NAME:

NO:

Boğaziçi University
Department of Economics
Fall 2015
EC 301 ECONOMICS OF INDUSTRIAL ORGANIZATION
MIDTERM 2
25/12/2015
Friday 17:30

- **Do not forget to write your full name and student number on the top.**
- **Turn off your cell phone and put it away. If you are seen with a cell phone, on or off, 50 points will be taken off immediately.**
- **Put away all your lecture notes, books, etc.**
- **There are 4 questions and 8 pages in the exam. Make sure you have them all.**
- **Please answer all of the questions in the space provided for each question.**
- **Show your work.**
- **You have 120 minutes.**

GOOD LUCK!!

1. **(22 pts)** In a market, there are two firms, A and B, competing in prices simultaneously. Firms are producing differentiated products. Each firm has a constant marginal cost, $c_A = c_B = 4c$ with $c > 0$. Firms have no fixed cost. Denoting the prices with p_A and p_B, firm A faces a demand given by $q_A(p_A, p_B) = 260 - p_A + p_B/2$, and firm B faces a demand given by $q_B(p_A, p_B) = 320 - 2p_B + p_A$.

   (a) (10 pts) Find the Nash equilibrium prices.

   Answer:

   The profit of firm A is

$$\pi_A = (260 - p_A + \frac{p_B}{2})(p_A - 4c)$$

   The first order condition gives us the best response,

$$p_A = 130 + 2c + \frac{p_B}{4}$$

   Likewise, the profit for firm B is

$$\pi_B = (320 - 2p_B + p_A)(p_B - 4c)$$

   The first order condition gives us the best response,

$$p_B = 80 + 2c + \frac{p_A}{4}$$

   Solving these two best responses we get

$$p_A = 160 + \frac{8c}{3}$$

$$p_B = 120 + \frac{8c}{3}$$

   (b) (12 pts) Now suppose that firm A is regulated through a price cap: it **cannot charge a price higher than** $\overline{p}_A$. Find the Nash equilibrium prices in terms of $\overline{p}_A$. Provide intuition for the effect of $\overline{p}_A$ on the equilibrium price of firm B.

   Answer:

   If $\overline{p}_A \geq 160 + \frac{8c}{3}$, then the equilibrium prices are same as in (a). If however $\overline{p}_A < 160 + \frac{8c}{3}$, then best response of firm A is given by

$$BR_A = \begin{cases} 130 + 2c + \frac{p_B}{4} & \text{if} \quad p_B \leq 4\overline{p}_A - 521 - 8c \\ \overline{p}_A & \text{if} \quad p_B > 4\overline{p}_A - 521 - 8c \end{cases}$$

   where $4\overline{p}_A - 521 - 8c$ solves for p_B in $\overline{p}_A = 130 + 2c + \frac{p_B}{4}$. Thus, when $\overline{p}_A < 160 + \frac{8c}{3}$ we have $p_B = 80 + 2c + \frac{\overline{p}_A}{4}$. To sum up, the equilibrium in terms of $\overline{p}_A$ is given by

$$(p_A, p_B) = \begin{cases} (160 + \frac{8c}{3}, 120 + \frac{8c}{3}) & \text{if} \quad \overline{p}_A \geq 160 + \frac{8c}{3} \\ (\overline{p}_A, 80 + 2c + \frac{\overline{p}_A}{4}) & \text{if} \quad \overline{p}_A < 160 + \frac{8c}{3} \end{cases}$$

2. **(30 pts)** In a market for a homogeneous product, there is an incumbent firm, Firm I, and there is a potential entrant, Firm E. The inverse demand function is given by $P = 270 - Q$, where $Q = q_I + q_E$ and q_i is the quantity level of firm i, $i = I, E$. One unit of production requires one unit of labor and one unit of capacity. Incumbent invests in capacity, k_I, and publicly announces it. Then, Firm E, after observing k_I, decides whether or not to enter. If Firm E enters, firms compete in quantities. During the quantity competition, firm I can expand its capacity if needed and firm E builds its capacity and produces its output at the same time. The cost of capacity is 40 and the cost of labor is 30, for both firms. Capacity cost is sunk. There is an entry cost for the potential entrant, given by $f > 0$. Also, the potential entrant has a constraint: if it enters, due to governmental regulations, it **has to produce at least an output level** $\underline{q}_E = 60$, which is public information.

(a) (10 pts) Find the best response function of the potential entrant in the quantity competition stage.

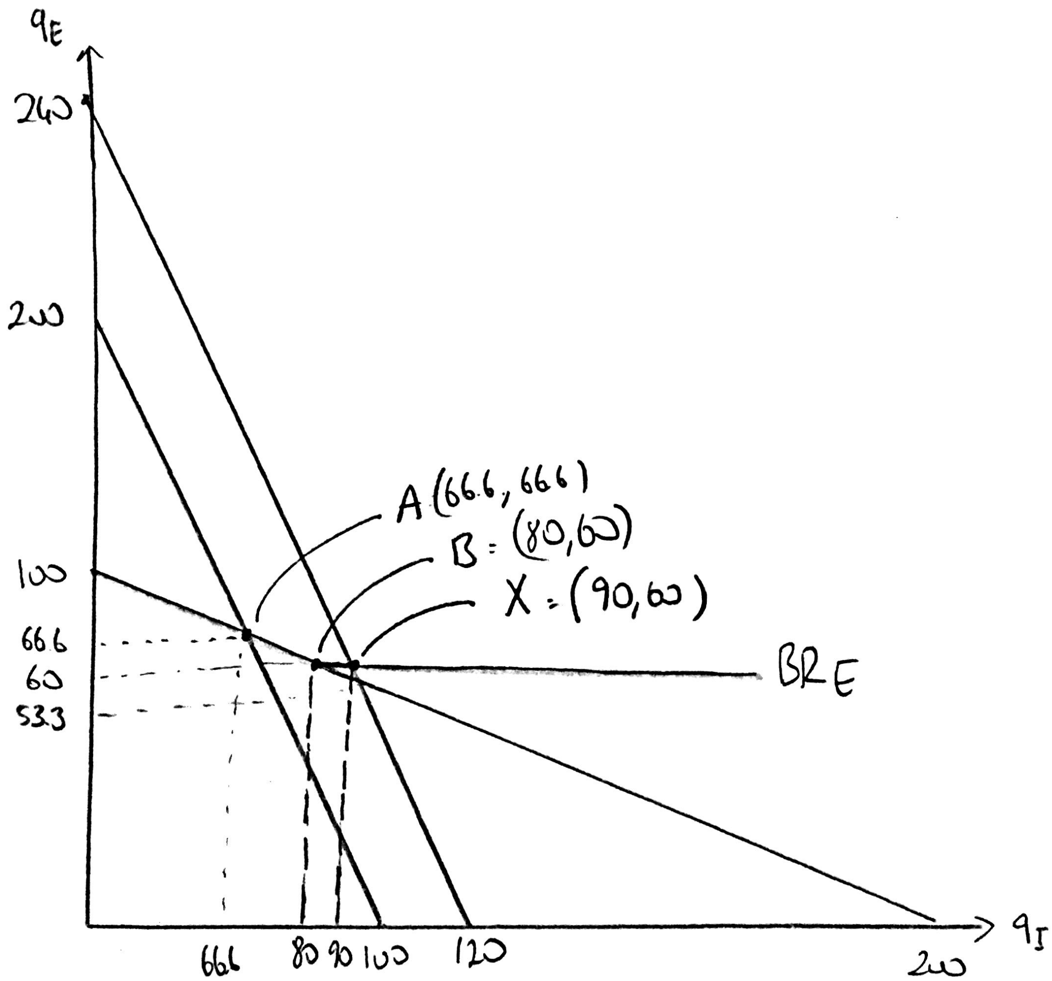
Answer:

The profit of the potential entrant is given by $\pi_E = (270 - q_I - q_E - 70)q_E - f$.

Maximizing this with respect to q_E we get $q_E = 100 - \frac{q_I}{2}$.

Then the best response is given by

$$BR_E = \begin{cases} 100 - \frac{q_I}{2} & \text{if} \quad q_I \leq 80 \\ 60 & \text{if} \quad q_I > 80 \end{cases}$$



(b) (10 pts) Find a range of values of f for which there is no entry regardless of k_I.

Answer:

At point A in the figure above, we have

$\pi_E = (270 - \frac{200}{3} - \frac{200}{3} - 70)\frac{200}{3} - f = (\frac{200}{3})^2 - f.$

If $\pi_E < 0$, then there is no entry regardless of k_I, that is, if $f > (\frac{200}{3})^2$, then there is no entry regardless of k_I.

(c) (10 pts) Let $f = 3000$. What will be the equilibrium capacity and the quantities?

Answer:

At point X above, $\pi_E(X) = (270 - 90 - 60 - 70)60 - 3000 = 3000 - 3000 = 0$

At point B above, $\pi_E(B) = (270 - 80 - 60 - 70)60 - 3000 = 3600 - 3000 = 600$

If $f = 3000$, then

(i) assuming potential entrant enters when her profit is 0, there will be entry regardless of k_I. Thus, we have with $k_I = q_I = 90$, the profit of the incumbent is 4500 ($q_E = 60$) and with $k_I = q_I = 80$, the profit of the incumbent is 4800 (again $q_E = 60$). Thus, the equilibrium capacity and quantities are $k_I^* = q_I^* = 80$ with $q_E^* = 60$.

(ii) assuming the potential entrant does not enter when her profit is 0, then $k \geq 90$ will deter entry. Since the monopoly output level is $q^M = 100$, the incumbent simply picks $k_I^* = q_I^* = 100$ and entry is deterred.

3. **(24 pts)** Consider a linear city that stretches along a $1km$ distance. Suppose N many consumers are distributed uniformly along the city which can be best represented by an interval $[0, 1]$. There are two firms, A and B, producing the same good. Firm A is located at the end point 0. Firm B is located at the other end point 1. Each firm has zero marginal cost and zero fixed cost. Each consumer wants to buy exactly one unit from the firm which has the smallest overall cost: sum of the firm's price and consumer's cost of going to and coming back from the firm. If a consumer is at a distance x to the firm she buys from, she incurs a transportation cost equal to $2tx$, which is the entire transportation cost of the round trip.

(a) (12 pts) Suppose firm A first announces its price, P_A, and then observing P_A, firm B announces its own price P_B. After both prices are announced, each consumer decides from which firm to buy one unit. Find the equilibrium prices, P_A and P_B. Is the equilibrium symmetric? What portion of the consumers buy from A? Provide intuition.

Answer:

At prices P_A and P_B, the indifferent buyer is located at x where $P_A + 2tx = P_B + 2t(1 - x)$, which gives

$x = \frac{P_B - P_A}{4t} + \frac{1}{2}$ and $1 - x = \frac{P_A - P_B}{4t} + \frac{1}{2}$

Then, the profit of B is given by $\pi_B = N(1 - x)P_B = N(\frac{P_A - P_B}{4t} + \frac{1}{2})P_B$

Maximizing π_B with respect to P_B we get $P_B = t + \frac{P_A}{2}$.

Then the profit of firm A is given by $\pi_A = N(x)P_A = N(\frac{P_B - P_A}{4t} + \frac{1}{2})P_A$

Inserting $P_B = t + \frac{P_A}{2}$ in π_A above we get

$\pi_A = N(\frac{(t + \frac{P_A}{2}) - P_A}{4t} + \frac{1}{2})P_A = N(\frac{3}{4} - \frac{P_A}{8t})P_A$

Maximizing π_A with respect to P_A we get $P_A = 3t$. Then inserting this into P_B we get $P_B = 5t/2$.

Thus, the equilibrium prices are $P_A^* = 3t$ and $P_B^* = 5t/2$.

And at these prices we have $x^* = \frac{P_B^* - P_A^*}{4t} + \frac{1}{2} = 3/8$.

(b) (12 pts) Now, suppose that the firms announce prices simultaneously and once the prices are announced each consumer decides from which firm to buy one unit. Also, now firm A has an option to introduce a shuttle service for the consumers who buy from it. The shuttle will be free for the consumers buying from firm A, but it will have an operating cost to the firm A, which is $f(r) = Nr$ where r is the ratio of the demand who buys from firm A. Firm B does not have this option, so the round trip to firm B is still $2tx$ for any consumer with a distance x to firm B. Will firm A choose to introduce the shuttle service when $t = 1$? How about when $t = 0.1$? Provide intuition.

Answer:

Case 1. No shuttle.

Again, at prices P_A and P_B, the indifferent buyer is located at x where $P_A + 2tx = P_B + 2t(1-x)$, which gives $x = \frac{P_B - P_A}{4t} + \frac{1}{2}$ and $1 - x = \frac{P_A - P_B}{4t} + \frac{1}{2}$.

Then the profit of firm A is given by $\pi_A = N(x)P_A = N(\frac{P_B - P_A}{4t} + \frac{1}{2})P_A$. Maximizing with respect to P_A we get the best response, $P_A = \frac{P_B}{2} + t$.

And, the profit of firm B is given by $\pi_A = N(1-x)P_B = N(\frac{P_A - P_B}{4t} + \frac{1}{2})P_B$. Maximizing with respect to P_B we get the best response, $P_B = \frac{P_A}{2} + t$.

Then the equilibrium prices are obtained by solving these two best responses at the same time and we get $P_A = P_B = 2t$ and $x = 1/2$.

Then $\pi_A = NxP_A = N(1/2)2t = Nt$.
When $t = 1$ the profit of A is $\pi_A = N$.
When $t = 0.1$, the profit of A is $\pi_A = 0.1N$.

Case 2. Shuttle.

Again, at prices P_A and P_B, when firm A has a free shuttle, the indifferent buyer is located at x where $P_A = P_B + 2t(1-x)$. Thus, $x = \frac{P_B - P_A}{2t} + 1$ and $1 - x = \frac{P_A - P_B}{2t}$.

Then, $\pi_A = NxP_A - Nx = Nx(P_A - 1) = N(\frac{P_B - P_A}{2t} + 1)(P_A - 1)$. Maximizing π_A with respect to P_A we get A's best response $P_A = \frac{P_B}{2} + t + \frac{1}{2}$

And $\pi_B = N(1-x)P_B = N(\frac{P_A - P_B}{2t})(P_B)$. Maximizing π_B with respect to P_B we get B's best response $P_B = \frac{P_A}{2}$

Solving $P_A = \frac{P_B}{2} + t + \frac{1}{2}$ and $P_B = \frac{P_A}{2}$ together we get $P_A = \frac{4t+2}{3}$ and $P_B = \frac{2t+1}{3}$ and $x = \frac{4t-1}{6t}$

When $t = 1$ we have $P_A = 2$ and $P_B = 1$ and $x = \frac{1}{2}$. $\pi_A(t = 1) = Nx(P_A - 1) = N(1/2)(2-1) = N/2 < N$ where $\pi_A(t = 1) = N$ is the no shuttle profit. Thus, no shuttle is better.

When $t = 0.1$ we have $P_A = 0.8$ and $P_B = 0.4$ and $x = 0$. $\pi_A(t = 1) = Nx(P_A - 1) = 0 < 0.1N$ where $\pi_A(t = 0.1) = 0.1N$ is the no shuttle profit. Thus, no shuttle is better.

4. **(24 pts)** Suppose that there are two firms, A and B, producing a homogenous product, and they are engaged in an **infinitely repeated game of price competition.** Both firms have zero marginal cost and no fixed cost. Each firm's action in a period is to choose a price from $[0, \infty)$. The lowest-priced firm gets all the demand, unless there is a tie. Whenever there is a tie in prices, A gets 3/4 portion of the demand and B gets 1/4 portion of the demand. They both have the same discount factor $\delta \in [0, 1]$.

   (a) (12 pts) Suppose the demand in each period is $Q(P) = 100 - P$. Consider the following grim-trigger strategy: play collusive price (monopoly price) in a period, as long as the other firm has played the collusive price so far; if the other firm has deviated to some other price in a period, t, then start the punishment phase (marginal cost pricing) from next period on with probability r, play collusive price with $1 - r$ probability. When you start the punishment phase at $t + 1$, keep it forever. If you do not start the punishment phase at $t + 1$, start it at $t + 2$ for sure. Find a condition on δ and r, which guarantees the collusive outcome to be played every period under this particular grim-trigger strategy. How does an increase in r affect collusion possibility?

   Answer:

   For B we need: $\frac{\pi^M}{4}(1 + \delta + \delta^2 + ...) \geq \pi^M + r \cdot \delta \cdot 0 + (1 - r)(\delta \frac{\pi^M}{4} + \delta^2 \cdot 0 + \delta^3 \cdot 0 + ...)$

   That is, $\frac{1}{4} \frac{1}{1-\delta} \geq 1 + (1 - r)\frac{\delta}{4}$, which is $1 \geq (1 - \delta)(4 + \delta(1 - r))$. $(\star)$

   For A we need: $\frac{3\pi^M}{4}(1 + \delta + \delta^2 + ...) \geq \pi^M + r \cdot \delta \cdot 0 + (1 - r)(\delta \frac{3\pi^M}{4} + \delta^2 \cdot 0 + \delta^3 \cdot 0 + ...)$

   That is, $\frac{3}{4} \frac{1}{1-\delta} \geq 1 + (1 - r)\frac{3\delta}{4}$, which is $1 \geq (1 - \delta)(\frac{4}{3} + \delta(1 - r))$. $(\star\star)$

   Since $(\star)$ implies $(\star\star)$, all we need is $1 \geq (1 - \delta)(4 + \delta(1 - r))$.

   Note that as r goes up the right hand side of the inequality decreases, thus it becomes easier to get cooperation.

(b) (12 pts) Now, suppose the demand in each period is stochastic and it can be either 0 or $Q(P) = 100 - P$, with probabilities 1/3 and 2/3, respectively. Suppose that the firms do not observe the demand realizations and the price of the other firm, within any period. Each firm observes only its own profit level at the end of a period. Consider the grim trigger strategy with the following punishment structure: Start with playing collusive price (monopoly price) in the first period and keep playing collusive price. But when you get a zero profit in a period t, play collusive price for exactly one more period at $t+1$, and then start the punishment phase (marginal cost pricing) at $t+2$, even if the profit in period $t+1$ is positive. When you start the punishment phase, keep it forever. Is there any δ value for which the collusive prices are sustainable under this particular strategy?

Answer:

$V^- = 0$

$V^4 = V = \frac{2}{3}(s\pi^M + \delta V) + \frac{1}{3}(0 + \frac{2}{3}\delta s\pi^M + \frac{1}{3}0 = \frac{2}{3}s\pi^M + \frac{2}{3}\delta V + \frac{2}{9}\delta s\pi^M.$

Solve $V = \frac{2}{3}s\pi^M + \frac{2}{3}\delta V + \frac{2}{9}\delta s\pi^M$ for V and get $V = \frac{2s\pi^M}{3}\frac{3+\delta}{3-2\delta}$.

We need $V \geq \frac{2}{3}\pi^M + \frac{2}{9}\delta s\pi^M$. Thus, we need $\frac{2s\pi^M}{3}\frac{3+\delta}{3-2\delta} \geq \frac{2}{3}\pi^M + \frac{2}{9}\delta s\pi^M$

That is, $3s \geq \frac{3-2\delta}{3+\delta}(3 + \delta s)$, or equivalently $2\delta^2 s + 6\delta + 9s \geq 9 \quad (\star)$

For $s = 3/4$, $(\star)$ becomes $2\delta^2 + 8\delta \geq 3$.

For $s = 1/4$, $(\star)$ becomes $2\delta^2 + 24\delta \geq 27$, but this never holds since even at $\delta = 1$, left hand side is at most 26.

Thus, there is no such δ for which collusive prices are sustainable under this strategy.