Dynamic Voluntary Contribution to a Public Project under Time Inconsistency*

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Abstract

We study a general voluntary public good provision model and introduce time inconsistency through β-δ preferences. There is a public project and finitely many agents where each agent is allowed to contribute any amount she likes in any period she likes before the project is completed. The agents have discontinuous preferences over the total contribution with a jump when the project is completed. There is complete information about the environment but imperfect information about others' individual actions: in each period, each agent observes only the total contribution made, not other agents' individual contributions. Assuming the agents are time-inconsistent and sophisticated, we characterize the set of equilibria. We compare the set of equilibrium outcomes under sophisticated time-inconsistent agents to that under time-consistent agents. More importantly, we show that for any given project that is completed in finite time by time-consistent agents, sophisticated time-inconsistent agents complete the project *earlier* than time-consistent agents.

Keywords : public good, dynamic voluntary contribution, time inconsistency, β-δ preferences, sophisticated agent. *JEL* **Classification Numbers:** D03, H41, C72

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1 Introduction

In public projects such as building a library collection, a public recreational facility, a railroad or subway, raising funding can take weeks, months or even years. Such time periods are required for many voluntary contribution plans that finance public projects. This paper addresses a public project/good that is socially desirable in the sense that its total benefit is more than its cost. The classic free-rider problem is present, that is, it is possible to benefit fully from the public good, by contributing little or nothing. When forming a contract over contributions in advance is not possible, a widely studied set of issues consists of whether the project will be completed and if so, how long it will take and what the contribution scheme will be for each agent.

When the nature of the problem has a dynamic component, agents' intertemporal preferences are important. There is evidence showing that agents tend to postpone costly actions, such as finishing a report or filing taxes, but they rarely postpone benefits. Thus, agents sometimes put less emphasis on future benefits and more on the immediate costs, yielding time-inconsistent preferences. In particular, agents can apply a higher discount between the current and the next period than the discount between any other two successive periods, that is, agents' intertemporal preferences may change over time.[1] Such a changing time preference may alter the contribution scheme of an agent. For this reason, above-mentioned issues are worth exploring under time-inconsistent preferences.

In their seminal paper, Marx and Matthews (2000), MM henceforth, consider a dynamic voluntary contribution model and show that there are equilibria where there is provision in a public good game with multiple contribution-rounds.[2] Thus, inefficiencies and the free-rider problem may disappear and the completion delay may vanish. Their theoretical results were later tested in an experiment by Duffy, Ochs, and Vesterlund (2007), where the behavior turned out to be inconsistent with the predictions of MM: the contributions were larger than the predicted contribution levels and the rate of contributions was decreasing over time, unlike MM's predicted contribution streams.[3] We provide a detailed discussion on this discrepancy between the behavior in that experiment and the predictions of MM, in Section 6, where we argue that the contribution levels in the experiment are not consistent with any calibration of a geometric discount factor. Therefore, there is room to consider alternative

[1] Frederick, Loewenstein, and O'donoghue (2002) provide an extensive overview of the literature on time inconsistency. Also see Loewenstein and Prelec (1992) for an extensive survey on anomalies in intertemporal choice.

[2] Also see Bergstrom, Blume, and Varian (1986) and Admati and Perry (1991)

[3] Regarding other experimental results on public good provision and free rider problem, see Smith (1976, 1981) and Marwell and Ames (1979, 1980, 1981) for evidence against free riding and Andreoni (1988) for evidence in favor of free riding. Also see Ledyard (1995) and Kim and Walker (1984).

discounting functions, for instance, β-δ discounting, which we adopt here.

We consider the dynamic voluntary contribution model introduced by MM, and introduce time inconsistency into the model.[4] We analyze the set of equilibria under time inconsistency and compare it to the set of equilibria under time-consistent agents. More importantly, we provide a sensible comparison of the two environments in terms of the number of periods it takes to complete a given project and show that time-inconsistent agents finish a given project earlier than time-consistent agents. Our contribution thus has two dimensions. First, we generalize an existing theory on dynamic public good provision and identify the effects of time inconsistency. Second, by showing that time-inconsistent agents provide the public good earlier/faster than time-consistent agents, we provide a stronger result than the one in MM, namely, even less free riding than expected and even larger contributions in a dynamic setting. Our results can also be interpreted as a contribution where time-inconsistency can help explain some of the behavior in the experiment in Duffy, Ochs, and Vesterlund (2007), since we predict higher contribution levels and a decreasing rate of contributions, which are, relative to MM, more in line with the behavior in their experiment. However, we do not provide an exhaustive explanation of their experimental data.

In our baseline model, each agent can contribute any amount she likes, in any period she likes, and the contributions are non-refundable. At the end of each period, each agent learns the total amount of contributions in that period, but they do not observe the individual contributions of other agents. The cost of the public good is common knowledge. Each agent has a discontinuous benefit function in the sense that there is a benefit jump when the public project is completed. Also, in each period, each agent has a marginal benefit from the total contribution made in that period, even when the public project is not completed.[5] Within this framework, we introduce time inconsistency on agents' intertemporal preferences, through β-δ preferences.[6] An agent with β-δ preferences has a discount factor $\beta\delta$ between the current and the next period, and a discount factor δ between any two future successive periods. If an agent is fully aware of her time inconsistency, she is a *sophisticated* time-inconsistent agent; otherwise, she is a *naïve* agent. We focus on the case of sophisticated agents and investigate the effect of agents' time inconsistency on the set of equilibria and on the equilibrium

[4] Admati and Perry (1991) also provide a similar model, where the contributions are made in a sequential order, unlike in MM.

[5] This marginal benefit can be zero or positive. Both cases are allowed in the model.

[6] Phelps and Pollak (1968) first developed β-δ preferences, which were then used in various settings. Among these settings, see for instance, Laibson (1997) and O'Donoghue and Rabin (1999a,b, 2001, 2008). Also see O'Donoghue and Rabin (2000) for a number of applications based on β-δ preferences, capturing time inconsistency.

number of periods to complete a given project, compared to those with time-consistent agents.

The Nash equilibria and the nearly efficient perfect Bayesian equilibria of the public good provision with time-consistent agents are characterized by MM. One of their main results is that by allowing agents to make contributions slowly over time, efficient outcomes can be constructed and the agents are willing to complete the project. Here we first focus on the set of equilibria under both agent types and compare them. We show that for any given project, any equilibrium outcome for time-consistent agents is also an equilibrium outcome for time-inconsistent agents. We also show that for any project, there is an equilibrium outcome for time-inconsistent agents that is not an equilibrium outcome for time-consistent agents. More importantly, we show that, for any given project that is completed in finitely many periods by time-consistent agents, time-inconsistent agents finish the same project (weakly) earlier than time-consistent agents.[7] Intuitively, since sophisticated agents are aware of their time inconsistency, they tend to guard themselves against their future selves by contributing higher amounts in earlier periods. They therefore reach the total contribution needed for completion of the project earlier than the time-consistent agents. In our main result, when comparing the number of completion periods for both types of agents, we construct the fastest equilibria for the two environments and compare them.[8] We also show that, in a symmetric environment, there are projects that are completed by time-inconsistent agents in finitely many periods, but not completed by the corresponding time-consistent agents.

The voluntary public good provision problem and its efficiency properties have been widely studied. In a sequential contribution game under complete information where the amount of the public good to be provided is continuous, Varian (1994) shows that the ability of the first mover to credibly commit to a certain level of contribution aggravates the free rider problem. Bag and Roy (2011), however, show that, under incomplete information, a sequential contribution mechanism may perform better than a simultaneous contribution game in terms of total expected contributions. Admati and

[7] Here there are a number of different time-consistent agents to which we can compare time-inconsistent agents. Following Chade, Prokopovych, and Smith (2008), for a given number of periods t, we consider a time-consistent agent with a discount factor $\hat{\delta}_t$, such that

$$1 + \hat{\delta}_t + \hat{\delta}_t^{\,2} + ... + \hat{\delta}_t^{\,t} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$$

Thus, for a given δ and β, there is a discrete set of corresponding time-consistent agents with a discount factor, depending on the number of periods, $\{\hat{\delta}_t\}_{t=2}^{\infty}$. We provide a general comparison by considering any possible $\hat{\delta}_t$ with $t = 2, ..., \infty$.

[8] In the Discussion and Conclusion section we discuss the efficiency induced in terms of how fast the project is completed. We argue that with time-inconsistent agents, for a given project, the equilibrium that completes the project fastest is efficient, that is, delay is welfare-reducing under certain conditions. In the same section, we also discuss other issues such as multiple equilibria, the case of $\beta > 1$, perfect Bayesian equilibria, and environments where both time-consistent and time-inconsistent agents are present.

Perry (1991) consider an alternating contribution game under both full-refund and no-refund cases. They investigate whether efficient equilibria still exist if the contributions are divided into small sums over time. The answer is negative; each of the two players has an incentive to let the other player contribute in the future. Fershtman and Nitzan (1991) also consider a dynamic public good provision problem with flow benefits. They too get negative results, which hinge on the fact that a player can sometimes increase the level of future contribution by lowering the current contribution, and this is a potential incentive for her to free ride on the future contributions of other players. On the contrary, if there is imperfect information about individual actions (aggregate contribution is observed but not the individual contributions) and if players can contribute more than once and at any time, efficiency is achieved under certain conditions, as MM show.[9] Compte and Jehiel (2003) introduce agent asymmetries to the model of Admati and Perry (1991) and show that their main result is not robust. In a model of private provision of public goods, Bagnoli and Lipman (1989) demonstrate that the set of undominated perfect equilibrium outcomes is identical to the core. The private provision mechanism for a discrete public good through both a contribution game and a subscription game that is studied by Barbieri and Malueg (2008a,b) yielded some interim inefficiency results. Barbieri and Malueg (2010) restrict attention to piecewise-linear equilibrium and show that a subscription game achieves the outcome of the optimal mechanism for a profit maximizing seller. Georgiadis (2015) shows, through a contracting problem in which a manager hires a team of agents to undertake a project, that the optimal contract compansates the agents when the project is completed.[10]

Time-inconsistent behavior, captured through β-δ preferences, has also been studied widely in various contexts, including ones where focus is on individual decision-making and those where focus is on strategic interaction. O'Donoghue and Rabin (1999a, 2000, 2001, 2008) focus on individual decision-making under β-δ preferences, with no strategic consideration. On the other hand, O'Donoghue and Rabin (1999b), DellaVigna and Malmendier (2004), Gilpatric (2008) and Yılmaz (2013, 2015) focus on contractual relationships under β-δ preferences. O'Donoghue and Rabin (1999b) introduce a moral hazard problem in the form of unobservable task-cost realizations and assume that the agent is risk-neutral. Gilpatric (2008) focuses on a contracting problem with time-inconsistent agents, assuming that profit is fully determined by the effort. DellaVigna and Malmendier (2004) consider a monopo-

[9] The horizon must be long, the players must have similar preferences and they must be patient enough.

[10] Also see Bliss and Nalebuff (1984), Laussel and Palfrey (2003), Ledyard (1995) and Ledyard and Palfrey (1999, 2002, 2007), Lu and Quah (2009), Morelli and Vesterlund (2000) for more on efficiency properties of the outcomes of the private provision of public goods.

listic firm that faces time-inconsistent agents for which they design an optimal two-part tariff. Yılmaz (2013) considers a repeated moral hazard problem with the standard trade-off between risk and incentives, and characterizes the optimal contract. Chade, Prokopovych, and Smith (2008) study infinitely repeated games where players have β-δ preferences. They characterize the equilibrium payoffs and show that the equilibrium payoff set is not monotonic in β or δ. Bisin, Lizzeri, and Yariv (2015) show that collective action may cause individual self-control problems to get bigger leading to excessive government debt.[11] To the best of our knowledge, time-inconsistent preferences have not been studied in a dynamic voluntary contribution problem. Our paper contributes to the literature in this aspect.

Section 2 provides the details of the model. We carry out the equilibrium analysis in Section 3. Section 4 provides a comparison of the set of equilibria of the two agent types. Section 5 provides a comparison on the equilibrium number of periods to complete a given project, together with some comparative statics, and Section 6 concludes and discusses a number of extensions, including heterogeneous environments, naivéte and continuous benefit function. Some of the proofs are given in the Appendix.

2 The Model

There are $n \geq 2$ players and a public good with a cost $\overline{X} > 0$. In each period $t \geq 0$, each player decides how much to contribute for the public good. Player i's contribution in period t is $z_i(t)$. Let $z(t) \equiv (z_1(t), ..., z_n(t))$ denote the contribution vector in period t. Let $z \equiv z(t)_{t=0}^{\infty}$ be the entire contribution sequence. The aggregate contribution in period t is given by $Z(t) \equiv \sum_{j=1}^{n} z_j(t)$. Each player can observe her own past contributions and the aggregate contribution of the other players' past contributions in each period. Hence, for player i, the history in the beginning of the period t is $h_i^{t-1} \equiv (z_i(\tau), Z_i(\tau))_{\tau=0}^{t-1}$ where $Z_i(\tau) \equiv Z(\tau) - z_i(\tau)$. There is a contributing horizon $\overline{T} \leq \infty$ such that contributions are not allowed after $\overline{T}$, and $z(\cdot)$ is called *feasible* if $z_i(t) \geq 0$ for $t \leq \overline{T}$ and $z_i(t) = 0$ for $t > \overline{T}$. A strategy for player i maps each h_i^{t-1} into a contribution, $z_i(t)$, that is feasible in the following period. Agents have infinite amount of private good to contribute, that is, the budget constraints are not binding. Also, the contributions are non-refundable.

Given a feasible contribution sequence, z, the completion period $T^*(z)$ is the smallest t with

[11] There are other related papers that examine time inconsistency in other contexts. These include Akin (2009, 2012), Sarafidis (2006) and Strotz (1955).

$\sum_{\tau=0}^{t} Z(\tau) \geq \overline{X}$. A contribution sequence $z(\cdot)$ is called *wasteless* if $Z(t) = 0$ for all $t > T^*(z)$ where $T^*(z)$ is the completion period under the sequence $z(\cdot)$. The cumulative contribution for player i at the end of period t is $x_i(t) \equiv \sum_{\tau=0}^{t} z_i(\tau)$. The aggregate cumulative contribution; *cumulation*, is $X(t) \equiv \sum_{j=1}^{n} x_j(t)$. Player i's total benefit from the project at any period t is determined by the the cumulation, $X(t)$, through $f_i(X(t))$, where f_i is the *benefit function*. As in MM, given $X(t) > 0$, if $f_i(X(t)) > 0$, the project generates a benefit in period t, even if $t < T^*$.[12] Alternatively, if $f_i(X(t)) = 0$ for all $t < T^*$, benefits are not generated until the project is completed. This is the case of a binary project such as the building of a bridge. We consider the following benefit function used by MM.

$$f_i(X) = \begin{cases} \lambda_i X & X < \overline{X} \\ V_i & X \geq \overline{X} \end{cases}$$

where λ_i is player i's marginal benefit from a non-completing aggregate cumulative contribution, V_i is the benefit for player i from the completed project.

Let b_i be the benefit jump received upon completion and every period following the completion period, that is $b_i \equiv V_i - \lambda_i \overline{X}$ as shown in the figure below.

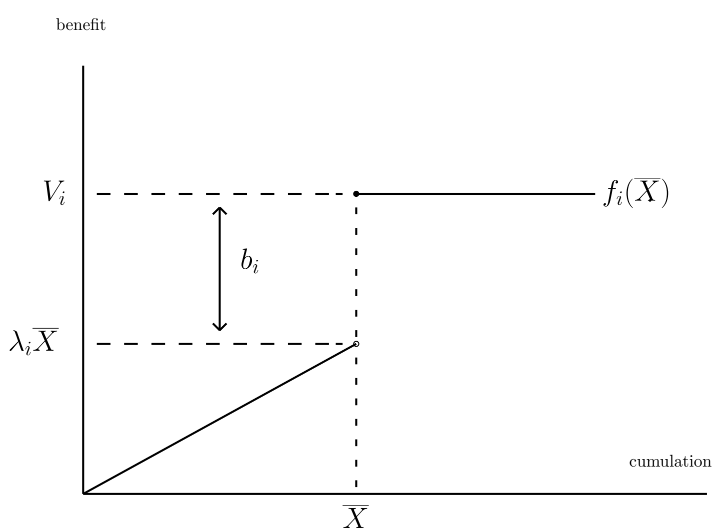

Figure 1. Benefit function.

We assume $1 > \lambda_i \geq 0$, and $b_i > 0$, which implies $0 \leq \lambda_i \leq V_i/\overline{X}$ for all $i \in N$. Note that $\lambda_i = 0$ yields the *binary benefit function* and $b_i = 0$ yields the *continuous benefit function*.

[12] As in MM, we abuse notation and write T^* instead of $T^*(z)$.

The cost function enters quasi-linearly and the cost of a contribution is borne at the period when it is made.

We focus on the case where no agent is willing to complete the project alone, but it's efficient to provide the public good. That is, for any player i, we have

$$V_i < \overline{X} < \sum_{j=1}^{n} V_j.$$

Players have β-δ preferences, that is, each player, at any period, discounts benefits and costs according to the discounting scheme $(1, \delta\beta, \delta^2\beta, \delta^3\beta, ..., \delta^t\beta, ...)$. $\beta\delta$ is the discount factor between the current period and the next period, however the discount factor between two adjacent periods in the future is δ. The agent is *time-inconsistent* when $\beta < 1$. A time-inconsistent agent can be fully aware, partially aware or fully unaware of his time inconsistency. Denote the agent's belief about his true β by $\widehat{\beta}$. As in the literature, a time-inconsistent agent is *sophisticated* when he is fully aware of his time inconsistency, that is, when $\widehat{\beta} = \beta < 1$. The agent is *partially naive* when $\beta < \widehat{\beta} < 1$ and *fully naive* when $\beta < \widehat{\beta} = 1$. The agent is *naive* when $\beta < \widehat{\beta} \leq 1$.

In this paper, we assume that the time-inconsistent agents are fully aware of their inconsistencies, that is, they are assumed to be **sophisticated**. We also assume that all agents are aware of the time-inconsistency of other agents, more specifically, we assume that it is common knowledge that every agent has β and all are sophisticated. The analysis with this specification is already cumbersome, thus we leave the analysis with **naïve** agents to be conducted in another paper. We do, however, discuss the case of naivete in the Discussion and Conclusion section.

When $\beta = 1$, the agent is *time-consistent* and the model becomes identical to the one in MM, which, thus, becomes a special case of our model. In MM, for a given feasible contribution sequence, z, player i with a discount factor $\hat{\delta}$, has a present discounted overall net payoff given by[13]

$$U_i(z) \equiv \sum_{t=0}^{\infty} \hat{\delta}^t [(1 - \hat{\delta}) f_i(X(t)) - z_i(t)]$$

Note that, the benefit function, not the costs, within each period is scaled by $(1 - \hat{\delta})$. This makes it possible to write the overall payoff in terms of period specific total contributions, rather than the cumulations. With the f_i specified above, the completion period of the project being T^*, and the

[13]We will be comparing the environment with time-inconsistent agents to the environment with time-consistent agents who have a discount factor, $\hat{\delta}$, which is not necessarily equal to δ.

contribution sequence z being both feasible and wasteless, this payoff can be written as

$$U_i(z) \equiv \sum_{t=0}^{T^*} \hat{\delta}^t[\lambda_i Z(t) - z_i(t)] + \hat{\delta}^{T^*} b_i$$

For our comparison of time-inconsistent environment and time-consistent environment to be reasonable, as in MM we also scale the benefit (not the costs) that a time-inconsistent agent gets in each period by the same scalar, $(1-\hat{\delta})$, which is used in the payoff function of a time-consistent agent with discount factor $\hat{\delta}$. When $\beta < 1$, for a given feasible contribution sequence, z, player i's present discounted overall net payoff is

$$U_i(z) \equiv (1-\hat{\delta}) f_i(X(0)) - z_i(0) + \sum_{t=1}^{\infty} \delta^t \beta[(1-\hat{\delta}) f_i(X(t)) - z_i(t)]$$

With the f_i specified above, the completion period of the project being T^*, and the contribution sequence z being both feasible and wasteless, this overall payoff, starting from period 0, can be written as

$$U_i(z,0) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i Z(0) - z_i(0) + \sum_{t=1}^{T^*} \delta^t \beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i Z(t) - z_i(t)] + \delta^{T^*}\beta(\frac{1-\hat{\delta}}{1-\delta})b_i$$

The discounted overall payoff, starting from a period $t < T^*$ is given by

$$U_i(z,t) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))\lambda_i Z(t) - z_i(t) + \sum_{\tau=t+1}^{T^*(z)} \delta^{\tau-t}\beta[(\frac{1-\hat{\delta}}{1-\delta})\lambda_i Z(\tau) - z_i(\tau)] + \delta^{T^*(z)-t}\beta(\frac{1-\hat{\delta}}{1-\delta})b_i$$

And the discounted overall payoff, starting from period $t = T^*(z)$ is given by

$$U_i(z,T^*) \equiv (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))[\lambda_i \overline{X} + b_i] - z_i(T^*)$$

Denoting $K_{\hat{\delta}} = \frac{1-\hat{\delta}}{1-\delta}$, and $\Delta = 1-\delta(1-\beta)$, the payoff function will be

$$U_i(z,t) = \begin{cases} K_{\hat{\delta}}\Delta\lambda_i Z(t) - z_i(t) + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i Z(\tau) - z_i(\tau)] + \delta^{T^*-t}\beta K_{\hat{\delta}} b_i & 0 \le t < T^* \\ K_{\hat{\delta}}\Delta[\lambda_i \overline{X} + b_i] - z_i(T^*) & t = T^* \end{cases}$$

The introduction of β-δ discounting together with the benefit function being appropriately scaled by $(1-\hat{\delta})$ changes the payoff function in a way that from any period t on, the next period's benefits and the costs are discounted to today by $\beta\delta$, but every other period is discounted by δ to the pervious one, again from period t's perspective. When arrived at the next period $t+1$, now the next period's benefits and costs are discounted to $t+1$ by only δ. This is the heart of the change in the agents' evaluation of costs and benefits compared to the time-consistent agents, who discount every period to the previous one by the same discount factor, regardless of the period they are looking at it. Also, note that when $\beta = 1$, the utility function above reduces to the one for the time-consistent agents with a discount factor δ.[14] In the Appendix, we provide details on how to get the utility function above, when β-δ discounting is introduced together with the benefit function scaled by $(1-\hat{\delta})$.

3 Equilibrium Analysis

As in MM, we first consider the static version of the game which we use to construct the set of equilibria in the dynamic version. A strategy profile in the static game, $(z_1, z_2, ..., z_n)$, yields an aggregate contribution $Z = \sum_{i=1}^{n} z_i$, and player i receives payoff $f_i(Z) - z_i$. Denote the total contribution of other players by $Z_i = Z - z_i$. Player i's best response to $Z_i < \overline{X}$, is either to finish the project by contributing the rest of the amount required, that is, $\overline{X} - Z_i$; or to contribute nothing.[15] The marginal benefit from completing the project is $f_i(\overline{X}) - f_i(Z_i) = V_i - \lambda_i Z_i$, and the marginal cost of completing the project is $\overline{X} - Z_i$. Marginal benefit is larger than the marginal cost if and only if the amount needed to complete the project is less than the *critical contribution*, which is the maximum amount that player i is willing to contribute. To get this critical contribution, we solve the equality $\overline{X} - Z_i = V_i - \lambda_i Z_i$ for $\overline{X} - Z_i$. That is, $\overline{X} - Z_i = V_i - \lambda_i Z_i + \lambda_i \overline{X} - \lambda_i \overline{X} = V_i + \lambda_i(\overline{X} - Z_i) - \lambda_i \overline{X}$, which implies $(1-\lambda_i)\overline{X} - Z_i = V_i - \lambda_i \overline{X}$. Thus, we get

$$c_i^* \equiv \overline{X} - Z_i = \frac{V_i - \lambda_i \overline{X}}{1-\lambda_i} = \frac{b_i}{1-\lambda_i}$$

The reaction function of player i, for $Z_i < \overline{X}$, is given by[16]

$$z_i^{BR}(Z_i) = \begin{cases} 0 & \overline{X} - Z_i > c_i^* \\ \overline{X} - Z_i & \overline{X} - Z_i \leq c_i^* \end{cases}$$

[14] With $\beta = 1$, we have $\Delta = 1$ and the corresponding time-consistent agent becomes the one with discount factor δ, thus we also have $K_{\hat{\delta}} = 1$, which will be clear in Section 4.

[15] Intermediate amounts are dominated, because the marginal benefit, λ_i, is less than 1.

[16] Note that, whenever $b_i = 0$, that is, whenever there is no benefit jump, the critical contribution level is zero, $c_i^* = 0$. So, contributing nothing is a dominant strategy when $b_i = 0$.

In the dynamic version of the game, let $g = (g_1(t), ..., g_n(t))_{t=0}^{\infty}$ be a feasible sequence of nonnegative contributions. Then, the corresponding aggregate contribution in period t is $G(t) \equiv \sum_{i=1}^{n} g_i(t)$ and the aggregate of all players' contributions other than player i's contribution in period t is $G_i(t) \equiv G(t) - g_i(t)$.

As in MM, for g to be a candidate equilibrium outcome, it has to be $feasible$ and $wasteless$. Recall, g is feasible if $g_i(t) \geq 0$ is for $t \leq \overline{T}$ and $z_i(t) = 0$ for $t > \overline{T}$, where $\overline{T}$ is the contributing horizon. g is wasteless if $G(t) = 0$ for all $t > T^*(g)$ where $T^*(g)$ is the completion period under the sequence g. We consider the strategies with maximal punishments, that is, the $grim - g$ strategy profile in which g is played in each period as long as $G(t)$ is observed, otherwise no player ever contributes again. Then, g is a Nash equilibrium outcome if and only if the $grim - g$ profile is a Nash equilibrium.

MM provide a necessary and sufficient condition, $under - contributing$ constraint, for players not to deviate to contributing zero as long as no deviation has occurred yet, when all players are time-consistent with a discount factor $\hat{\hat{\delta}}$, which is given by

$$\lambda_i G_i(t) \leq \hat{\delta}^{T^*-t} b_i + [\lambda_i G(t) - g_i(t)] + \sum_{\tau=t+1}^{T^*} \hat{\delta}^{\tau-t} [\lambda_i G(\tau) - g_i(\tau)]$$

for all $i \in N$ and $t \leq T^*$.

In our setting, in a feasible and wasteless sequence of non-negative contributions, $g = (g_1(t), ..., g_n(t))_{t=0}^{\infty}$, a time-inconsistent player i does not deviate to contributing zero at a given $t < T^*$, if

$$U_i(g, t) \geq K_{\hat{\delta}} \Delta \lambda_i G_i(t)$$

where the right hand side is the discounted payoff (from period t on) when every other player sticks to the grim-g profile, and player i deviates to no contribution at t, where $G_i(t) \equiv G(t) - g_i(t)$. This condition is rewritten as

$$K_{\hat{\delta}} \Delta \lambda_i G_i(t) \leq \delta^{T^*-t} K_{\hat{\delta}} \beta b_i + K_{\hat{\delta}} \Delta \lambda_i G(t) - g_i(t) + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [K_{\hat{\delta}} \lambda_i G(\tau) - g_i(\tau)] \tag{1}$$

for all $i \in N$ and for $t < T^*$. If player i deviates to zero in period t (under-contributing), her benefit net of the benefits from the previous contributions is given by the left hand side. The terms on the right hand side constitute her benefit from the current period t on and thereafter, net of previous benefits resulting from the earlier contributions, discounted appropriately via inconsistent discounting. Given that the other players do not deviate (play $grim - g$ strategies), player i prefers to contribute according to g in period t and thereafter, rather than to deviate to zero, if and only if (1) holds. Rearrangement of (1) yields

$$[1 - \lambda_i K_{\hat{\delta}} \Delta] g_i(t) \leq \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [K_{\hat{\delta}} \lambda_i G(\tau) - g_i(\tau)] \tag{2}$$

for all $i \in N$ and $t < T^*$, which is the $under - contributing$ constraint, and it guarantees that players do not deviate downward, that is, they do not free ride.

Introduction of β-δ discounting changes the under-contributing constraint in a way that the continuation payoff when the agent does not deviate is discounted with β as well, whereas the current net cost of not deviating is not discounted with β. This is intuitive since any comparison between the current period and the very next period, from the current period's perspective will induce an extra discounting with β. The factor $K_{\hat{\delta}}$ emerges due to the scaling of benefit function by $1 - \hat{\delta}$, which naturally appears only in front of the benefit terms.

When $t = T^*$, this condition becomes $U_i(g, T^*) \geq K_{\hat{\delta}} \Delta \lambda_i G_i(T^*)$, that is, $K_{\hat{\delta}} \Delta [\lambda_i \overline{X} + b_i] - g_i(T^*) \geq K_{\hat{\delta}} \Delta \lambda_i G_i(T^*)$, equivalently,

$$[1 - \lambda_i \Delta K_{\hat{\delta}}] g_i(T^*) \leq \Delta K_{\hat{\delta}} b_i \tag{3}$$

where the left-side of (4) is player i's net cost of contributing $g_i(t)$ and the right-side is interpreted as the payoff she gives up by not contributing $g_i(t)$.[17]

There is also another incentive for players to deviate, which is to complete the project prematurely. This upward deviation incentive is deterred by another constraint, the $over - contributing$ constraint. In MM, this is given by

$$(\lambda_i - 1) \left(\overline{X} - \sum_{\tau=0}^{t} G(\tau) \right) + b_i \leq \hat{\delta}^{T^*-t} b_i + \sum_{\tau=t+1}^{T^*} \hat{\delta}^{\tau-t} [\lambda_i G(\tau) - g_i(\tau)]$$

for all $i \in N$ and $t < T^*$. In our setting, the corresponding constraint is given by

$$(\lambda_i K_{\hat{\delta}} \Delta - 1) \left(\overline{X} - \sum_{\tau=0}^{t} G(\tau) \right) + \Delta K_{\hat{\delta}} b_i \leq \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \tag{4}$$

for all $i \in N$ and $t < T^*$. This constraint guarantees that player i does not find it profitable to

[17]For details on how to get the under-contributing constraint, please see the Appendix.

contribute the extra amount to complete the project before it is supposed to be finished. The left hand side is the extra payoff from completing the project by contributing $g_i(t) + \overline{X} - \sum_{\tau=0}^{t} G(\tau)$, on top of what she gets when she contributes according to g. The right hand side is the continuation payoff she loses if she deviates.[18]

Now, we show that these two constraints, over and under contributing constraints, are necessary and sufficient for a grim-g outcome g to be a Nash equilibrium outcome. That is, these two conditions are enough to not just deter two kinds of deviations but also deter any other kinds of deviations. This is shown in MM for time-consistent agents. We modify their proof to accommodate time inconsistency.

Theorem 1 *Assume $1 > \lambda_i \Delta K_{\hat{\delta}}$ for all i. Then, a grim-g outcome g is a Nash equilibrium outcome if and only if it satisfies over-contributing and under contributing constraints, in the time-inconsistent environment.*

Proof. See the Appendix. ■

MM also show that the over-contributing constraint is implied by the under-contributing constraint under certain conditions. We show that this result still holds in our model with time-inconsistent agents.

Corollary 1 *Assume $1 > \lambda_i \Delta K_{\hat{\delta}}$ for all i. Then, for a candidate outcome g, let T^* be the the number of periods the project is completed under the grim-g profile. If g satisfies under-contributing constraint, then it also satisfies over-contributing constraint if either (i) $T^* = 0$ or (ii) $c_i^* = 0$ for all i or (iii) $T^* < \infty$ and $g_i(T-1) + G(T) \geq c_i^*$ for all i.*

Proof. See the Appendix.[19] ■

The condition we assume in each of the two results above, $1 > \lambda_i \Delta K_{\hat{\delta}}$, makes sure not only that the best possible non-completing deviation is 0 (used in the proof), but also that the critical contribution levels defined later in Section 4 are well-defined. The reason this condition emerges is both due to the scaling of the benefits by $1-\hat{\delta}$ and the β-δ discounting. It is explicitly written as $1 > \lambda_i \frac{1-\hat{\delta}}{1-\delta}(1-\delta+\delta\beta))$, where $\hat{\delta}$ is the discount factor of the time-consistent agent we will base our comparison on. For a given $\hat{\delta}$, when β is smaller, that is, when present bias is larger, the right hand side of the condition becomes smaller. Thus, the larger the present bias, the easier the condition holds.[20]

[18]The change in the over-contributing constraint when we introduce β-δ discounting is similar to the one explained above for the change in under-contributing constraint.

[19]The proofs of Theorem 1 and Corollary 1 are similar to the ones in MM. We modify them to account for the β-δ discounting.

[20]In Section 4 below, we will define a discount factor for a time-consistent agent, which will correspond to the average

4 Comparing the Equilibria

We are interested in the comparison of equilibrium outcomes under time consistency and those under time inconsistency, in terms of how fast they finish a given project. To do this, we restrict attention to the projects completed in finite time by both type of agents. For such an equilibrium to exist when players are time-consistent, MM provide sufficient conditions: some player must have a discontinuous benefit function, the contributing horizon, $\overline{T}$, must be long enough and the discount factor must be large enough. We assume $b_i > 0$ for all i, thus, each player has a discontinuous benefit function. We also focus on infinite horizon, that is, $\overline{T} = \infty$. Therefore, the first two conditions are automatically met in the current setting. Regarding the last condition, we implicitly assume that time-consistent agents we compare to have a high enough discount factor, $\hat{\delta}$.[21] Thus, we will not focus on this existence result in our environment, instead we take it as given by assuming that these conditions hold, and then compare the number of periods time-inconsistent agents finish a given project with that for time-consistent agents.

We construct the equilibrium profile recursively; as in MM, starting with the completion period T^*, in which the under-contributing constraint is $g_i(T^*) \leq \frac{\Delta K_{\hat{\delta}}}{1-\lambda_i \Delta K_{\hat{\delta}}} b_i = c_i^*$ for player i. Define $c_i(0) \equiv c_i^*$. Given that $(c_1(0), ..., c_n(0))$ is contributed in period T^*, then binding under-contributing constraints give us a sequence $c_i(k)_{k=0}^{\infty}$ for each i where

$$c_i(0) = \frac{\Delta K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} b_i$$

$$c_i(1) = \frac{\delta \beta \lambda_i K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(0) + \frac{\delta(1-\beta)}{1 - \lambda_i \Delta K_{\hat{\delta}}} \frac{\delta \beta}{\Delta} c_i(0)$$

$$c_i(2) = \frac{\delta \beta \lambda_i K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(1) + \frac{\delta(1-\beta)}{1 - \lambda_i \Delta K_{\hat{\delta}}} [1 - \lambda_i K_{\hat{\delta}}(1-\delta)] c_i(1)$$

and

$$c_i(k) = \frac{\delta \beta \lambda_i K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} \sum_{j \neq i} c_j(k-1) + \frac{\delta(1-\beta)}{1 - \lambda_i \Delta K_{\hat{\delta}}} [1 - \lambda_i K_{\hat{\delta}}(1-\delta)] c_i(k-1)$$

discount factor of the sophisticated time-inconsistent agent. This corresponding discount factor will depend on the length of the horizon we pick. If the horizon is infinite, then we will have $\Delta K_{\hat{\delta}} = 1$ (see the proof of Lemma 2), and our condition above will always hold since $\lambda_i < 1$ for any i.

[21] We are more explicit about this condition below, after we define the discount factor $\hat{\delta}$, which we use for the comparison.

for all $T^* - 1 \geq k \geq 2$. Equivalently,

$$c_i(0) = \frac{\Delta K_{\hat{\delta}}}{1 - \lambda_i \Delta K_{\hat{\delta}}} b_i \quad \text{and} \quad c_i(1) = \frac{\delta\beta}{1 - \lambda_i \Delta K_{\hat{\delta}}} \left[\lambda_i K_{\hat{\delta}} \sum_{j \neq i} c_j(0) + \frac{\delta(1-\beta)}{1 - \delta(1-\beta)} c_i(0) \right]$$

$$c_i(k) = \frac{\delta\beta}{1 - \lambda_i \Delta K_{\hat{\delta}}} \left[\lambda_i K_{\hat{\delta}} \sum_{j \neq i} c_j(k-1) + \frac{\delta(1-\beta)}{\delta\beta} [1 - \lambda_i K_{\hat{\delta}}(1-\delta)] c_i(k-1) \right]$$

for all $T^* - 1 \geq k \geq 2$.

When we introduce β-δ preferences, an interesting feature emerges: the current critical contribution level of agent i, $c_i(k)$, depends on the next period critical level of the same agent, $c_i(k-1)$. This is not present in the time-consistent case. An intuition for this change is that when the agent is time-inconsistent and sophisticated, the agent is playing against other players and also against her own future self.

In an equilibrium outcome g^*, in the very first period, t=0, each player i contributes a fraction of c_i^* and in every other period $t > 0$, each player i contributes the amount $c_i(T^* - t)$.[22] Thus,

$$g_i^*(t) \equiv \begin{cases} \frac{\overline{X} - R(T^*-1)}{R(T^*) - R(T^*-1)} c_i(T^*) & \text{for } t = 0 \\ c_i(T^* - t) & \text{for } 0 < t \leq T^* \\ 0 & \text{for } t > T^* \end{cases}$$

where $R(k) = \sum_{\tau=0}^{k} \sum_{i \in N} c_i(\tau)$. Note that, $g^*(t)$ satisfies the under-contributing constraint by construction. If T^* is finite, then since $g_i^*(T^*-1) + G^*(T^*) \geq c_i^* = c_i(0)$ for all i, Corollary 1 implies that $g^*(t)$ also satisfies the over-contributing constraint. Thus, it is a Nash equilibrium outcome.

Now to proceed to the comparison of the outcome with time-inconsistent agents and the one with time-consistent agents, we consider plausible discount factors for time-consistent agents. We follow the analysis in Chade, Prokopovych, and Smith (2008) to find the time-consistent agent who has a discount factor that corresponds to the average discount factor of the sophisticated time-inconsistent agent. That is, for a given number of periods t, we consider a time-consistent agent with a discount

[22] Player i is willing contribute at most $c_i(k)$ in period $T^* - k$, provided that each player j contributes exactly $c_j(T^* - \tau)$ in period $\tau > t$. In the equilibrium we construct, player i contributes these critical amounts, $c_i(k)$, for every period $T^* - k$, except in the very first period. In the first period, period 0, players may need to contribute less. When each player contributes $c_i(\tau)$ in period $T^* - \tau$, the remaining total contribution to be made from $T^* - k$ on is given by $R(k) = \sum_{\tau=0}^{k} \sum_{i \in N} c_i(\tau)$. Thus, at $t = 0$, the remaining amount of contributions, $R(T^*)$, must be at least as big as $\overline{X}$. That is, T^* satisfies $R(T^* - 1) < \overline{X} \leq R(T^*)$.

factor $\hat{\delta}_t$, such that

$$1 + \hat{\delta}_t + \hat{\delta}_t^{\,2} + ... + \hat{\delta}_t^{\,t} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$$

Thus, for a given δ and β, there is a discrete set of corresponding time-consistent agents with discount factor depending on the number of periods, $\{\hat{\delta}_t\}_{t=1}^{\infty}$. Note that when $t = \infty$, we have $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta+\delta\beta} = \frac{\delta\beta}{\Delta}$. And when $t = 2$, $\hat{\delta}_2 = \delta\beta$. Thus, for any $t > 2$ we have $\delta\beta < \hat{\delta}_t < \frac{\delta\beta}{\Delta}$, where $\Delta = 1 - \delta(1-\beta) < 1$.

Even in an equilibrium which completes the project in finitely many periods, V_i is received every period following the completion period. Thus, the relevant corresponding time-consistent agent seems to be the one with $\hat{\delta}_\infty$. Nevertheless, we carry out our comparison with respect to any possible corresponding time-consistent agent who has the discount factor $\hat{\delta}_t$ with $t = 2, ..., \infty$. Thus, we not only provide a comparison using $\hat{\delta}_\infty$, but also any possible $\hat{\delta}_t$.

The condition that is needed to ensure the existence of a completing equilibrium in the time-consistent environment is $\hat{\delta} > \delta^*$, where δ^* is the threshold discount factor provided in MM. To meet this condition we assume that δ and β are such that $\delta\beta \geq \delta^*$. This ensures that $\hat{\delta}_t > \delta^*$ for any $t > 2$.

Assumption 1 *$\delta^* \leq \delta\beta$.*[23]

Now, we prove two useful lemmas. Recall $K_{\hat{\delta}} = \frac{1-\hat{\delta}}{1-\delta}$.

Lemma 1 *$\hat{\delta}_t$ is strictly increasing in t.*

Proof. See the Appendix. ■

Lemma 2 *For any $\hat{\delta}_t$ with $t \leq \infty$, $\Delta K_{\hat{\delta}_t} \geq 1$.*

Proof. See the Appendix. ■

Before we carry out the main comparison on how fast a project is completed in each environment, we prove two useful propositions relating the set of equilibria under each environment. We show that any equilibrium outcome for the time-consistent agents (with any discount factor, $\hat{\delta}_t$) is also an equilibrium outcome for the sophisticated time-inconsistent agents. Moreover, there are strategy profiles which are Nash equilibria under the setting with time-inconsistent agents while they are not under the time-consistent setting with a discount factor equal to the average discount factor of the

[23] Alternatively, in the comparison, if we use time-consistent agents with the discount factor $\hat{\delta}_\infty$, then the following weaker assumption suffices: $\delta^* \leq \frac{\delta\beta}{1-\delta+\delta\beta}$. Also, note that $\hat{\delta}_t > \delta\beta$ for all $t > 2$. Thus, $\hat{\delta}_t > \delta^*$ for all $t > 2$.

sophisticated time-inconsistent agent. Note that since an outcome g is a Nash equilibrium outcome if and only if the $grim - g$ profile is a Nash equilibrium, we can simply focus on $grim - g$ profiles. Then, by Theorem 1, any contribution scheme will be a Nash equilibrium outcome if it satisfies under-contributing constraints and over-contributing constraints.

Proposition 1 *For all $\overline{X}$,δ,β and $\hat{\delta}_k$, any equilibrium outcome $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ for the time-consistent agents with discount factor $\hat{\delta}_k$, is also an equilibrium outcome for the time-inconsistent agents.*

Proof. Let $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ be an arbitrary equilibrium outcome for a given project size $\overline{X}$ and $\hat{\delta}_k$, for time-consistent agents with discount factor $\hat{\delta}_k$. Suppose g completes the project in T^* periods. Note that, a contribution scheme g is a Nash equilibrium outcome if and only if the grim-g is a Nash equilibrium strategy profile. Thus, the grim-g is a Nash equilibrium strategy profile. Then, the under contributing constraints must be satisfied. That is,

$$(1-\lambda_i)g_i(t) \leq \hat{\delta}_k^{T^*-t}b_i + \sum_{\tau=t+1}^{T^*} \hat{\delta}_k^{\tau-t}(\lambda_i G(\tau) - g_i(\tau))$$

is satisfied for all $t \in \{0, 1, 2, .., T^* - 1\}$, and for period T^*, we have

$$g_i(T^*) \leq \frac{b_i}{1-\lambda_i}$$

Now, we check whether this sequence g satisfies the under contributing constraints of time-inconsistent agents. For period T^*, the condition is $[1 - \lambda_i \Delta K_{\hat{\delta}}]g_i(T^*) \leq \Delta K_{\hat{\delta}} b_i$ which is satisfied since $\frac{b_i}{1-\lambda_i} \leq \frac{\Delta K_{\hat{\delta}_t} b_i}{1-\lambda_i \Delta K_{\hat{\delta}_t}}$, which is because $\Delta K_{\hat{\delta}_t} \geq 1$ by Lemma 2. Now, we have

$$\begin{aligned}
[1-\lambda_i\Delta K_{\hat{\delta}}]g_i(t) &\leq (1-\lambda_i)g_i(t) \\
&\leq \hat{\delta}_k^{T^*-t}b_i + \sum_{\tau=t+1}^{T^*} \hat{\delta}_k^{\tau-t}(\lambda_i G(\tau) - g_i(\tau)) \\
&\leq \frac{\delta^{T^*-t}\beta}{\Delta}b_i + \sum_{\tau=t+1}^{T^*} \frac{\delta^{\tau-t}\beta}{\Delta}(\lambda_i G(\tau) - g_i(\tau)) \\
&\leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]
\end{aligned}$$

for all $t \in \{0, 1, 2, .., T^* - 1\}$. The first inequality follows from Lemma 2. The second inequality is simply the under-contributing constraint for time-consistent agent. To see the third inequality, note that $\hat{\delta}_t \leq \frac{\delta\beta}{1-\delta+\delta\beta}$ and $\hat{\delta}_t < \delta$ for any t. Thus, we have $\hat{\delta}_t^s \leq \frac{\delta^s\beta}{1-\delta+\delta\beta} = \frac{\delta^s\beta}{\Delta}$ for any $s \geq 1$, which brings the third inequality. And the last inequality follows from Lemma 2 and from the fact that $1/\Delta \geq 1$. Thus, for $t < T^* - 1$, we get

$$[1 - \lambda_i \Delta K_{\hat{\delta}}] g_i(t) \leq \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

Thus, grim-g is also a Nash equilibrium strategy profile for time-inconsistent agents, that is, g is a Nash equilibrium outcome for time-inconsistent agents. ∎

Assumption 1 implies $\hat{\delta}_k > \delta^*$ for any $k > 2$, where δ^* is the threshold discount factor in MM, above which the existence of a completing equilibrium in finite periods in the time-consistent environment is ensured. Thus, under Assumption 1, a completing equilibrium in the time-consistent environment exists, which is also an equilibrium in the time-inconsistent environment by Proposition 1 above. Thus, the existence of an equilibrium in the time-inconsistent environment is also established.

Proposition 2 *For all $\overline{X}$,δ,β, there exists a sequence of nonnegative contributions, $g \equiv \{g_i(t)\}_{t=0}^{\infty}$, such that g is a Nash equilibrium outcome for the time-inconsistent agents while it is not a Nash equilibrium outcome for the corresponding time-consistent agents.*

Proof. Let $g \equiv \{g_i(t)\}_{t=0}^{\infty}$ be an arbitrary equilibrium outcome for a given project size $\overline{X}$, for time-inconsistent agents, where g completes the project in T^* periods, such that the under contributing constraint binds for some $t' \in \{0, 1, 2, ..., T^* - 2\}$. Since grim-$g$ strategy profile is a Nash equilibrium strategy profile, the under contributing constraint holds, that is,

$$[1 - \lambda_i \Delta K_{\hat{\delta}}] g_i(t) \leq \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

for all $t \in \{0, 1, 2, .., T^*\}$. Moreover for period t' we have,

$$[1 - \lambda_i \Delta K_{\hat{\delta}}] g_i(t') = \delta^{T^*-t'} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t'+1}^{T^*} \delta^{\tau-t'} \beta [(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

For the corresponding time-consistent agents with $\hat{\delta}_k$, the under contributing constraint at period t'

$$
\begin{aligned}
(1-\lambda_i)g_i(t') &= \frac{1-\lambda_i}{1-\lambda_i \Delta K_{\hat{\delta}}}\delta^{T^*-t'}\beta K_{\hat{\delta}} b_i + \frac{1-\lambda_i}{1-\lambda_i \Delta K_{\hat{\delta}}}\sum_{\tau=t'+1}^{T^*}\delta^{\tau-t'}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \\
&\geq \delta^{T^*-t'}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t'+1}^{T^*}\delta^{\tau-t'}\beta[(\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \\
&= \frac{\delta^{T^*-t'}\beta}{\Delta} K_{\hat{\delta}}\Delta b_i + \sum_{\tau=t'+1}^{T^*}\frac{\delta^{\tau-t'}\beta}{\Delta}[(\lambda_i K_{\hat{\delta}}\Delta G(\tau) - \Delta g_i(\tau)] \\
&> \frac{\delta^{T^*-t'}\beta}{\Delta} b_i + \sum_{\tau=t'+1}^{T^*}\frac{\delta^{\tau-t'}\beta}{\Delta}[(\lambda_i G(\tau) - g_i(\tau)] \\
&> \hat{\delta}_k^{T^*-t'} b_i + \sum_{\tau=t'+1}^{T^*}\hat{\delta}_k^{\tau-t'}[(\lambda_i G(\tau) - g_i(\tau)]
\end{aligned}
$$

where the first equality follows from the binding under contributing constraint for time-inconsistent agent at period t'. The first inequality follows from the fact that $K_{\hat{\delta}_k}\Delta \geq 1$ by Lemma 2. The first strict inequality follows from both $K_{\hat{\delta}_k}\Delta \geq 1$ and that $\Delta < 1$. And the last strict inequality follows from the fact that $\hat{\delta}_k^s < \frac{\delta^s\beta}{\Delta}$ for all finite $s > 1$, which follows from $\hat{\delta}_k^s < \frac{\delta\beta}{\Delta}$ and $\hat{\delta}_k < \delta$. Thus, at period t', the under contributing constraint for time-consistent agents does not hold. Thus, grim-g is not a Nash equilibrium strategy profile, hence not an equilibrium outcome, for time-consistent agents. ■

These two propositions show that the set of Nash equilibrium outcomes expands as we move from time-consistent environment to time-inconsistent environment. The reason for this result is that the equilibrium contribution levels of the corresponding time-consistent agents satisfy the under-contributing condition of the sophisticated time-inconsistent agents. The intuition is that for time-inconsistent agents, their net cost of contributing the same amount $g_i(t)$ is smaller compared to the time-consistent agents who have the same average discounting. This is due to the present bias and is reflected in $\lambda_i \Delta K_{\hat{\delta}}$, where $\Delta K_{\hat{\delta}} \geq 1$. Thus, there is more room to contribute for time-inconsistent agents in the equilibrium. In the next section, we analyze the number of periods it takes to finish a given project for both environments.

5 Completion Period

Now, we show our main result. When it comes to compare time-inconsistent agents to time-consistent agents with a discount factor $\hat{\delta}_t$, in the sense of completion period of any given project, surprisingly;

sophisticated time-inconsistent agents finish the project no later than time-consistent agents do.

Let $T^*_{SO}(\overline{X})$ be the minimum number of periods that sophisticated time-inconsistent agents finish the project of size $\overline{X}$. Let $T^*_{TC(\hat{\delta}_k)}(\overline{X})$ be the minimum number of periods that the corresponding time-consistent agents, with the discount factor $\hat{\delta}_k$, finish the project of size $\overline{X}$. In Theorem 2 below, we show that time-inconsistent agents finish a given project (weakly) faster than time-consistent agents.

Theorem 2 *For any $\overline{X}$, δ, β, $\hat{\delta}_k$, $\lambda_i < 1$, $b_i > 0$ and $n \geq 2$, with $1 > \lambda_i K_{\hat{\delta}_k} \Delta$ for all i, we have $T^*_{SO}(\overline{X}) \leq T^*_{TC(\hat{\delta}_k)}(\overline{X})$.*

Proof. First, we show that a grim-g strategy profile with binding critical values is the fastest, that is, $T^*_i(\overline{X})$, is the number of periods induced by the grim-g with binding critical values, where $i \in \{SO, TC(\hat{\delta}_k)\}$. To see this, suppose otherwise, that is, there is another equilibrium which finishes the project faster than the equilibrium profile with the binding critical values. Thus, at least one player at some period must be contributing more than her critical value for that period, violating the under contributing constraint. Thus, by Theorem 1, this profile is not a Nash equilibrium outcome. Thus, the grim-g equilibrium outcome with binding critical values is the fastest.

Now, we show that for any δ, β, $\hat{\delta}_k$ and for any given values of λ_i and n, we have

$$\sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t) \tag{5}$$

for any $T^* \geq 1$, where $c_i^{\delta\beta}(t)$ is the critical value of the time-inconsistent agent i in period t, and $c_i^{\hat{\delta}_k}(t)$ is the critical value of the corresponding time-consistent agent i, with the discount factor $\hat{\delta}_k$, in period t. To see this, we use the critical values for the time-inconsistent agent.

$$c_i^{\delta\beta}(0) = \frac{\Delta K_{\hat{\delta}_k}}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} b_i$$

$$c_i^{\delta\beta}(1) = \frac{\delta\beta}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} \left[\lambda_i K_{\hat{\delta}_k} \sum_{j \neq i} c_j^{\delta\beta}(0) + \frac{1-\Delta}{\Delta} c_i^{\delta\beta}(0) \right]$$

$$c_i^{\delta\beta}(s) = \frac{\delta\beta}{1 - \lambda_i \Delta K_{\hat{\delta}_k}} \left[\lambda_i K_{\hat{\delta}_k} \sum_{j \neq i} c_j^{\delta\beta}(s-1) + \frac{1-\Delta}{\Delta - (1-\delta)} [1 - \lambda_i K_{\hat{\delta}_k}(1-\delta)] c_i^{\delta\beta}(s-1) \right]$$

for all $s \geq 2$. The critical values of the corresponding time-consistent agent with a discount factor $\hat{\delta}_k$

are

$$c_i^{\hat{\delta}_k}(0) = \frac{b_i}{1-\lambda_i}$$

$$c_i^{\hat{\delta}_k}(s) = \hat{\delta}_k \frac{\lambda_i}{1-\lambda_i} \sum_{j \neq i} c_j^{\hat{\delta}_k}(s-1)$$

for all $s \geq 1$. Now, we compare the critical values. First note that, since $\Delta K_{\hat{\delta}_k} \geq 1$ for any k, we have

$$c_i^{\hat{\delta}_k}(0) = \frac{b_i}{1-\lambda_i} \leq \frac{\Delta K_{\hat{\delta}_k}}{1-\lambda_i \Delta K_{\hat{\delta}_k}} b_i = c_i^{\delta\beta}(0)$$

Also,

$$\begin{aligned}
c_i^{\delta\beta}(1) &= \frac{\delta\beta}{\Delta} \frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i \Delta K_{\hat{\delta}_k}} \left[\sum_{j \neq i} c_j^{\delta\beta}(0) + \frac{1-\Delta}{\lambda_i \Delta K_{\hat{\delta}_k}} c_i^{\delta\beta}(0) \right] \\
&\geq \frac{\delta\beta}{\Delta} \frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i \Delta K_{\hat{\delta}_k}} \sum_{j \neq i} c_j^{\delta\beta}(0) \\
&\geq \hat{\delta}_k \frac{\lambda_i}{1-\lambda_i} \sum_{j \neq i} c_j^{\hat{\delta}_k}(0) = c_i^{\hat{\delta}_k}(1)
\end{aligned}$$

which follows from the fact that $1-\Delta > 0$, $\frac{\delta\beta}{\Delta} \geq \hat{\delta}_k$ and $\Delta K_{\hat{\delta}_k} \geq 1$ for any k, and the fact that $c_i^{\hat{\delta}_k}(0) \leq c_i^{\delta\beta}(0)$. Similarly,

$$\begin{aligned}
c_i^{\delta\beta}(s) &= \frac{\delta\beta}{1-\lambda_i \Delta K_{\hat{\delta}_k}} \left[\lambda_i K_{\hat{\delta}_k} \sum_{j \neq i} c_j^{\delta\beta}(s-1) + \frac{1-\Delta}{\Delta-(1-\delta)} [1-\lambda_i K_{\hat{\delta}_k}(1-\delta)] c_i^{\delta\beta}(s-1) \right] \\
&> \frac{\delta\beta}{1-\lambda_i \Delta K_{\hat{\delta}_k}} \lambda_i K_{\hat{\delta}_k} \sum_{j \neq i} c_j^{\delta\beta}(s-1) \\
&= \frac{\delta\beta}{\Delta} \frac{\lambda_i \Delta K_{\hat{\delta}_k}}{1-\lambda_i \Delta K_{\hat{\delta}_k}} \sum_{j \neq i} c_j^{\delta\beta}(s-1) \\
&\geq \hat{\delta}_k \frac{\lambda_i}{1-\lambda_i} \sum_{j \neq i} c_j^{\hat{\delta}_k}(s-1) = c_i^{\hat{\delta}_k}(s)
\end{aligned}$$

for all $s \geq 1$, where the first inequality follows from the fact that $1-\Delta > 0$ and $\Delta-(1-\delta) = \delta\beta > 0$ and $1-\lambda_i K_{\hat{\delta}_k}(1-\delta) > 1-\lambda_i K_{\hat{\delta}_k}\Delta > 0$. And the last inequality follows from the fact that $c_j^{\delta\beta}(s-1) \geq$

$c_j^{\hat{\delta}_k}(s-1)$ for all j and for all $s \geq 1$. Thus, we get $c_i^{\delta\beta}(t) \geq c_i^{\hat{\delta}_k}(t)$ for all i, t and k. Thus,

$$\sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t)$$

Thus, for the cumulative contributions we get

$$\sum_i \sum_{t=0}^{T^*} c_i^{\delta\beta}(t) \geq \sum_i \sum_{t=0}^{T^*} c_i^{\hat{\delta}_k}(t)$$

This implies that time-inconsistent agents always achieve a weakly higher cumulative contribution at any period. Thus, they will always achieve a given $\overline{X}$ weakly earlier than any corresponding time-consistent agents, that is, $T^*_{SO}(\overline{X}) \leq T^*_{TC(\hat{\delta}_k)}(\overline{X})$. ■

A direct corollary to Theorem 2 above is the following.

Corollary 2 *For any $\overline{X}$, δ, β, $\lambda_i < 1$, $b_i > 0$ and $n \geq 2$, we have $T^*_{SO}(\overline{X}) \leq T^*_{TC(\hat{\delta}_\infty)}(\overline{X})$.*

Proof. With $\hat{\delta}_\infty$ the condition $1 > \lambda_i K_{\hat{\delta}_k}\Delta$ turns into $1 > \lambda_i$, since $K_{\hat{\delta}_\infty}\Delta = 1$, which is shown in the proof of Lemma 2 and the result directly follows from Theorem 2 above. ■

The intuition behind this result is that time-inconsistent agents are sophisticated, thus they know that their future selves may tend to postpone contributions and cause the project be finished later. Thus, current selves of time-inconsistent agents contribute more, relative to time-consistent agents, in early periods in order to guard themselves against future selves. Thus, with achieving higher cumulative contribution levels in the early periods, time-inconsistent agents manage to finish the project earlier than time-consistent agents.

Below, we provide an example in which time-inconsistent agents finish the project **strictly** earlier than time-consistent agents. Thus, our main result is not vacuous.

Example 1 *Consider the set of parameters given below.*

δ	β	b	λ	n	$\overline{X}$
0.8	0.7	10	$\frac{1}{8}$	4	66.50

Table 1. Parameter values.

Given these values of the parameters, the critical contribution levels are

	k=0	k=1	k=2	k=3	k=4	Total
$c^{\delta\beta}(k)$	11.4285	5.918	3.4388	1.9981	1.1610	95.7779
$c^{\hat{\delta}_\infty}(k)$	11.4285	3.6098	1.1402	0.3601	0.1138	66.6099

Table 2. Contributions for both time-inconsistent and time-consistent agents.

The required amount $\overline{X} = 66.50$ *will be collected by the time-inconsistent agents within the first 2 periods, that is,* $T^*_{SO} = 1$. *However, the maximum amount that the corresponding time-consistent agents, with a discount factor* $\hat{\delta}_\infty$ *defined above, are willing to contribute in this environment is the summation of the second row in Table 2, which is* 66.6099. *Hence, the project, which requires an amount of* 66.50, *will be provided by these time-consistent agents within 5 periods, that is,* $T^*_{TC(\hat{\delta}_\infty)} = 4$. *Thus,* $T^*_{SO}(\overline{X}) < T^*_{TC(\hat{\delta}_\infty)}(\overline{X})$.[24]

Let $\overline{X}_{SO}$ be the infinite sum of critical contributions of time-inconsistent agents and let $\overline{X}_{TC}$ be the infinite sum of critical contributions of the corresponding time-consistent agents.

Corollary 3 *If* $\overline{X}_{TC}$ *is finite, then* $\overline{X}_{SO} > \overline{X}_{TC}$.

Proof. This immediately follows from the comparison of the critical contribution levels of the two types of agents for each period, in the proof of Theorem 2. ■

In Example 1, time-inconsistent agents finish a project that costs 95 in finitely many periods, however, the largest amount that the corresponding time-consistent agents can collect in this environment is 66.824, which can only happen if they contribute over an infinite number of periods. Thus, any project that costs more than 66.824 will not be completed by time-consistent agents, that is, the project that costs 95 is finished in finitely many periods by the time-inconsistent agents but not finished by the corresponding time-consistent agents. This gives rise to the possibility that for a range of cost values, time-inconsistent agents finish a given project with a cost from that range, but the corresponding time-consistent agents do not. Below, we provide a result regarding the projects that are finished in finite number of periods by time-inconsistent agents but not with time-consistent agents, in symmetric environments.

[24]If we would get the corresponding time-consistent agent as the one with a discount factor $\hat{\delta}_5$ for instance, it would be even easier to satisfy the condition, since the critical contributions would be smaller in that case. Thus, they finish the project no earlier than $T^* = 4$. Thus, we have $T^*_{SO}(\overline{X}) < T^*_{TC(\hat{\delta}_k)}(\overline{X})$, for any k in this example.

Proposition 3 *Assume that* $\lambda_i = \lambda$ *and* $b_i = b > 0$ *for all* i. *If* $\frac{\delta\beta}{1-\delta(1-\beta)}\frac{\lambda}{1-\lambda}(N-1) < 1$, *then any project with a cost* $\overline{X}$ *such that* $\overline{X}_{SO} > \overline{X} > \overline{X}_{TC}$ *is finished by time-inconsistent agents in finitely many periods but not with the corresponding time-consistent agents with a discount factor* $\hat{\delta}_\infty$.

Proof. First note that $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$. Thus, $\frac{\delta\beta}{1-\delta(1-\beta)}\frac{\lambda}{1-\lambda}(N-1) < 1$ implies that $\hat{\delta}_\infty\frac{\lambda}{1-\lambda}(N-1) < 1$. Thus, the critical contributions are given by $c^{\hat{\delta}_\infty}(s) = \hat{\delta}_\infty\frac{\lambda}{1-\lambda}(N-1)c^{\hat{\delta}_\infty}(s-1) = [\hat{\delta}_\infty\frac{\lambda}{1-\lambda}(N-1)]^s c^{\hat{\delta}_\infty}(0)$. Since $\sum_{s=1}^{\infty}[\hat{\delta}_\infty\frac{\lambda}{1-\lambda}(N-1)]^s$ is finite, $\overline{X}_{TC}$ is finite. Now, take a project with $\overline{X}$ such that $\overline{X}_{SO} > \overline{X} > \overline{X}_{TC}$. Clearly, the corresponding time-consistent agents with $\hat{\delta}_\infty$ cannot finish $\overline{X}$ in finitely many periods. We show that $\overline{X}$ is completed by time-inconsistent agents in finitely many periods. This follows from the following claim.

Claim. Let $\sum_{t=0}^{\infty} c(t) = Y$ where $Y \in \mathbb{R}_{++}$, then for any $Y' < Y$, there exists a finite T such that $\sum_{t=0}^{T} c(t) = Y'$.

Proof. Define $Y_n = \sum_{t=0}^{n} c(t)$. Thus, $lim_{n\to\infty}Y_n = Y$. Note that $Y > Y_n$ for all n. For any $\epsilon > 0$, there exists an N such that $Y - Y_n < \epsilon$ for all $n > N$. Thus, for $\epsilon = Y - Y' > 0$, there is a $N(Y')$ such that $Y - Y_n < Y - Y'$ for all $n > N_{Y'}$, that is, $Y_n > Y'$ for all $n > N_{Y'}$. Thus, for some $T \leq N_{Y'}$, we must have $Y_T \geq Y'$. □

Thus, for some finite number of periods T, we have $N\sum\limits_{t=0}^{T} c^{\delta\beta}(t) \geq \overline{X}$, that is, time-inconsistent agents finish the project with a cost $\overline{X}$ in finitely many periods. ■

Note that if the assumption we have in Proposition 3, namely, $\frac{\delta\beta}{1-\delta(1-\beta)}\frac{\lambda}{1-\lambda}(N-1) < 1$ does not hold, we have a sequence of critical contributions for the corresponding time-consistent agents, which is non-decreasing. Thus, the infinite sum will be infinite, in which case the project would be completed in both environments, for which we already have two relevant results: See Theorem 2 and Corollary 2.

One can interpret our result in Proposition 3 in terms of the condition provided by MM regarding the agents to be sufficiently patient. Here, we argue that this result can be seen as an improvement on this sufficiently patient condition. To see this, fix a δ and a β and find the discount factor for the corresponding time-consistent agents, that is, $\hat{\delta}_\infty$. Then, calculate $\overline{X}_{TC}$ for $\hat{\delta}_\infty$. Now, a project with $\overline{X}_{TC}$ cannot be completed in finitely many periods by time-consistent agents with $\hat{\delta}_\infty$. Thus, for the time-consistent agents to finish this project in finitely many periods, they must have a larger discount factor, $\tilde{\delta} > \hat{\delta}_\infty$. However, time-inconsistent agents finish the same project in finitely many periods. Since the corresponding discount factor for these β-δ time-inconsistent agents is $\hat{\delta}_\infty$, we have a weaker sufficiently patient condition, a smaller threshold on the discount factor, a larger interval to get positive results.

Below, we also provide a result regarding the case where the time-inconsistent agents finish a given project strictly earlier than the time-consistent agents, as in Example 1. We focus on symmetric environment and assume that the discount factor for the time-consistent agents is $\hat{\delta}_\infty$. First we prove a useful lemma.

Lemma 3 *If $\sum_{k=0}^{1} c^{\delta\beta}(k) \geq \sum_{k=0}^{2} c^{\hat{\delta}_\infty}(k)$, then $\sum_{k=0}^{t-1} c^{\delta\beta}(k) \geq \sum_{k=0}^{t} c^{\hat{\delta}_\infty}(k)$ for all $t > 2$.*

Proof. See the Appendix. ■

Lemma 3 says that, if the maximum amount time-inconsistent agents contribute in two periods, for $t = 0, 1$, is more than the maximum amount that the corresponding time-consistent agents contribute in three periods, for $t = 0, 1, 2$, then for each period after $t = 2$, the maximum amount corresponding time-consistent agents would collect can always be collected in the earlier periods by the time-inconsistent agents. Using this lemma we prove the following result, which provides sufficient conditions for the time-inconsistent agents to finish strictly earlier than the corresponding time-consistent agents.

Proposition 4 *Assume that $\lambda_i = \lambda$, and $b_i = b > 0$ for all i. For any δ and β, there exist a threshold λ^*, such that if $\lambda < \lambda^*$ and $\overline{X} > N\sum_{k=0}^{1} c^{\hat{\delta}_\infty}(k)$, then $T^*_{SO}(\overline{X}) < T^*_{TC(\hat{\delta}_\infty)}(\overline{X})$.*

Proof. If the cost of the project, $\overline{X}$, is such that $\overline{X} \leq N\frac{b}{1-\lambda}$, then in both environments, the project is finished immediately. Also, if $N\frac{b}{1-\lambda} \leq \overline{X} \leq N\sum_{k=0}^{1} c^{\hat{\delta}_\infty}(k)$, then the project is finished in two periods in both environments. Thus, we must have $\overline{X} > N\sum_{k=0}^{1} c^{\hat{\delta}_\infty}(k)$. Now, if the premise of Lemma 3 also holds, then the time-inconsistent agents finish the project strictly earlier than the time-consistent agents. Note that $\sum_{k=0}^{1} c^{\delta\beta}(k) \geq \sum_{k=0}^{2} c^{\hat{\delta}_\infty}(k)$ is equivalent to $c^{\delta\beta}(1) \geq c^{\hat{\delta}_\infty}(2) + c^{\hat{\delta}_\infty}(1)$. After some algebra, using the critical contribution levels in the proof of Theorem 2, we get $c^{\delta\beta}(1) \geq c^{\hat{\delta}_\infty}(2) + c^{\hat{\delta}_\infty}(1)$ if and only if

$$\delta(1-\beta) \geq \frac{\delta\beta}{1-\delta(1-\beta)}\frac{\lambda^2}{1-\lambda}(N-1)^2 \tag{6}$$

Note that right-hand side of (6) is increasing in λ. When, $\lambda = 1$, the right-hand side is greater than 1 while the left-hand side is always less than 1. Also, when $\lambda = 0$, the right-hand side collapses to 0. Thus, for any given nonzero δ and β, there exists a $\lambda^* \in (0,1)$ such that for all $\lambda < \lambda^*$, the inequality in (6) holds. Thus, if the cost of the project is high enough, that is, $\overline{X} > N\sum_{k=0}^{1} c^{\hat{\delta}_\infty}(k)$, and if λ is low enough, that is, $\lambda < \lambda^*$, then the time-inconsistent agents finish the project strictly earlier than the corresponding time-consistent agents. ■

Therefore, if the partial benefit, λ, is low enough, and if the project cost, $\overline{X}$, is high enough, then the time-inconsistent agents finish the project strictly earlier than the time-consistent agents. This is in line with our intuition. The time-inconsistent agent wants to guard herself against future selves, who may contribute less than the current self wants, and thus the current self contributes more today. When the partial benefit parameter is lower, any delay is even worse for the time-inconsistent current self. Thus, she feels the urge to contribute more today even more strongly, relative to the case with a higher partial benefit.

5.1 Comparative Statics with respect to δ and β

We provide comparative statics with respect to changes in δ and β. To make it tractable we consider the discount factor $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$, thus we have $K_{\hat{\delta}_\infty}\Delta = 1$, thus the critical levels for the time-inconsistent agents are given by

$$c_i(0) = \frac{b_i}{1-\lambda_i} \quad \text{and} \quad c_i(1) = \frac{\delta\beta}{1-\lambda_i}\left[\lambda_i K_{\hat{\delta}_\infty}\sum_{j\neq i} c_j(0) + \frac{\delta(1-\beta)}{1-\delta(1-\beta)}c_i(0)\right]$$

$$c_i(k) = \frac{\delta\beta}{1-\lambda_i}\left[\lambda_i K_{\hat{\delta}_\infty}\sum_{j\neq i} c_j(k-1) + \frac{\delta(1-\beta)}{\delta\beta}[1-\lambda_i K_{\hat{\delta}_\infty}(1-\delta)]c_i(k-1)\right]$$

for all $T^*-1 \geq k \geq 2$, where $K_{\hat{\delta}_\infty} = \frac{1}{\Delta} = \frac{1}{1-\delta(1-\beta)}$. Note that the critical $c_i(0)$ is not affected by either of δ or β.

The critical level $c_i(1)$ is increasing in δ. This is immediate since both $K_{\hat{\delta}_\infty}$ and $\frac{\delta(1-\beta)}{1-\delta(1-\beta)}$ are increasing in δ. The critical level $c_i(k)$ is also increasing in δ. To see this, assume that $c_i(k-1)$ is increasing in δ. Since $c_i(k-1)$ and $K_{\hat{\delta}_\infty}$ are increasing in δ and $K_{\hat{\delta}_\infty}(1-\delta) = \frac{1-\delta}{1-\delta(1-\beta)}$ is decreasing in δ, we get that $c_i(k)$ is also increasing in δ. By induction, we get that $c_i(k)$ increases for all $k > 0$. This translates into that the more patient agent has higher critical contribution levels, which is plausible.

The comparative statics with respect to β, however, is not straightforward. To make it easier, we assume symmetry, that is, $\lambda_i = \lambda$ and $b_i = b$ for all i, and drop the i subscript, and N denotes the number of agents.

The critical level $c(1)$, the contribution level for the period right before completion, can be written as

$$c(1) = \frac{\delta c(0)}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right] \tag{7}$$

The first term in the parentheses, $\frac{\beta}{1-\delta(1-\beta)}$, increases as β increases, however, whether the second term, $\frac{\beta(1-\beta)}{1-\delta(1-\beta)}$, increases or decreases depends on β and δ. Thus, the overall effect of an increase in β is ambiguous. However, for large enough λ or N, the increase in the first term will dominate a potential decrease due to the second term. Thus, assuming large λ or large N, we can say that an increase in β will result in an increase in $c(1)$.

For other critical levels we have,

$$c(k) = \frac{\delta c(k-1)}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta(1-\delta)(1-\lambda)\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right] \tag{8}$$

Again for large λ and N, this critical level will increase when β increases. However, because of the multiplier $(1-\delta)(1-\lambda) < 1$ in the second term which is absent in critical level $c(1)$, the increase will be weaker for $c(k)$, for k>1 relative to the increase in $c(1)$.

Thus, for large λ or N, as β decreases, the contribution levels will decrease, that is, the more time-inconsistent the agents are (a lower β), the smaller the contributions are. However, the earlier contributions will not decrease as much as the later contributions. When β decreases, the corresponding time-consistent agent's discount factor, $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$ also decreases. Thus, the contribution levels for the time-consistent agents will also decrease. Thus, the decrease in the contribution levels of the time-inconsistent agents is also expected, which is due to the larger overall discounting. The interesting part is that when the inconsistency gets larger, the later contributions decrease more than the earlier contributions, which is in line with our intuition: The sophisticated agent guards herself with relatively higher earlier contribution levels (relative to the consistent agents' earlier contributions), resulting in a faster finish of the project. To illustrate, consider a case with $\beta > \beta'$ with critical levels, $c(0)$ and $c(1)$ for β and $c'(0)$ and $c'(1)$ for β'. Our intuition suggests that the sophisticated agent with β' must guard herself better, that is, we expect $c'(1) - c'(0) > c(1) - c(0)$, which is parallel to the analysis we provide here.

5.2 Slope of the contribution scheme: A comparison

Here, we provide an analysis on the slope of the contribution schemes of the time-consistent agents and the time-inconsistent agents. To make it tractable, we assume symmetry, $\lambda_i = \lambda$ and $b_i = b$ for all i, and drop the i subscript. And we conduct the comparison with the time-consistent agents with a discount factor $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$. Then, the contribution levels for the time-inconsistent agents are given by Equations 7 and 8 above. We rewrite them here as follows: $c(1) = Ac(0)$ and $c(k) = Bc(k-1)$ for $k \geq 2$, where

$A = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right]$ and
$B = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta(1-\delta)(1-\lambda)\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right]$.

Then, for $k \geq 2$, we have, $c(k) - c(k-1) = AB^{k-2}(B-1)c(0)$, for the time-inconsistent agents, where $c(0) = \frac{b}{1-\lambda}$, since $K_{\hat{\delta}_\infty}\Delta = 1$. Now, for the time-consistent agents with $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$, the critical contributions for $k \geq 2$ are given by

$$\begin{aligned}
c^{\hat{\delta}_\infty}(k) &= \hat{\delta}_\infty \frac{\lambda}{1-\lambda}(N-1)c^{\hat{\delta}_\infty}(k-1) \\
&= \frac{\delta\beta}{1-\delta(1-\beta)}\frac{\lambda}{1-\lambda}(N-1)c^{\hat{\delta}_\infty}(k-1) \\
&= \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)}\right]c^{\hat{\delta}_\infty}(k-1) \\
&= Dc^{\hat{\delta}_\infty}(k-1) \\
&= D^k c^{\hat{\delta}_\infty}(0)
\end{aligned}$$

for any $k \geq 2$, where $D = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)}\right]$. Then we have,

$$c^{\hat{\delta}_\infty}(k) - c^{\hat{\delta}_\infty}(k-1) = (D^k - D^{k-1})c^{\hat{\delta}_\infty}(0) = DD^{k-2}(D-1)c(0)$$

where $c^{\hat{\delta}_\infty}(0) = c(0) = \frac{b}{1-\lambda}$.

Thus, we need to compare $AB^{k-2}(B-1)$ and $DD^{k-2}(D-1)$. Clearly, $A > D$ and $B > D$. When $B > 1$, we get $c(k) - c(k-1) > c^{\hat{\delta}_\infty}(k) - c^{\hat{\delta}_\infty}(k-1)$ for any $k \geq 2$. For $k = 1$, we have $c(1) - c(0) = (A-1)c(0)$ and $c^{\hat{\delta}_\infty}(1) - c^{\hat{\delta}_\infty}(0) = (D-1)c^{\hat{\delta}_\infty}(0) = (D-1)c(0)$. Clearly, $c(1) - c(0) > c^{\hat{\delta}_\infty}(1) - c^{\hat{\delta}_\infty}(0)$. Thus, for any $k \geq 1$, we have $c(k) - c(k-1) > c^{\hat{\delta}_\infty}(k) - c^{\hat{\delta}_\infty}(k-1)$.[25]

Thus, the contribution scheme for the time-inconsistent agents has a higher slope than the contribution scheme for the time-consistent agents. More precisely, for the time-inconsistent agents, earlier contributions are relatively higher than the later contribution levels, when compared to the consistent agents. This is parallel to our intuition, that earlier contributions are larger for the sophisticated agent, since she guards herself against the future self by contributing more in earlier periods, when compared

[25] When $\lambda N > 1$ and δ is large enough, $B > 1$. If $\lambda N < \beta$, however, we have $A < 1$, thus $B < 1$. In this case, we get $c(k) - c(k-1) < c^{\hat{\delta}_\infty}(k) - c^{\hat{\delta}_\infty}(k-1)$ for all $k \geq 1$. This follows from three facts: (1) both $c(k)$ and $c^{\hat{\delta}_\infty}(k)$ are monotonically decreasing in k, (2) $c(0) = c^{\hat{\delta}_\infty}(0)$ and (3) $c(k) > c^{\hat{\delta}_\infty}(k)$ for all $k \geq 1$.

to the consistent agent.

6 Discussion and Conclusion

The model of voluntary contribution, that we studied in spirit of MM, lets the players contribute any amount in any period as they like, by observing the aggregate contribution at each period. We introduced sophisticated time-inconsistent agents with linear discontinuous preferences to the model and studied the Nash equilibria as well as the shortest time period in which the project is finished.

Our main result is concerned with the comparison of completion period of a given project. We find that any project is completed by time-inconsistent agents weakly earlier than any corresponding time-consistent agents. This is surprising because time-inconsistent agents tend to postpone costly actions. Intuitively, a time-inconsistent agent commits to higher contributions in the earlier periods in order to guard herself against the potentially low contribution levels of her future selves. We believe that this result is not only surprising but also important since time inconsistency is usually perceived as a source of inefficiency, yet in a very well known voluntary contribution environment, it is no longer the case: time inconsistency may result in more efficient outcomes. Our result may be a stepping stone to think of more general environments and conditions where time inconsistency may bring higher efficiency levels than time consistency.

Although our predictions (higher contribution levels and a decreasing rate of contributions) are more in line with the behavior in the experiment of Duffy, Ochs, and Vesterlund (2007), we do not provide an exhaustive explanation of their experimental data. Our model does not explain their result that shows that the higher levels of contributions in the dynamic version do not critically depend on the benefit jump.

Now, we discuss a number issues and extensions.

Experimental Evidence. In their experimental study, Duffy, Ochs, and Vesterlund (2007) show that, in a dynamic setting similar to the one in MM, players contribute larger amounts than in a static set-up and higher level of contributions in the dynamic version does not critically depend on the benefit jump which is present in MM. In their treatment when there is a positive benefit jump upon completion of the public good, there are 38 groups (groups always consist of three players) who provide the public good on or before 4 periods and do not contribute further after completion.[26] Of these 38 groups, that

[26] The parameters used in the experiment are $\lambda = 0, 5$, $b = 1$ and $\overline{X} = 12$, all in chips, the unit used in the experiment.

is, 114 players, the average contribution levels are 1.68, 0.91, 0.83 and 0.7 for the first, second, third and fourth periods, respectively.[27] For any discount factor, $\delta \geq 0.63$, using the parameters employed in the experiment, the contribution levels according to MM are such that the public good is provided in the very first period, which occurs just once in 38 groups. For discount factors, $0.63 > \delta \geq 0.5$, the provision is achieved in two periods according to MM, however, provision in the second period occurs in only 1 out of 38 groups. For discount factors, $0.5 > \delta \geq 0.405$, the provision is achieved in three periods according to MM, however, provision occurs in the third period in only 4 out of 38 groups. If we consider smaller discount factors and calculate the contribution sequences that provide the public good in four periods, we find that the discount factor should be such that $0.405 > \delta \geq 0.27$.[28] However, for all these discount factors, the contribution sequence is also inconsistent with the observed sequence of contributions in the experiment, since the one in the experiment is decreasing, whereas the one MM predicts is increasing.[29] Thus, for both large and small discount factors, it is hard to argue that the contribution levels in the experiment are consistent with any reasonable calibration of a geometric discount factor. Therefore, it is reasonable to consider alternative discounting functions like β-δ discounting. In fact, we show that when β-δ discounting is introduced, the earlier contributions are larger relative to the time-consistent players' early contributions. In the experiment, the average contribution levels of the first and second periods are 1.68 and 0.91, respectively. MM's prediction for the first and second periods are, however, 0.31 and 0.58, respectively, when $\delta = 0.27$. Thus, our result is consistent with the discrepancy between these two contribution sequences.

Delay and efficiency. Our main comparison between the two environments, one with time-consistent agents and the other with time-inconsistent agents, was concerned with the speed of the completion of the project. Given a project that is completed by time-consistent agents, we showed that the same project is completed in less (or same) amount of time by time-inconsistent agents, that is, time-inconsistent agents are (weakly) faster. Although, this does not necessarily mean that time-inconsistent agents are more efficient, we can argue that delay is usually inefficient and the fastest equilibrium is efficient. To see this, let's consider an equilibrium where the project is completed in T^*

[27] Among these 38 groups, only 1 group completes the public good in the first period, only 1 group completes the public good in the second period, and 4 groups complete in the third period, and the rest, 32 groups, complete in the fourth period. In fact, there are more groups who finish on or before the fourth period, but we eliminated those who keep contributing to the public good even after completion, which is clearly suboptimal.

[28] If $\delta < 0.27$, there is no provision within the first four periods.

[29] For instance, with $\delta = 0.27$, the sequence MM predicts is 0.31, 0.58, 1.08 and 2 for the first, second, third and fourth periods, respectively. When $\delta = 0.4$, the sequence MM predicts is 1.02, 1.28, 1,60 and 2 for the first, second, third and fourth periods, respectively.

periods. For simplicity let's assume that $\lambda_i = \lambda$ and $b_i = b$ for all $i = 1, 2, ..., N$. Now, let's consider a deviation that causes a delay. Let's say, one of the agents does not contribute her equilibrium amount at period T^*, but contributes $\epsilon > 0$ amount less than what she was supposed to contribute, but contributes ϵ more in period $T^* + 1$. Thus, there is a delay of exactly one period. The overall gain from this delay (from period T^* point of view) is due to postponing an ϵ amount of contribution one period, that is, $\epsilon - \delta\beta\epsilon$. The overall loss is $N[\lambda\overline{X} + b - \lambda(\overline{X} - \epsilon)]$. Thus, a delay of this kind will be inefficient if $\epsilon(1 - \delta\beta) < N[\lambda\overline{X} + b - \lambda(\overline{X} - \epsilon)]$, which is $Nb > (1 - \delta\beta - N\lambda)\epsilon$.[30] Therefore, when the number of agents is large enough, or when the benefit jump, b, is large enough, or when the marginal benefit of a non-finished project, λ, is large enough, this kind of delay will be inefficient. Thus, under any of these conditions, the fastest completing equilibria will be efficient.

Multiple equilibria. In a given environment (time-consistent or time-inconsistent), the Nash equilibrium we considered entails the critical levels, that is, the contribution levels that make the under-contributing constraints bind in every period $0 < t = T^*$. In the proof of Theorem 2, we showed that this type of equilibrium is the fastest. Therefore, if delay is inefficient as argued above, we have compared the (fastest) Nash equilibrium that is efficient in the time-inconsistent environment to the (fastest) Nash equilibrium that is efficient in the time-consistent environment. However, there are other Nash equilibria. For instance where the under-contributing constraints do not bind, we can still construct a grim-g strategy profile with non-binding critical contribution levels. But, these type of equilibria will be slower compared to the one we consider with binding critical levels.

Perfect Bayesian equilibria. As in MM, the grim-g strategy that we have constructed may not be sequentially rational when the cumulative contribution is close enough to finish the project. In this case, the strategy profile we have constructed leads to no contribution at all, when there is a deviation. However, the sequentially rational behavior would be finishing the project immediately for the player whom the cumulation so far is in their critical set defined as $C_i \equiv (\overline{X} - c_i^*, \overline{X})$. Nonetheless, we believe that, the grim-g outcome can be reached as a PBE outcome in our model as well, under some specific conditions, as in MM.

When $\beta > 1$. There is empirical and experimental evidence showing that there may be future bias, which, in our context, would translate into the time inconsistency parameter β being larger than 1.[31]

[30] For other kinds of delays, we can find similar conditions.

[31] See Takeuchi (2011) and Sayman and Öncüler (2009) for instance.

When $\beta > 1$, we believe that an opposite result may be obtained, that is, the time-inconsistent agents (with future bias) may always finish a given project (weakly) later than the time-consistent agents. This would be inline with our main intuition though. The current self realizes that her future selves will contribute more than the time-consistent future selves. Thus, current self contributes less today relative to the time-consistent players, ending up in a delay with respect to the time-consistent environment. However, delay in this case may be desirable since there is a future bias.

Heterogeneous environment. We have considered two environments, one with only time-consistent agents and one with only time-inconsistent agents. Although we have heterogeneity within a given environment in terms of different benefit jumps and different marginal benefits, b_i and λ_i, one can think of environments with a mix of time-consistent and time-inconsistent agents. However, the results are robust in the sense that the more inconsistent agent in the environment, the more contributions over time. Therefore, fixing everything else, if we compare two environments with different distribution of time-consistent and time-inconsistent agents, the one with a higher ratio of time-inconsistent agents will be more likely to finish a given project earlier than the environment with a lower ratio of time-inconsistent agents. This is due to the intuition we provided earlier, that is, time-inconsistent agents contribute more today in order to offset the possible delay that may emerge from future selves, who are likely to postpone contributions.

To see this, we assume symmetry in terms of benefit jump and partial benefit, that is, $b_i = b$, and $\lambda_i = \lambda$ for all $i \in N$, for simplicity. We compare three environments: (1) the one with only sophisticated time-inconsistent agents, (2) the one with only time-consistent agents with a discount factor $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$, and (3) the one with a mixture of sophisticated time-inconsistent and time-consistent agents (with a discount factor $\hat{\delta}_\infty$), where m of them are time-inconsistent while remaining n of them are time-consistent where $m + n = N$.

First notice that in each environment, $c_i(0) = \frac{b}{1-\lambda}$ for all $i \in N$, regardless of other agents being time-consistent or inconsistent. Thus, the under-contributing constraint for a time-inconsistent agent in environment 3 does not change relative to the environment 1. Likewise, the under-contributing constraint for a time-consistent agent in environment 3 does not change relative to the environment 2. Thus, we get the following critical contribution levels for environment 3, for both time-inconsistent and time-consistent agents, where the subscript h is used to denote environment 3, the heterogeneous environment. For time-inconsistent agents we have

$$c_h^{\delta\beta}(0) = \frac{b}{1-\lambda}$$

$$c_h^{\delta\beta}(1) = \frac{\delta\beta\lambda}{(1-\lambda)\Delta}((m-1)c_h^{\delta\beta}(0) + nc_h^{\hat{\delta}_\infty}(0)) + \frac{\delta(1-\beta)}{1-\lambda}\frac{\delta\beta}{\Delta}c_h^{\delta\beta}(0)$$

$$c_h^{\delta\beta}(k) = \frac{\delta\beta\lambda}{(1-\lambda)\Delta}((m-1)c_h^{\delta\beta}(k-1) + nc_h^{\hat{\delta}_\infty}(k-1)) + \frac{\delta(1-\beta)}{1-\lambda}[1-\lambda\frac{(1-\delta)}{\Delta}]c_h^{\delta\beta}(k-1)$$

for all $T^* - 1 \geq k \geq 2$. For time-consistent agents we get

$$c_h^{\hat{\delta}_\infty}(0) = \frac{b}{1-\lambda}$$

$$c_h^{\hat{\delta}_\infty}(1) = \frac{\delta\beta\lambda}{(1-\lambda)\Delta}(mc_h^{\delta\beta}(0) + (n-1)c_h^{\hat{\delta}_\infty}(0))$$

$$c_h^{\hat{\delta}_\infty}(k) = \frac{\delta\beta\lambda}{(1-\lambda)\Delta}(mc_h^{\delta\beta}(k-1) + (n-1)c_h^{\hat{\delta}_\infty}(k-1))$$

for all $T^* - 1 \geq k \geq 1$. Notice that $(m-1)c_h^{\delta\beta}(0) + nc_h^{\hat{\delta}_\infty}(0) = mc_h^{\delta\beta}(0) + (n-1)c_h^{\hat{\delta}_\infty}(0) = (N-1)\frac{b}{1-\lambda}$. Therefore, we have $c_h^{\delta\beta}(t) = c^{\delta\beta}(t)$ and $c_h^{\hat{\delta}_\infty}(t) = c^{\hat{\delta}_\infty}(t)$ for $t = \{0, 1\}$. But given that $c^{\hat{\delta}_\infty}(1) < c^{\delta\beta}(1)$, we have $Nc^{\hat{\delta}_\infty}(1) < mc_h^{\delta\beta}(1) + nc_h^{\hat{\delta}_\infty}(1) < Nc^{\delta\beta}(1)$. This generalizes for all $k \geq 1$, that is, we have $Nc^{\hat{\delta}_\infty}(k) < mc_h^{\delta\beta}(k) + nc_h^{\hat{\delta}_\infty}(k) < Nc^{\delta\beta}(k)$. This establishes our conjecture above.

Naivéte. A naive time-inconsistent agent is not aware of her time inconsistency. Denoting the agent's belief about his true β by $\widehat{\beta}$, a *partially naive* agent has $\beta < \widehat{\beta} < 1$ and a *fully naive* agent has $\beta < \widehat{\beta} = 1$. The agent is *naive* when $\beta < \widehat{\beta} \leq 1$. Here, we discuss the case of **fully naive** agents and their contribution schemes compared to those of the time-consistent agents.

Since the environment we consider here has only naive time-inconsistent agents, there is no chance of inferring any new information about own time inconsistency after observing other agents' actions. Thus, learning about own time inconsistency is possible only through some exogenous mechanism. We consider two cases: (1) there is no learning at all, and (2) there is learning at some period $t > 0$. Note that learning at $t = 0$, that is, immediate learning, corresponds to the sophisticated agent case.

Case 1. When there is no learning at all, a fully naive agent always believes that her β is 1, thus uses the $(1, \delta, \delta^2, ...)$ discounting scheme. Thus, the naive agents contribute amounts that are equal to the amounts contributed by the time-consistent agents with a discount factor δ. The critical contribution levels for time-consistent agents with a discount factor δ are

$$c_i^\delta(0) = \frac{b_i}{1-\lambda_i} \quad \text{and} \quad c_i^\delta(t) = \delta \frac{\lambda_i}{1-\lambda_i} \sum_{j \neq i} c_j^\delta(t-1)$$

for all $t \geq 1$. Clearly, these critical contribution levels are increasing in δ. Thus, the critical contribution levels for time-consistent agents with a discount factor $\hat{\delta}_k$ will be smaller than the contribution levels of the naive agents above since $\delta > \hat{\delta}_k$ for every $k > 2$.[32] Thus, naive agents contribute more than than the corresponding time-consistent agents, when there is no learning.

Case 2. When the naive agents learn about their true β at some period $t > 0$, then from that period on they become sophisticated and contribute according to the sophisticated agent's critical contribution levels. In all of the periods that are prior to learning the true β, the naive agents contribute more than the corresponding time-consistent agents. Thus, at the period when learning occurs, the time-inconsistent agents' total contribution is larger than the time-consistent agents. Once the naive agents become sophisticated their contribution levels are, as we know from our main result, larger than the time-consistent agents' contributions, as well. Thus, we expect that even with learning, the naive agents contribute more than the corresponding time-consistent agents.

Therefore, we argue that the present bias effect is the main driving force behind our main result, since it continues to hold even when there is no sophistication or only partial sophistication (learning about true β at an intermediate period). However, sophistication effect, as O'Donoghue and Rabin (1999a) call it, weakens the present bias effect, letting the contributions fall relative to the naivéte case.

Continuous benefit function with b=0. When there is no benefit jump, that is, when $b = 0$, the benefit function becomes continuous. The analysis and the critical contribution levels in MM and in our setting for the completion in the finite horizon both depend on the positive benefit jump assumption, $b > 0$. When $b = 0$, the critical contributions are all zero: $c(0) = b/(1-\lambda) = 0$ and thus $c(t) = 0$ for all t. However, for asymptotical completion, MM provide a result that provides the contribution levels that would asymptotically complete a given project. Here, we show that in the symmetric environment, for some parameter values, the initial contribution level is larger in our setting than the initial contribution level in MM's setting when $b = 0$.

[32] To see this, note that when $k = \infty$, we have $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta+\delta\beta}$. By Lemma 1, $\hat{\delta}_k$ is increasing in k, thus we have $\hat{\delta}_k \leq \hat{\delta}_\infty = \frac{\delta\beta}{1-\delta+\delta\beta} < \delta$, for any k.

In MM, assuming a symmetric environment with $\lambda_i = \lambda$ and $b_i = 0$, for all i, the equilibrium outcome is symmetric, $g_i(t) = g_j(t) = g(t)$ for each i and j, where $g(t) = [\frac{1-\lambda}{\delta\lambda(N-1)}]^t[1 - \frac{1-\lambda}{\delta\lambda(N-1)}]\frac{\overline{X}}{N}$ and this $\{g(t)\}_t$ completes the project asymptotically if $\delta > \frac{1-\lambda}{\lambda(N-1)}$. MM calculate $g(t)$ by solving the binding under-contributing constraints under symmetry, subject to $N\sum_{t=0}^{\infty} g(t) = \overline{X}$.

Assuming that the corresponding time-consistent agents have a discount factor equal to $\hat{\delta}_\infty = \frac{\delta\beta}{1-\delta(1-\beta)}$, we solve the binding under-contributing constraints of our sophisticated time-inconsistent agents, under symmetry and subject to $N\sum_{t=0}^{\infty} g_s(t) = \overline{X}$, and we get

$$g_s(0) = \left[\frac{\beta[K\lambda N - 1]}{\beta[K\lambda N - 1] + (1-\lambda)} - \frac{(1-\lambda)(1-\delta)}{\delta(1-\lambda) + \delta\beta[K\lambda N - 1]}\right]\frac{\overline{X}}{N}$$

where $K = \frac{1}{1-\delta(1-\beta)}$.

When the discount factor of the time-consistent agents is $\hat{\delta}_\infty$, we have $g(0) = [1 - \frac{1-\lambda}{\hat{\delta}_\infty\lambda(N-1)}]\frac{\overline{X}}{N}$.

Since $\delta > \hat{\delta}_\infty$, we have $g(0) = [1 - \frac{1-\lambda}{\hat{\delta}_\infty\lambda(N-1)}]\frac{\overline{X}}{N} < [1 - \frac{1-\lambda}{\delta\lambda(N-1)}]\frac{\overline{X}}{N}$. Thus, to show $g(0) < g_s(0)$, it is enough to show

$$1 - \frac{1-\lambda}{\delta\lambda(N-1)} < \frac{\beta[K\lambda N - 1]}{\beta[K\lambda N - 1] + (1-\lambda)} - \frac{(1-\lambda)(1-\delta)}{\delta(1-\lambda) + \delta\beta[K\lambda N - 1]}$$

Denote $\beta[K\lambda N - 1] = Y$. Thus, $g(0) < g_s(0)$ if

$$1 - \frac{1-\lambda}{\delta\lambda(N-1)} < \frac{Y}{Y + 1 - \lambda} - \frac{1}{\delta}\frac{(1-\lambda)(1-\delta)}{(1-\lambda) + Y}$$

Arranging this inequality simplifies to $\lambda N < 1 + Y$. Plugging $Y = \beta[K\lambda N - 1]$ and $K = \frac{1}{1-\delta(1-\beta)}$, we get $\lambda N(1-\delta) + \delta(1-\beta) < 1$. For large δ, small λ or small N, this inequality holds, which implies $g(0) < g_s(0)$. For instance, if $\delta = 1$, the inequality holds for any β, λ and N. Thus, the earlier contribution level is higher when the agents are time-inconsistent, which is in line with our main result and intuition.[33]

[33] Solving for $g_s(t)$ for $t \geq 1$ is quite cumbersome and does not add much here.

7 Appendix

Derivation of the under-contributing condition and the critical levels: First, for the time-inconsistent agents, we describe how we get the discounted overall payoff from the perspective of any period t in terms of within period total contributions, as in MM. Then, we derive the under and over contributing constraints, together with the critical contribution levels for the time-inconsistent agents. In what follows we use feasible and wasteless contribution sequence.

The discounted overall payoff for a time-consistent agent with a discount factor $\hat{\delta}$ is

$$U_i(z) \equiv \sum_{t=0}^{\infty} \hat{\delta}^t [(1-\hat{\delta}) f_i(X(t)) - z_i(t)]$$

which is in terms of cumulative contribution levels. This payoff function can be written in terms of within period total contributions as follows

$$\begin{aligned}
U_i(z) &= (1-\hat{\delta})\lambda_i X(0) - z_i(0) + \hat{\delta}(1-\hat{\delta})\lambda_i X(1) - \hat{\delta} z_i(1) + \hat{\delta}^2(1-\hat{\delta})\lambda_i X(2) - \hat{\delta}^2 z_i(2) \\
&+ \hat{\delta}^3(1-\hat{\delta})\lambda_i X(3) - \hat{\delta}^3 z_i(3) + ... + \hat{\delta}^{T^*-1}(1-\hat{\delta})\lambda_i X(T^*-1) - \hat{\delta}^{T^*-1} z_i(T^*-1) \\
&+ \hat{\delta}^{T^*}(1-\hat{\delta})[\lambda_i X(T^*) + b_i] - \hat{\delta}^{T^*} z_i(T^*) + \hat{\delta}^{T^*+1}(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + ... \\
&= (1-\hat{\delta})\lambda_i X(0) + \hat{\delta}(1-\hat{\delta})\lambda_i X(1) + ... + \hat{\delta}^{T^*-1}(1-\hat{\delta})\lambda_i X(T^*-1) \\
&+ \hat{\delta}^{T^*}(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + \hat{\delta}^{T^*+1}(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + ... \\
&- [z_i(0) + \hat{\delta} z_i(1) + \hat{\delta}^2 z_i(2) + ... + \hat{\delta}^{T^*} z_i(T^*)] \\
&+ \hat{\delta}^{T^*}(1-\hat{\delta})[\lambda_i[Z(0) + Z(1) + ... + Z(T^*)] + b_i] \\
&= (1-\hat{\delta})\lambda_i Z(0) + \hat{\delta}(1-\hat{\delta})\lambda_i[Z(0) + Z(1)] + ... + \hat{\delta}^{T^*-1}(1-\hat{\delta})\lambda_i[Z(0) + Z(1) + ... + Z(T^*-1)] \\
&+ \hat{\delta}^{T^*+1}(1-\hat{\delta})[\lambda_i[Z(0) + Z(1) + ... + Z(T^*)] + b_i] + ... - \sum_{t=0}^{T^*} \hat{\delta}^t z_i(t) \\
&= \sum_{t=0}^{\infty}(1-\hat{\delta})\hat{\delta}^t \lambda_i Z(0) + \hat{\delta}\sum_{t=0}^{\infty}(1-\hat{\delta})\hat{\delta}^t \lambda_i Z(1) + ... + \hat{\delta}^{T^*}\sum_{t=0}^{\infty}(1-\hat{\delta})\hat{\delta}^t \lambda_i Z(T^*) + \hat{\delta}^{T^*}\sum_{t=0}^{\infty}(1-\hat{\delta}) b_i \\
&- \sum_{t=0}^{T^*} \hat{\delta}^t z_i(t) \\
&= \sum_{t=0}^{T^*} \hat{\delta}^t [\lambda_i Z(t) - z_i(t)] + \hat{\delta}^{T^*} b_i
\end{aligned}$$

Now, we write the discounted payoff of a time-inconsistent agent from the perspective of period

$t = 0$, in terms of within period total contributions, using the same benefit function $(1-\hat{\delta})f_i(X(t))$.

$$
\begin{aligned}
U_i(z,0) &\equiv (1-\hat{\delta})f_i(X(0)) - z_i(0) + \sum_{t=1}^{\infty}\delta^t\beta[(1-\hat{\delta})f_i(X(t)) - z_i(t)] \\
&= (1-\hat{\delta})\lambda_i X(0) - z_i(0) + \delta\beta(1-\hat{\delta})\lambda_i X(1) - \delta\beta z_i(1) + \delta^2\beta(1-\hat{\delta})\lambda_i X(2) - \delta^2\beta z_i(2) \\
&+ \delta^3\beta(1-\hat{\delta})\lambda_i X(3) - \delta^3\beta z_i(3) + ... + \delta^{T^*-1}\beta(1-\hat{\delta})\lambda_i X(T^*-1) - \delta^{T^*-1}\beta z_i(T^*-1) \\
&+ \delta^{T^*}\beta(1-\hat{\delta})[\lambda_i X(T^*) + b_i] - \delta^{T^*}\beta z_i(T^*) + \delta^{T^*}\beta(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + ... \\
&= (1-\hat{\delta})\lambda_i X(0) + \delta\beta(1-\hat{\delta})\lambda_i X(1) + ... + \delta^{T^*-1}\beta(1-\hat{\delta})\lambda_i X(T^*-1) \\
&+ \delta^{T^*}\beta(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + \delta^{T^*+1}\beta(1-\hat{\delta})[\lambda_i X(T^*) + b_i] + ... \\
&- [z_i(0) + \delta\beta z_i(1) + \delta^2\beta z_i(2) + ... + \delta^{T^*}\beta z_i(T^*)] \\
&= (1-\hat{\delta})\lambda_i Z(0) + \delta\beta(1-\hat{\delta})\lambda_i[Z(0)+Z(1)] + ... + \delta^{T^*-1}\beta(1-\hat{\delta})\lambda_i[Z(0)+Z(1)+...+Z(T^*-1)] \\
&+ \delta^{T^*}\beta(1-\hat{\delta})[\lambda_i[Z(0)+Z(1)+...+Z(T^*)] + b_i] \\
&+ \delta^{T^*+1}\beta(1-\hat{\delta})[\lambda_i[Z(0)+Z(1)+...+Z(T^*)] + b_i] + ... - \sum_{t=0}^{T^*}\delta^t\beta z_i(t) \\
&= (1+\delta\beta+\delta^2\beta+\delta^3\beta+...)[(1-\hat{\delta})\lambda_i Z(0)] + \delta\beta\sum_{t=0}^{\infty}\delta^t(1-\hat{\delta})\lambda_i Z(1) \\
&+ \delta^2\beta\sum_{t=0}^{\infty}\delta^t(1-\hat{\delta})\lambda_i Z(2) + ... + \delta^{T^*}\beta\sum_{t=0}^{\infty}\delta^t(1-\hat{\delta})\lambda_i[Z(T^*) + b_i] - \sum_{t=0}^{T^*}\delta^t\beta z_i(t) \\
&= \left(\frac{1-\hat{\delta}}{1-\delta}\right)(1-\delta(1-\beta))\lambda_i Z(0) - z_i(0) + \sum_{t=1}^{T^*}\delta^t\beta\left[\left(\frac{1-\hat{\delta}}{1-\delta}\right)\lambda_i Z(t) - z_i(t)\right] + \delta^{T^*}\beta\left(\frac{1-\hat{\delta}}{1-\delta}\right)b_i \\
&= K_{\hat{\delta}}\Delta\lambda_i Z(0) - z_i(0) + \sum_{t=1}^{T^*}\delta^t\beta[K_{\hat{\delta}}\lambda_i Z(t) - z_i(t)] + \delta^{T^*}\beta K_{\hat{\delta}} b_i
\end{aligned}
$$

where $K_{\hat{\delta}} = \frac{1-\hat{\delta}}{1-\delta}$ and $\Delta = 1-\delta(1-\beta)$. Similarly, the discounted overall payoff of a time-inconsistent agent starting from a period t with $0 < t < T^*$, in terms of within period total contributions can be written as follows.

$$
\begin{aligned}
U_i(z,t) &= \left(\frac{1-\hat{\delta}}{1-\delta}\right)(1-\delta(1-\beta))\lambda_i Z(t) - z_i(t) + \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta\left[\left(\frac{1-\hat{\delta}}{1-\delta}\right)\lambda_i Z(\tau) - z_i(\tau)\right] + \delta^{T^*(z)-t}\beta\left(\frac{1-\hat{\delta}}{1-\delta}\right)b_i \\
&= K_{\hat{\delta}}\Delta\lambda_i Z(t) - z_i(t) + \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i Z(\tau) - z_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}} b_i
\end{aligned}
$$

Finally, the discounted overall payoff, starting from period $t = T^*(z)$ is given by

$$\begin{aligned} U_i(z, T^*) &= (1-\hat{\delta})f_i(X(T^*)) - z_i(T^*) + \sum_{t=1}^{\infty}\delta^t\beta(1-\hat{\delta})f_i(X(T^*)) \\ &= (1+\delta\beta+\delta^2\beta+\delta^3\beta+...)(1-\hat{\delta})[\lambda_i\overline{X}+b_i] - z_i(T^*) \\ &= (\frac{1-\hat{\delta}}{1-\delta})(1-\delta(1-\beta))[\lambda_i\overline{X}+b_i] - z_i(T^*) \\ &= K_{\hat{\delta}}\Delta[\lambda_i\overline{X}+b_i] - z_i(T^*) \end{aligned}$$

Thus, we get the utility function $U_i(z,t)$ for any $0 \leq t \leq T^*$, specified at the end of Section 2,

Now, we provide details on how to derive the under-contributing constraint. For a feasible and wasteless sequence of nonnegative contributions, $g = (g_1(t), ..., g_n(t))_{t=0}^{\infty}$, for player i not to deviate to contributing zero at a given $t < T^*$, we need $U_i(g,t) \geq U_i(g^{dev}, t)$ where

$$\begin{aligned} & U_i(g,t) \geq U_i(g^{dev}, t) \\ \iff\quad & K_{\hat{\delta}}\Delta\lambda_i G(t) - g_i(t) + \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}}b_i \geq K_{\hat{\delta}}\Delta\lambda_i G_i(t) \\ \iff\quad & \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}}b_i \geq g_i(t) + K_{\hat{\delta}}\Delta\lambda_i G_i(t) - K_{\hat{\delta}}\Delta\lambda_i G(t) \\ \iff\quad & \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}}b_i \geq g_i(t) - K_{\hat{\delta}}\Delta\lambda_i[G(t) - G_i(t)] \\ \iff\quad & \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}}b_i \geq g_i(t) - K_{\hat{\delta}}\Delta\lambda_i g_i(t) \\ \iff\quad & \sum_{\tau=t+1}^{T^*(z)}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-t}\beta K_{\hat{\delta}}b_i \geq g_i(t)[1 - \lambda_i K_{\hat{\delta}}\Delta] \end{aligned}$$

for all $i \in N$ and $t < T^*$, which is the inequality 2 in Section 3, where $K_{\hat{\delta}}\Delta\lambda_i G_i(t)$ is the discounted payoff (from period t on) when every other player sticks to the grim-g profile, but player i deviates to no contribution at t, where $G_i(t) \equiv G(t) - g_i(t)$.

When $t = T^*$, this condition becomes $U_i(g, T^*) \geq U_i(g^{dev}, T^*)$, which is equivalent to

$$\begin{aligned}
& K_{\hat{\delta}}\Delta[\lambda_i\overline{X} + b_i] - g_i(T^*) \geq K_{\hat{\delta}}\Delta\lambda_i G_i(T^*) \\
\iff \quad & K_{\hat{\delta}}\Delta[\lambda_i\overline{X} + b_i] - g_i(T^*) \geq K_{\hat{\delta}}\Delta\lambda_i[\overline{X} - g_i(T^*)] \\
\iff \quad & K_{\hat{\delta}}\Delta b_i - g_i(T^*) \geq -K_{\hat{\delta}}\Delta\lambda_i g_i(T^*) \\
\iff \quad & K_{\hat{\delta}}\Delta b_i \geq [1 - \lambda_i\Delta K_{\hat{\delta}}]g_i(T^*)
\end{aligned}$$

where the last inequality is the inequality 4 in Section 3.

Now we give details on how to derive the critical contribution levels. The under-contributing constraint for $t = T^*$ is $g_i(T^*) \leq \frac{\Delta K_{\hat{\delta}}}{1-\lambda_i\Delta K_{\hat{\delta}}}b_i$ for player i. We define $c_i(0) \equiv \frac{\Delta K_{\hat{\delta}}}{1-\lambda_i\Delta K_{\hat{\delta}}}b_i$. Given that $(c_1(0), ..., c_n(0))$ is contributed in period T^*, then the binding under-contributing constraint for $t = T^* - 1$ for player i is given by

$$\delta\beta[K_{\hat{\delta}}\lambda_i G(T^*) - g_i(T^*)] + \delta\beta K_{\hat{\delta}}b_i = g_i(T^* - 1)[1 - \lambda_i K_{\hat{\delta}}\Delta]$$

that is,

$$\delta\beta[K_{\hat{\delta}}\lambda_i \sum_j c_j(0) - c_i(0)] + \delta\beta c_i(0)\frac{1 - \lambda_i\Delta K_{\hat{\delta}}}{\Delta} = c_i(1)[1 - \lambda_i K_{\hat{\delta}}\Delta]$$

Solving for $c_i(1)$, we get

$$c_i(1) = \frac{\delta\beta}{1 - \lambda_i\Delta K_{\hat{\delta}}}[\lambda_i K_{\hat{\delta}} \sum_{j\neq i} c_j(0) + \frac{1-\Delta}{\Delta}c_i(0)]$$

Then, the binding under-contributing constraint for $t = T^* - 2$ for player i is given by

$$\sum_{\tau=T^*-1}^{T^*} \delta^{\tau-(T^*-2)}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*(z)-(T^*-2)}\beta K_{\hat{\delta}}b_i = g_i(T^* - 2)[1 - \lambda_i K_{\hat{\delta}}\Delta]$$

which implies

$$\delta\beta[K_{\hat{\delta}}\lambda_i G(T^* - 1) - g_i(T^* - 1)] + \delta^2\beta[K_{\hat{\delta}}\lambda_i G(T^*) - g_i(T^*)] + \delta^2\beta K_{\hat{\delta}}b_i = g_i(T^* - 2)[1 - \lambda_i K_{\hat{\delta}}\Delta]$$

$$\delta\beta[K_{\hat{\delta}}\lambda_i \sum_j c_j(1) - c_i(1)] + \delta^2\beta[K_{\hat{\delta}}\lambda_i \sum_j c_j(0) - c_i(0)] + \delta^2\beta K_{\hat{\delta}}b_i = c_i(2)[1 - \lambda_i K_{\hat{\delta}}\Delta]$$

$$\delta\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_iK_{\hat\delta})c_i(1)]+\delta^2\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(0)-(1-\lambda_iK_{\hat\delta})c_i(0)+K_{\hat\delta}b_i]=c_i(2)[1-\lambda_iK_{\hat\delta}\Delta]$$

which gives

$$c_i(2)(1-\lambda_iK_{\hat\delta}\Delta)=\delta\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_iK_{\hat\delta})c_i(1)]+\delta^2\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(0)-(1-\lambda_iK_{\hat\delta})c_i(0)+\frac{c_i(0)(1-\lambda_iK_{\hat\delta}\Delta)}{\Delta}]$$

$$c_i(2)(1-\lambda_iK_{\hat\delta}\Delta)=\delta\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_iK_{\hat\delta})c_i(1)]+\delta^2\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(0)+c_i(0)\frac{1-\Delta}{\Delta}]$$

$$c_i(2)(1-\lambda_iK_{\hat\delta}\Delta)=\delta\beta K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(1)-\delta\beta(1-\lambda_iK_{\hat\delta})c_i(1)+\delta^2\beta[c_i(1)\frac{(1-\lambda_iK_{\hat\delta}\Delta)}{\delta\beta}]$$

$$c_i(2)=\frac{\delta\beta K_{\hat\delta}\lambda_i}{1-\lambda_iK_{\hat\delta}\Delta}\sum_{j\neq i}c_j(1)+c_i(1)[\delta-\frac{\delta\beta(1-\lambda_iK_{\hat\delta})}{1-\lambda_iK_{\hat\delta}\Delta}]$$

which boils down to

$$c_i(2)=\frac{\delta\beta\lambda_iK_{\hat\delta}}{1-\lambda_i\Delta K_{\hat\delta}}\sum_{j\neq i}c_j(1)+\frac{\delta(1-\beta)}{1-\lambda_i\Delta K_{\hat\delta}}[1-\lambda_iK_{\hat\delta}(1-\delta)]c_i(1)$$

Before we provide the generalization for all $T^*-1\geq k\geq 2$, we find the critical levels for one more round. The binding under-contributing constraint for $t=T^*-3$ for player i is given by

$$\sum_{\tau=T^*-2}^{T^*}\delta^{\tau-(T^*-3)}\beta[K_{\hat\delta}\lambda_iG(\tau)-g_i(\tau)]+\delta^{T^*(z)-(T^*-3)}\beta K_{\hat\delta}b_i=g_i(T^*-3)[1-\lambda_iK_{\hat\delta}\Delta]$$

which implies

$$\begin{aligned}g_i(T^*-3)[1-\lambda_iK_{\hat\delta}\Delta] &= \delta\beta[K_{\hat\delta}\lambda_iG(T^*-2)-g_i(T^*-2)]\\ &+ \delta^2\beta[K_{\hat\delta}\lambda_iG(T^*-1)-g_i(T^*-1)]\\ &+ \delta^3\beta[K_{\hat\delta}\lambda_iG(T^*)-g_i(T^*)+K_{\hat\delta}b_i]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_iK_{\hat\delta}\Delta] &= \delta\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_iK_{\hat\delta})c_i(2)]\\ &+ \delta^2\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_iK_{\hat\delta})c_i(1)]\\ &+ \delta^3\beta[K_{\hat\delta}\lambda_i\sum_{j\neq i}c_j(0)-(1-\lambda_iK_{\hat\delta})c_i(0)+c_i(0)\frac{1-\lambda_iK_{\hat\delta}\Delta}{\Delta}]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta^2\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_i K_{\hat{\delta}})c_i(1)]\\ &+ \delta^3\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(0)+c_i(0)[\frac{1-\lambda_i K_{\hat{\delta}}\Delta}{\Delta}-(1-\lambda_i K_{\hat{\delta}})]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta^2\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_i K_{\hat{\delta}})c_i(1)]\\ &+ \delta^3\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(0)+c_i(0)\frac{1-\Delta}{\Delta}]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta^2\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_i K_{\hat{\delta}})c_i(1)]\\ &+ \delta^3\beta[c_i(1)\frac{1-\lambda_i K_{\hat{\delta}}\Delta}{\delta\beta}]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta^2\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)-(1-\lambda_i K_{\hat{\delta}})c_i(1)+\delta c_i(1)\frac{1-\lambda_i K_{\hat{\delta}}\Delta}{\delta\beta}]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta^2\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)+c_i(1)(\frac{1-\lambda_i K_{\hat{\delta}}\Delta}{\beta}-(1-\lambda_i K_{\hat{\delta}}))]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3)[1-\lambda_i K_{\hat{\delta}}\Delta] &= \delta\beta[K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(2)-(1-\lambda_i K_{\hat{\delta}})c_i(2)]\\ &+ \delta[\delta\beta K_{\hat{\delta}}\lambda_i\sum_{j\neq i}c_j(1)+c_i(1)\delta\beta(\frac{1-\lambda_i K_{\hat{\delta}}\Delta}{\beta}-(1-\lambda_i K_{\hat{\delta}}))]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3) &= \frac{\delta\beta K_{\hat{\delta}}\lambda_i}{1-\lambda_i K_{\hat{\delta}}\Delta}\sum_{j\neq i} c_j(2) - \delta\beta\frac{1-\lambda_i K_{\hat{\delta}}}{1-\lambda_i K_{\hat{\delta}}\Delta}c_i(2) \\ &+ \delta[\frac{\delta\beta K_{\hat{\delta}}\lambda_i}{1-\lambda_i K_{\hat{\delta}}\Delta}\sum_{j\neq i} c_j(1) + c_i(1)[\delta - \frac{\delta\beta(1-\lambda_i K_{\hat{\delta}})}{1-\lambda_i K_{\hat{\delta}}\Delta}]]\end{aligned}$$

$$\begin{aligned}\Rightarrow c_i(3) &= \frac{\delta\beta K_{\hat{\delta}}\lambda_i}{1-\lambda_i K_{\hat{\delta}}\Delta}\sum_{j\neq i} c_j(2) - \delta\beta\frac{1-\lambda_i K_{\hat{\delta}}}{1-\lambda_i K_{\hat{\delta}}\Delta}c_i(2) + \delta c_i(2) \\ &= \frac{\delta\beta K_{\hat{\delta}}\lambda_i}{1-\lambda_i K_{\hat{\delta}}\Delta}\sum_{j\neq i} c_j(2) + c_i(2)[\delta - \frac{\delta\beta(1-\lambda_i K_{\hat{\delta}})}{1-\lambda_i K_{\hat{\delta}}\Delta}] \\ &= \frac{\delta\beta K_{\hat{\delta}}\lambda_i}{1-\lambda_i K_{\hat{\delta}}\Delta}\sum_{j\neq i} c_j(2) + c_i(2)\frac{\delta(1-\beta)}{1-\lambda_i\Delta K_{\hat{\delta}}}[1-\lambda_i K_{\hat{\delta}}(1-\delta)]\end{aligned}$$

Thus, we get the critical values as follows

$$c_i(k) = \frac{\delta\beta\lambda_i K_{\hat{\delta}}}{1-\lambda_i\Delta K_{\hat{\delta}}}\sum_{j\neq i} c_j(k-1) + \frac{\delta(1-\beta)}{1-\lambda_i\Delta K_{\hat{\delta}}}[1-\lambda_i K_{\hat{\delta}}(1-\delta)]c_i(k-1)$$

for all $T^*-1 \geq k \geq 2$.

Proof of Theorem 1. We modify the proof in MM. Let T^* be the number of periods the project is completed under the grim-g profile. Then, the payoff of player i, under the grim g profile at $t=0$ is

$$U_i^*(g,0) = K_{\hat{\delta}}\Delta\lambda_i G(0) - g_i(0) + \sum_{\tau=1}^{T^*}\delta^\tau\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*}K_{\hat{\delta}}\beta b_i$$

and her payoff from period t on is

$$U_i^*(g,t) = K_{\hat{\delta}}\Delta\lambda_i X(t) - g_i(t) + \sum_{\tau=t+1}^{T^*}\delta^{\tau-t}\beta[K_{\hat{\delta}}\lambda_i G(\tau) - g_i(\tau)] + \delta^{T^*-t}K_{\hat{\delta}}\beta b_i$$

Since, in our setting, the discounting changes over time, we need to check the deviations for any period t, *from the perspective of that period.* Suppose player i contributes a non-completing $z_i \neq g_i(t)$ in period t and then never contributes again. Then, her payoff from such deviation, from period t on, is

$$U_i'(z_i,t) = K_{\hat{\delta}}\Delta\lambda_i[X(t) - g_i(t) + z_i] - z_i = K_{\hat{\delta}}\Delta\lambda_i[X(t) - g_i(t)] - (1-K_{\hat{\delta}}\Delta\lambda_i)z_i$$

Since $1 - K_{\hat{\delta}}\Delta\lambda_i > 0$ and $z_i \geq 0$, we have $U_i'(z_i,t) \leq U_i'(0,t)$. For this type of deviation not to be profitable, we need $U_i^*(g,t) \geq U_i'(0,t)$, which, after arranging, is equivalent to the under-contributing constraint:

$$[1-\lambda_i \Delta K_{\hat{\delta}}]g_i(t) \leq \delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)]$$

for each $t < T^*$ and for all i. And for $t = T^*$, the condition $U_i^*(g,T^*) \geq U_i'(0,T^*)$ is equivalent to

$$K_{\hat{\delta}}\Delta[\lambda_i \overline{X} + b_i] - g_i(T^*) \geq K_{\hat{\delta}}\Delta\lambda_i[\overline{X} - g_i(T^*)]$$

which, once arranged, is equivalent to $[1-\lambda_i\Delta K_{\hat{\delta}}]g_i(T^*) \leq \Delta K_{\hat{\delta}} b_i$.

Now, suppose player i contributes an amount $\hat{z}_i(t) = g_i(t) + \overline{X} - \sum\limits_{\tau=0}^{t} G(\tau)$ in period $t < T$, hence finishes the project at period $t < T$ immediately. Then, her payoff from period t on, is

$$\begin{aligned} U_i''(\hat{z}_i,t) &= K_{\hat{\delta}}\Delta\lambda_i[\overline{X}] + \Delta K_{\hat{\delta}} b_i - \hat{z}_i \\ &= K_{\hat{\delta}}\Delta\lambda_i[X(t) - g_i(t) + \hat{z}_i] + \Delta K_{\hat{\delta}} b_i - \hat{z}_i \\ &= K_{\hat{\delta}}\Delta\lambda_i[X(t) - g_i(t)] + \Delta K_{\hat{\delta}} b_i - (1 - K_{\hat{\delta}}\Delta\lambda_i)\hat{z}_i \end{aligned}$$

Then, the condition, for no such deviation to occur at $t < T^*$, namely $U_i^*(g,t) \geq U_i''(\hat{z}_i,t)$, is equivalent to

$$\delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \geq \Delta K_{\hat{\delta}} b_i - (1-\lambda_i K_{\hat{\delta}}\Delta)(\hat{z}_i - g_i(t))$$

which is

$$\delta^{T^*-t}\beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t}\beta[\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] \geq \Delta K_{\hat{\delta}} b_i - (1-\lambda_i K_{\hat{\delta}}\Delta)(\overline{X} - X(t))$$

since $\hat{z}_i(t) - g_i(t) = \overline{X} - \sum\limits_{\tau=0}^{t} G(\tau) = \overline{X} - X(t)$. Finally, consider a deviation in which player i contributes an amount $z_i \neq g_i(t)$ in period $t < T$, which does not complete the project in period t, and then contributes a positive amount at some $t' > t$ as well. Note that $U_i''(\hat{z}_i,t) = U_i'(0,t) + \Delta K_{\hat{\delta}} b_i - (1 - K_{\hat{\delta}}\Delta\lambda_i)\hat{z}_i$. Thus, this type of deviation is dominated by contributing zero in every period following period t, whenever $\Delta K_{\hat{\delta}} b_i \leq (1 - K_{\hat{\delta}}\Delta\lambda_i)\hat{z}_i$. Whenever $\Delta K_{\hat{\delta}} b_i > (1 - K_{\hat{\delta}}\Delta\lambda_i)\hat{z}_i$, it is again dominated, this time by contributing exactly $\hat{z}_i(t) = g_i(t) + \overline{X} - \sum_{\tau=0}^{t} G(\tau)$ in period t.

Note that the three type of deviations considered above are exhaustive, that is, there is no other deviation we need to check. Thus, for player i, any kind of deviation becomes non-profitable if and only if $U_i'(0,t) \leq U_i^*(g)$ for each $t \leq T^*$ and $U_i''(t) \leq U_i^*(g)$ for each $t < T^*$. These two conditions for each i are equivalent to the under-contributing and over-contributing constraints as shown above. Thus, under-contributing and over-contributing constraints are necessary and sufficient for the grim-g outcome to be a Nash equilibrium outcome. ■

Proof of Corollary 1. If $T^* = 0$, over-contributing constraint is vacuously satisfied. Let $T^* > 0$ and $t < T^*$. Then, condition (ii) and (iii) independently imply $g_i(t) + \sum_{\tau=t+1}^{T^*} G(\tau) \geq c_i^*$ for all i. Since, $1 > \lambda_i \Delta K_{\hat{\delta}}$, $c_i^* = \frac{\Delta K_{\hat{\delta}}}{1-\lambda_i \Delta K_{\hat{\delta}}} b_i$ and $\overline{X} \geq \sum_{\tau=0}^{T^*} G(\tau)$, we get

$$(\lambda_i K_{\hat{\delta}} \Delta - 1)(\overline{X} - \sum_{\tau=0}^{t} G(\tau)) + K_{\hat{\delta}} \Delta b_i \leq (1 - \lambda_i K_{\hat{\delta}} \Delta) g_i(t)$$

Then, the under-contributing constraint implies

$$\begin{aligned} \delta^{T^*-t} \beta K_{\hat{\delta}} b_i + \sum_{\tau=t+1}^{T^*} \delta^{\tau-t} \beta [\lambda_i K_{\hat{\delta}} G(\tau) - g_i(\tau)] &\geq [1 - \lambda_i \Delta K_{\hat{\delta}}] g_i(t) \\ &\geq \Delta K_{\hat{\delta}} b_i - (1 - \lambda_i K_{\hat{\delta}} \Delta)(\overline{X} - X(t)) \end{aligned}$$

which is the over-contributing constraint. ■

Proof of Lemma 1. First note that $1 + \hat{\delta}_t + \hat{\delta}_t^{\ 2} + ... + \hat{\delta}_t^{\ t} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$ implies $\hat{\delta}_t(1 + \hat{\delta}_t + ... + \hat{\delta}_t^{\ t-1}) = \beta\delta(1 + \delta + ... + \delta^{t-1})$. Since, $\delta > \hat{\delta}_t$, we have $\hat{\delta}_t > \delta\beta$. Also, $\beta\delta^t > \hat{\delta}_t^{\ t}$. To see this, suppose otherwise, that is, assume $\hat{\delta}_t^{\ t} \geq \beta\delta^t$. Then, $\frac{\beta\delta^t}{\delta} < \frac{\hat{\delta}_t^t}{\hat{\delta}}$ since $\delta > \hat{\delta}_t$. Thus, $\hat{\delta}_t^{t-1} > \beta\delta^{t-1}$. Repeating this argument we get $\hat{\delta}_t^s > \beta\delta^s$ for all $s = 1, ..., t$, which implies $1 + \hat{\delta}_t + \hat{\delta}_t^{\ 2} + ... + \hat{\delta}_t^{\ t} > 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$, which is a contradiction. Thus, we get $\hat{\delta}_t^{\ t} < \beta\delta^t$. Now, to see $\hat{\delta}_t$ is strictly increasing in t, add $\hat{\delta}_t^{t+1}$ to the left hand side, and $\beta\delta^{t+1}$ to the right hand side of

$$1 + \hat{\delta}_t + \hat{\delta}_t^2 + ... + \hat{\delta}_t^t = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t$$

and get

$$1 + \hat{\delta}_t + \hat{\delta}_t^2 + ... + \hat{\delta}_t^t + \hat{\delta}_t^{t+1} < 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t + \beta\delta^{t+1}$$

since $\hat{\delta}_t^{t+1} < \beta\delta^{t+1}$, which is because $\hat{\delta}_t < \delta$ and $\hat{\delta}_t^{\,t} < \beta\delta^t$. Thus, to get the equality

$$1 + \hat{\delta}_{t+1} + \hat{\delta}_{t+1}^2 + ... + \hat{\delta}_{t+1}^t + \hat{\delta}_{t+1}^{t+1} = 1 + \beta\delta + \beta\delta^2 + ... + \beta\delta^t + \beta\delta^{t+1}$$

we must have $\hat{\delta}_{t+1} > \hat{\delta}_t$. ■

Proof of Lemma 2. First note that $K_{\hat{\delta}_\infty} = \frac{1-\hat{\delta}_\infty}{1-\delta} = \frac{1}{1-\delta}(1 - \frac{\delta\beta}{1-\delta+\delta\beta}) = \frac{1}{1-\delta}\frac{1-\delta}{1-\delta+\delta\beta} = 1/\Delta$, that is, $\Delta K_{\hat{\delta}_\infty} = 1$. Also note that $K_{\hat{\delta}_t} = \frac{1-\hat{\delta}_t}{1-\delta}$ is strictly decreasing in $\hat{\delta}_t$, thus strictly decreasing in t by Lemma 1 above. Thus, $K_{\hat{\delta}_t} > K_{\hat{\delta}_\infty} = 1/\Delta$. Thus, $\Delta K_{\hat{\delta}_t} \geq 1$ for any t. ■

Proof of Lemma 3. We are done if we show that $\sum_{k=0}^{t} c^{\hat{\delta}_\infty}(k) - \sum_{k=0}^{t-1} c^{\delta\beta}(k)$ decreases in t, for all $t \geq 2$. That is, we need to show, for all $t \geq 2$

$$\sum_{k=0}^{t} c^{\hat{\delta}_\infty}(k) - \sum_{k=0}^{t-1} c^{\delta\beta}(k) > \sum_{k=0}^{t+1} c^{\hat{\delta}_\infty}(k) - \sum_{k=0}^{t} c^{\delta\beta}(k)$$

This inequality holds if and only if $c^{\delta\beta}(t) > c^{\hat{\delta}_\infty}(t+1)$, for each $t \geq 2$. We have $c^{\hat{\delta}_\infty}(0) = c^{\delta\beta}(0) = \frac{b}{1-\lambda}$. Also,

$$c^{\delta\beta}(1) = Ac(0)$$
$$c^{\delta\beta}(t) = AB^{t-1}c(0) \quad \forall t \geq 2$$
$$c^{\hat{\delta}_\infty}(t) = D^t c(0) \quad \forall t \geq 1$$

where
$A = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right]$,
$B = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)} + \delta(1-\delta)(1-\lambda)\frac{\beta(1-\beta)}{1-\delta(1-\beta)}\right]$,
$D = \frac{\delta}{1-\lambda}\left[\lambda(N-1)\frac{\beta}{1-\delta(1-\beta)}\right]$.

Thus, we have $c^{\delta\beta}(t) > c^{\hat{\delta}_\infty}(t+1)$ if and only if $AB^{t-1} > D^{t+1}$. Also note that $A > B > D > 0$. Therefore, if $D \leq 1$, then $AB^{t-1} > D^{t+1}$ clearly holds. If $D > 1$, on the other hand; since $B > D$, if $AB \geq D^3$, then $AB^{t-1} > D^{t+1}$ for all $t > 2$. Our assumption, $\sum_{k=0}^{1} c^{\delta\beta}(k) \geq \sum_{k=0}^{2} c^{\hat{\delta}_\infty}(k)$, implies $A \geq D + D^2$, which implies $AB \geq BD + BD^2 > BD^2 > D^3$. Thus, we have shown $AB^{t-1} > D^{t+1}$, which implies $c^{\delta\beta}(t) > c^{\hat{\delta}_\infty}(t+1)$, which finishes the proof. ■

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