, $\tau : M_E \to \Delta R$.[16]

After E learns its private information and both firms E and N observe the mechanism, they simultaneously decide whether to accept the mechanism (y) or not (n). After this *ratification stage*, the firms observe the decisions of each other. There are four possible combinations of ratification decisions: $(y, y), (y, n), (n, y), (n, n)$. The first combination is the only one where the mechanism comes into effect. In this case, E sends its message to the mechanism and functions χ_E, χ_N and τ determine the production and transfer levels. The mechanism is vetoed by the remaining three combinations of ratification decisions. When the mechanism is vetoed, the firms observe who has rejected the mechanism and the game proceeds to a continuation game where the firms choose their production levels in the Cournot game.

7.3 Strategies and the Equilibrium

Given the proposed mechanism, the resulting interaction between the two firms can be thought as a sequential game. We now specify the Bayesian (behavior) strategies of these two firms in this sequential game. We start with N. N has to decide the probability that it will accept the mechanism:

$$\sigma_N \in [0, 1].$$

If the mechanism is accepted by both firms, then the output and side transfer levels are determined by E's message. In case that the mechanism is vetoed,

[16] The model can be easily extended to allow for correlation between the randomizations over production and side transfer levels.

N should decide its (randomization over) production level after observing the identity of the vetoers. As we mentioned above, there are three different combinations of ratification decisions leading to a veto of the mechanism. Technically, a firm can condition its continuation behavior to which of these three cases has realized. However, since N does not have any private information, its ratification behavior does not reveal any payoff relevant information for the continuation game. Therefore we concentrate on continuation strategies contingent only on the ratification behavior of E:

$$q_N : \{y, n\} \to \Delta R_+.$$

σ_N and q_N specify the strategy of N. E also faces similar ratification and production level choices. However, unlike N, it can condition these choices on its cost level:

$$\begin{aligned} \sigma_E &: \{l, h\} \to [0, 1], \\ q_E &: \{y, n\} \times \{l, h\} \to \Delta R_+. \end{aligned}$$

As in the definition of N's strategy, set $\{y, n\}$ above refers to the possible ratification choices of E only. In addition to the above choices, E also decides what message (or what randomization over messages) to send to the mechanism in case that the mechanism is accepted unanimously:

$$m_E : \{l, h\} \to \Delta M_E.$$

Functions σ_E, q_E, and m_E specify the strategy of E.

To talk about a Perfect Bayesian equilibrium (PBE), we should also consider a system of beliefs. The relevant belief here is what N thinks of E's cost level after observing the latter firm's ratification behavior. To formalize these beliefs we define β_y as the probability that N attributes to E having low cost after observing that E accepted the mechanism. Similarly, β_n is the probability N assigns to the same event in case that E rejects the mechanism.

Definition 1 *A Perfect Bayesian Equilibrium is a collection of strategies and beliefs* $\left(\sigma_N^*, \sigma_E^*, q_N^*, q_E^*, m_E^*, \beta_y^*, \beta_n^*\right)$ *which together satisfy the conditions listed below:*

i) For $\theta = l, h$*, message (or any message in the support of randomization)* $m_E^*(\theta)$ *maximizes the expected profit of E with type* θ *among the messages in* M_E*.*

ii) For $r_E = y, n$, functions $q_N^*(r_E)$ and $q_E^*(r_E, \cdot)$ constitute a BE of the Cournot game under the belief $\beta_{r_E}^*$.

iii) σ_N^* maximizes the expected continuation payoff of N, given the rival firm's ratification behavior (σ_E^*) and the continuation strategies of both firms (q_N^*, q_E^*, m_E^*). Similarly, for $\theta = l, h$, $\sigma_E^*(\theta)$ maximizes the expected continuation payoff of E with type θ, given the rival firm's ratification behavior (σ_N^*) and the continuation strategies of both firms (q_N^*, q_E^*, m_E^*).

iv) β_y^* and β_n^* are derived by the Bayes formula on the equilibrium path. That is, $\beta_y^* = \frac{\sigma_E^*(l)}{\sigma_E^*(l)+\sigma_E^*(h)}$ if $\sigma_E^*(l) + \sigma_E^*(h) > 0$ and $\beta_n^* = \frac{1-\sigma_E^*(l)}{2-\sigma_E^*(l)-\sigma_E^*(h)}$ if $\sigma_E^*(l) + \sigma_E^*(h) < 2$.

7.4 The Mechanisms

Now we turn to proving that the firm behavior we refer to in certain parts of the paper can indeed be rationalized as equilibrium behavior given the corresponding mechanism.

- Mechanism $\mathcal{M}_1$: This mechanism has the binary message set $M_E = \{L, H\}$. Functions χ_i and τ are defined as below:

$$\chi_N(m) = \begin{cases} 0 & \text{if } m = L \\ 0.15 & \text{if } m = H \end{cases}, \chi_E(m) = \begin{cases} 0.175 & \text{if } m = L \\ 0 & \text{if } m = H \end{cases},$$
$$\tau(m) = \begin{cases} (0.175)^2 - (0.1)^2 & \text{if } m = L \\ 0 & \text{if } m = H \end{cases}.$$

  In the sequential game following the announcement of this mechanism, we will show that there exists a PBE such that

  - $\sigma_N^* = \sigma_E^*(l) = \sigma_E^*(h) = 1$,
  - $m_E^*(l) = L$, and $m_E^*(h) = H$,
  - $\beta_y^* = 0.5$, $q_N^*(y) = \frac{17}{149}$ and $q_E^*(y, l) = \frac{4}{35}$, and $q_E^*(y, h) = 0$,
  - $\beta_n^* = 0$, $q_N^*(n) = 0.15$, $q_E^*(n, l) = 0.1$, and $q_E^*(n, h) = 0$.

To prove this, we need to establish that the strategies and the beliefs above satisfy the four conditions of PBE:

i) Once the mechanism is unanimously accepted, the low cost E produces 0.175 and earns a profit of $(0.1)^2$ by sending the message L. The high cost E

produces 0 and makes a profit of 0 by sending the message H. Either type would be worse off if it imitated the other type.

ii) For $r_E = y$, it follows from our discussion of the BE of the Cournot game that there is a unique BE under the interim belief $\beta_y^* = 0.5$ with output levels $\frac{17}{149}$, $\frac{4}{35}$, and 0 for N, E with low cost, and E with high cost respectively.

For $r_E = n$, the belief $\beta_n^* = 0$ dictates that N produces the monopoly output level 0.15 and expects the profit $(0.15)^2$. As a best response, E with low cost produces 0.1 and receives the profit $(0.1)^2$, E with high cost produces 0 and receives 0 profit.

iii) N receives the BE expected payoff of $\left(\frac{17}{149}\right)^2$ if it rejects the mechanism. If it accepts, it receives the average of its monopoly profit $(0.15)^2$ and monetary transfer $(0.175)^2 - (0.1)^2$, which is larger than the expected rejection profit. So it is optimal for N to accept.

On the other hand, E with low cost receives $(0.1)^2$ and E with high cost receives 0 regardless of accepting or rejecting the mechanism. So either type is indifferent. Therefore accepting the mechanism with probability one is optimal for E as well.

iv) Both types of E accept the mechanism with probability one. Bayes rule dictates that $\beta_y^* = 0.5$. Since rejection is off the equilibrium path, there is no restriction on β_n^*.

- Mechanism $\mathcal{M}_2$: Message set M_E is singleton, $\chi_N = 0$, $\chi_E = 0.175$, $\tau = \left(\frac{1}{12}\right)^2$.

  In the sequential game following the announcement of this mechanism, there exists a PBE such that

  - $\sigma_N^* = \sigma_E^*(l) = 1$ and $\sigma_E^*(h) = 0$,
  - $\beta_y^* = 1$, $q_N^*(y) = \frac{1}{12}$, $q_E^*(y,l) = \frac{2}{15}$ and $q_E^*(y,h) = 0$,
  - $\beta_n^* = 0$, $q_N^*(n) = 0.15$, $q_E^*(n,l) = 0.1$, and $q_E^*(n,h) = 0$.

Condition (i) of PBE is satisfied trivially since M_E is singleton. We check for the other conditions below:

ii) For $r_E = y$, the belief $\beta_y^* = 1$ implies that $q_N^*(y)$ and $q_E^*(y,l)$ are equal to the unique *complete information* Cournot competition output levels ($\frac{1}{12}$ and $\frac{2}{15}$) when firms have cost levels 0.7 and 0.65. Moreover, $q_E^*(y,h) = 0$ is the dominant output level for E with high cost.

For $r_E = n$, the belief $\beta_n^* = 0$ dictates that N produces the monopoly output level 0.15 and expects the profit $(0.15)^2$. As a best response, E with low cost produces 0.1 and receives the profit $(0.1)^2$, E with high cost produces 0 and receives 0 profit.

iii) By rejecting the mechanism, N receives the average of $\left(\frac{1}{12}\right)^2$ and the monopoly profit $(0.15)^2$. By accepting the mechanism, N guarantees exactly the same payoff. Since N is indifferent, accepting the mechanism with probability one is an optimal ratification behavior.

E with low cost receives $(0.175)^2 - \left(\frac{1}{12}\right)^2$ by accepting the mechanism and $(0.1)^2$ by rejecting it. Since acceptance brings a larger payoff, $\sigma_E^*(l) = 1$ is optimal. E with high cost receives a negative payoff by accepting the mechanism and 0 by rejecting it. Since rejection brings a higher payoff to this type, $\sigma_E^*(h) = 0$ is optimal.

iv) The ratification behavior separates the two types of E. Bayes rule dictates that $\beta_y^* = 1$ and $\beta_n^* = 0$.

- Mechanism $\mathcal{M}_3$: Message set M_E is singleton, $\chi_N = 0.15$, $\chi_E = 0$, $\tau = 0$.

  In the sequential game following the announcement of this mechanism, there exists a PBE such that

  - $\sigma_N^* = \sigma_E^*(h) = 1$ and $\sigma_E^*(l) = 0$,
  - $\beta_y^* = 0$, $q_N^*(y) = 0.15$, $q_E^*(y, l) = 0.1$ and $q_E^*(y, h) = 0$,
  - $\beta_n^* = 1$, $q_N^*(n) = \frac{1}{12}$, $q_E^*(n, l) = \frac{2}{15}$, and $q_E^*(n, h) = 0$.

Condition (i) of PBE is satisfied trivially since M_E is singleton. We check for the other conditions below:

ii) For $r_E = y$, the belief $\beta_y^* = 0$ implies that N produces the monopoly output level 0.15 and expects the profit $(0.15)^2$. As a best response, E with low cost produces 0.1 and receives the profit $(0.1)^2$, E with high cost produces 0 and receives 0 profit.

For $r_E = n$, the belief $\beta_n^* = 1$ dictates that $q_N^*(n)$ and $q_E^*(n, l)$ are equal to the unique *complete information* Cournot competition output levels ($\frac{1}{12}$ and $\frac{2}{15}$) when firms have cost levels 0.7 and 0.65. Moreover, $q_E^*(n, h) = 0$ is the dominant output level for E with high cost.

iii) By rejecting the mechanism, N receives the average of $\left(\frac{1}{12}\right)^2$ and the monopoly profit $(0.15)^2$. By accepting the mechanism, N guarantees exactly the same payoff. Since N is indifferent, accepting the mechanism with probability one is an optimal ratification behavior.

E with low cost receives 0 profit by accepting the mechanism and $\left(\frac{2}{15}\right)^2$ by rejecting it. Since rejection brings a larger payoff, $\sigma_E^*(l) = 0$ is optimal. E with high cost receives 0 payoff regardless of the ratification decision.

iv) The ratification behavior separates the two types of E. Bayes rule dictates that $\beta_y^* = 0$ and $\beta_n^* = 1$.

- Mechanism $\mathcal{M}_4$: Message set $M_E = \{L, H\}$, and

$$\begin{aligned}\chi_N(m) &= \begin{cases} 0 & \text{if } m = L \\ 0.15 & \text{if } m = H \end{cases}, \chi_E(m) = \begin{cases} 0.175 & \text{if } m = L \\ 0 & \text{if } m = H \end{cases}, \\ \tau(m) &= \begin{cases} 6.875 \times 10^{-3} & \text{if } m = L \\ 0 & \text{if } m = H \end{cases}.\end{aligned}$$

  In the sequential game following the announcement of this mechanism, there exists a PBE such that

  - $\sigma_N^* = \sigma_E^*(l) = 1, \sigma_E^*(h) = 0.25$,
  - $m_E^*(l) = L$, and $m_E^*(h) = H$,
  - $\beta_y^* = 0.8$, $q_N^*(y) = 0.1$, $q_E^*(y, l) = 0.125$, and $q_E^*(y, h) = 0$,
  - $\beta_n^* = 0$, $q_N^*(n) = 0.15$, $q_E^*(n, l) = 0.1$, and $q_E^*(n, h) = 0$.

Now we check for the conditions of PBE:

i) Once the mechanism is accepted, the low cost E produces 0.175 and makes a profit of $(0.175)^2 - 6.875 \times 10^{-3} = 2.375 \times 10^{-2}$ by sending the message L. The high cost E produces 0 and makes a profit of 0 by sending the message H. Either type would be worse off if it imitated the other type.

ii) For $r_E = y$, the two firms play the Cournot game under the belief $\beta_y^* = 0.8$. The unique BE here stipulates that N produces 0.1, E with low cost produces 0.125, and E with high cost produces 0.

For $r_E = n$, the belief $\beta_n^* = 0$ dictates that N produces the monopoly output level 0.15 and receives the profit $(0.15)^2$. As a best response, E with low cost produces 0.1 and receives the profit $(0.1)^2$, E with high cost produces 0 and receives 0 profit.

iii) If N rejects the mechanism, what it expects to receive in the Cournot game to follow depends on whether E accepted or rejected the mechanism.

If E rejected (which happens with probability $\frac{1}{2}\left(1-\sigma_E^*\left(h\right)\right)=\frac{3}{8}$), then N receives the monopoly profit $\left(0.15\right)^2$. If E accepted (with probability $\frac{5}{8}$), the BE expected payoff for N is $\left(0.1\right)^2$. On the other hand, by accepting the mechanism, N receives either the monopoly profit $\left(0.15\right)^2$ (if its rival has high cost) or the side transfer 6.875×10^{-3} (if its rival has low cost). The weighted average of profits under acceptance is equal to the weighted average under rejection. Since N is indifferent, accepting the mechanism with probability one is an optimal decision.

Similarly, E with high cost produces 0 and receives 0 profit regardless of accepting or rejecting the mechanism. Therefore randomizing between these options according to $\sigma_E^*\left(h\right)=0.25$ is optimal for this type. Finally, E with low cost receives 2.375×10^{-2} by accepting the mechanism and $\left(0.1\right)^2$ by rejecting it. Since acceptance payoff is larger, E with low cost accepts the mechanism with probability one.

iv) Bayes rule dictates that $\beta_y^*=\frac{\sigma_E^*(l)}{\sigma_E^*(l)+\sigma_E^*(h)}=\frac{1}{1+0.25}=0.8$ and $\beta_n^*=\frac{1-\sigma_E^*(l)}{2-\sigma_E^*(l)-\sigma_E^*(h)}=\frac{0}{0.75}=0.$

7.5 Strategies and the Equilibrium for the Informed Principal Model

One other modeling choice we considered in this paper had player E in the role of an informed principal, offering a mechanism to player N. A mechanism here is defined the same way as in the previous designer-initiated mechanism version of the model. However, since the firms are making different decisions the definitions of their strategies should be revised. We start with E. The first decision this firm has to make is the choice of the mechanism. Let $\mathcal{C}$ be the set of all mechanisms that E can offer. The mechanism choice can be conditioned on the cost level of E :

$$c_E:\{l,h\}\rightarrow\Delta\mathcal{C}.$$

If N accepts the mechanism, E decides what message to send to the mechanism. Recall that the set of available messages is determined by the mechanism choice of E. Let $M_E\left(c_E\right)$ denote the message set for mechanism c_E. The message choice of E can be represented with function

$$m_E:\mathcal{C}\times\{l,h\}\rightarrow\Delta\cup_{c_E\in\mathcal{C}}M_E\left(c_E\right),$$

such that $m_E(c_E, \theta) \in \Delta M_E(c_E)$ for all c_E and θ. If N rejects the mechanism, E decides the production level in the Cournot game, which can still be conditioned on the rejected mechanism:

$$q_E : \mathcal{C} \times \{l, h\} \to \Delta R_+.$$

On the other hand, N decides whether to accept the offered mechanism and what output level to produce in case that it rejects the mechanism:

$$\begin{aligned} \sigma_N &: \mathcal{C} \to [0,1], \\ q_N &: \mathcal{C} \to \Delta R_+. \end{aligned}$$

The relevant belief here is the probability N assigns to the event that E has the low cost after learning the offered mechanism. We denote this probability by function

$$\beta : \mathcal{C} \to [0,1].$$

Now we are ready to define the solution concept for the informed principal version of our model.

Definition 2 *A Perfect Bayesian Equilibrium in the informed principal game is a collection of strategies and beliefs* $(c_E^*, \sigma_N^*, q_N^*, q_E^*, m_E^*, \beta^*)$ *which together satisfy the conditions listed below:*

i) For $\theta = l, h$ *and* $c_E \in \mathcal{C}$, *message (or any message in the support of the randomization)* $m_E^*(c_E, \theta)$ *maximizes the expected profit of* E *with type* θ *among the messages in* $M_E(c_E)$.

ii) For $c_E \in \mathcal{C}$, *functions* $q_N^*(c_E)$ *and* $q_E^*(c_E, \cdot)$ *constitute a BE of the Cournot game under the belief* $\beta^*(c_E)$.

iii) σ_N^* *maximizes the expected continuation payoff of* N, *given the belief* $\beta^*(c_E)$ *and the continuation strategies of both firms* (q_N^*, q_E^*, m_E^*).

iv) $c_E^*(\theta)$ *maximizes the expected continuation payoff of* E *with type* θ, *given the continuation strategies of both firms* $(\sigma_N^*, q_N^*, q_E^*, m_E^*)$.

v) β^* *is derived by the Bayes formula on the equilibrium path given* c_E^*.

There exists a PBE of this game such that

$$\begin{aligned}
c_E^*(l) &= \mathcal{M}_2,\ c_E^*(h) = \mathcal{M}_3, \\
\beta^*(c_E) &= \begin{cases} 1 & \text{if } c_E = \mathcal{M}_2 \\ 0 & \text{otherwise} \end{cases}, \\
\sigma_N^*(\mathcal{M}_2) &= \sigma_N^*(\mathcal{M}_3) = 1, \\
q_E^*(c_E, h) &= 0 \text{ for all } c_E, \\
q_E^*(c_E, l) &= \begin{cases} \frac{2}{15} & \text{if } c_E = \mathcal{M}_2 \\ 0.1 & \text{otherwise} \end{cases}, \\
q_N^*(c_E) &= \begin{cases} \frac{1}{12} & \text{if } c_E = \mathcal{M}_2 \\ 0.15 & \text{otherwise} \end{cases},
\end{aligned}$$

where mechanisms $\mathcal{M}_2$ and $\mathcal{M}_3$ are as defined in the main body of the paper. E's equilibrium path message choice is trivial since both equilibrium path mechanisms have singleton message sets. Nevertheless, the above functions do not constitute a full characterization of the equilibrium since they are silent about N's ratification and E's message choice decisions for off the equilibrium path mechanisms.

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