Boğaziçi University
Department of Economics
Fall 2016
EC 301 ECONOMICS of INDUSTRIAL ORGANIZATION
Problem Set 4 - Answer Key

1. Suppose there are two firms competing in quantities over infinitely many repeated periods where the market demand function is given by $p(Q) = p(q_1 + q_2)$, and π^M and π^C denote the monopoly and Cournot profits, respectively. Suppose that firms have no fixed cost, and they both have zero marginal cost. Find a condition that ensures that the grim trigger strategies with T-period punishments (*play collusion output until you see the other firm deviating to some other output, and once there is deviation play Cournot output from the next period on for exactly T periods and then go back to collusion output*) constitute an equilibrium. How does the threshold discount factor depend on T? Elaborate.

   Answer: If they both play this strategy, each gets a discounted profit of $\frac{\pi^M/2}{1-\delta}$. Suppose (given the other firm sticks to this strategy) one firm deviates to q^D (which is his best response to $q^M/2$ in the one shot game) at period t. Say, this deviation brings π^D for that period of deviation. Then, for the next T periods this firm gets π^C, and then starts getting $\pi^M/2$ again and gets this till the end. So, the present value of the payoff from this deviation is

   $$V^D = (\pi^M/2)\frac{1-\delta^{t-1}}{1-\delta} + \delta^{t-1}\pi^D + \delta^t\pi^C + \delta^{t+1}\pi^C + ... + \delta^{T+t-1}\pi^C + \delta^{T+t}(\pi^M/2) + \delta^{T+t}(\pi^M/2) + ...$$

   For this deviation to be not profitable we need the following

   $$\delta^{t-1}\pi^D + \delta^t\pi^C + \delta^{t+1}\pi^C + ... + \delta^{T+t-1}\pi^C \leq \delta^{t-1}(\pi^M/2) + \delta^t(\pi^M/2) + ... + \delta^{T+t-1}(\pi^M/2)$$

   which reduces to the following condition

   $$(1-\delta)\pi^D + \delta(1-\delta^T)\pi^C \leq (1-\delta^{T+1})\pi^M/2$$

   Thus for all such δ that satisfies the above condition, we get an equilibrium. Let the threshold discount factor be $\overline{\delta}$ such that

   $$(1-\overline{\delta})\pi^D + \overline{\delta}(1-\overline{\delta}^T)\pi^C = (1-\delta^{T+1})\pi^M/2$$

   So for any $\delta \geq \overline{\delta}$ we have an equilibrium. Note that as T goes up the punishment length is longer, thus there is bigger incentives to not deviate. Thus, with a larger T, it must be easier to sustain collusion, thus the threshold discount factor should decrease (hence for a larger set of discount factors, it'll be an equilibrium).

2. Suppose there are two firms competing in quantities over infinitely many repeated periods where the market demand function is given by $p(Q) = a - Q = a - (q_1 + q_2)$. Suppose that firms have no fixed cost, and they both have zero marginal cost. Find the threshold discount factor that would ensure that the grim trigger strategies (play collusion output until you see the other firm deviating to some other output, and once there is deviation play Cournot output from the next period on) constitute an equilibrium. Does it depend on a?

   Answer: We know $\delta \geq \bar{\delta} = \dfrac{\pi^D - \dfrac{\pi^M}{2}}{\pi_D - \pi_C}$ is the condition needed to sustain cooperation in the equilibrium. So we only need to find π^M, π^C and π^D ; monopoly profit, Cournot profit and profit from one-shot deviation, respectively.
   $p = a - Q \; MC = 0$
   $\pi^M : q^M = a/2 \Longrightarrow \pi^M = (a - q^M)q^M = a^2/4$
   $\pi^C : q^C = a/3, Q^C = 2a/3, p^C = a/3 \Longrightarrow \pi^C = a^2/9$
   π^D : Given other firm is playing the cooperation output $(\dfrac{q^M}{2})$, what is the deviation output?
   $q_1^D = \dfrac{a - q_2}{2} = \dfrac{a - q^M/2}{2} = \dfrac{a - a/4}{2} = \dfrac{3a}{8}$. Then
   $p^D = a - Q^D = a - q_1^D - q_2 = a - 3a/8 - a/4 = \dfrac{3a}{8}$
   So, $\pi^D = p^D q^D = \dfrac{3a}{0}\dfrac{3a}{8} = \dfrac{9a^2}{64}$
   Thus, $\bar{\delta} = \dfrac{9a^2/64 - a^2/8}{9a^2/64 - a^2/9} = \dfrac{9/64 - 1/8}{9/64 - 1/9} \cong 0.53$ and it does not depend on a.

3. Suppose there are two firms competing in quantities over infinitely many repeated periods where the market demand function is given by $p(Q) = a - Q = a - (q_1 + q_2)$. Suppose that the firms have no fixed cost, and they both have zero marginal cost. Suppose that they use the following strategy: At $t = 0$, start with collusion quantity, then at any later period $t > 0$, pick the other firm's $t - 1$ quantity. Find the threshold discount factor that would ensure that this strategy constitutes an equilibrium.

   Answer: If they both play this strategy, each gets a discounted profit of $\frac{\pi^M/2}{1-\delta}$. Suppose (given the other firm sticks to this strategy) one firm deviates to q^D (which is his best response to $q^M/2$ in the one shot game) at period t. This deviation brings $\pi^M/2$ for first $t - 1$ periods, π^D for period t. So π^D is the profit of deviating firm where he picks q^D and the other firm picks $q^M/2$. Suppose the other firm gets π^W at these quantities. Now, at period $t + 1$ going back to the equilibrium strategy, they do exactly what the other firm has done in t, and so on. So at $t + 1$, the deviating firm picks $q^M/2$ and the other firm picks q^D, at $t + 2$ they alternate the quantities. So the deviating firm gets π^W at $t + 1, t + 3, t + 5, ...$ and π^D at $t, t + 2, t + 4, ...$ This stream of profits discounted is given by:

$$V^D = (\pi^M/2)\frac{1-\delta^{t-1}}{1-\delta} + \pi^D\frac{\delta^{t-1}}{1-\delta^2} + \pi^W\frac{\delta^t}{1-\delta^2}$$

Comparing this discounted deviation payoff with the equilibrium payoff we get:

$$\delta \geq \frac{\pi^D - \pi^M/2}{\pi^M/2 - \pi^W}$$

Note that $\pi^D > \pi^M/2 > \pi^W$. In fact it's easy to check that $q^M = a/2$, hence $\pi^M = a^2/4$, hence $\pi^M/2 = a^2/8$. And best response to $q^M/2 = a/4$ is $3a/8$. Thus, $\pi^D = 9a^2/64$, and $\pi^W = 3a^2/32$. Thus the inequality above becomes $\delta \geq \frac{a^2/64}{2a^2/64} = 1/2$

So for all such $\delta \geq 1/2$ we get an equilibrium, hence the strategy specified is sustained and the outcome is the collusive outcome.

4. Suppose that two firms, A and B, are competing in prices over infinitely many time periods, $t = 1, 2, ...$ For simplicity suppose that they can only set 2 prices in each period, p_H and p_L, with $p_H > p_L > 0$. The payoffs from possible price combinations are given in the table below. The first payoff in brackets is of firm A, and the second one is of firm B.

		B: p_L	B: p_H
A	p_L	$(100, 100)$	$(500, 0)$
	p_H	$(0, 300)$	$(200, 250)$

Let the grim-trigger strategy be

$$p^t = \begin{cases} p_H & \text{if the other firm has played } p_H \text{ until period } t-1 \\ p_L & \text{otherwise} \end{cases}$$

where p^t is the price in period t.

Note $\delta \geq \dfrac{\pi^D - \pi^{Coop}}{\pi^D - \pi^{Comp}} = \bar{\delta}$ where π^{Coop}:cooperation profit per firm , π^{Comp} : competition profit per firm, π^D : deviation profit per firm

(a) Find the minimum threshold discount factor for firm A to play the grim-trigger strategy assuming that the firm B is playing grim-trigger strategy.
Answer: $\bar{\delta}_A = \dfrac{500-200}{500-100} = \dfrac{3}{4}$

(b) Find the minimum threshold discount factor for firm B to play the grim-trigger strategy assuming that the firm A is playing grim-trigger strategy.
Answer: $\bar{\delta}_B = \dfrac{300-250}{300-100} = \dfrac{1}{4}$

5. Consider Dixit's entry deterrence model we discussed in class. Let the inverse demand function be given by $p(Q) = a - bQ = a - b(q_1 + q_2)$, where q_1 is the incumbent firm's

output and q_2 is the potential entrant's output. Let w, r be the cost of one unit of labor and one unit of capacity, respectively. Let f be the fixed cost for both firms. One unit of production requires one unit of labor and one unit of capacity. Capacity cost is sunk. Find the minimum capacity that would blockade entry as a function of a, b, w, r and f.

Answer: You can verify that the best response function of firm 2 is given by
$q_2 = \dfrac{a-w-r}{2b} - \dfrac{k_1}{2}$. Then profit of firm 2 is

$$\pi_2 = (a - bk_1 - bq_2 - w - r)q_2 - f$$
$$= \left(a - bk_1 - b\left(\frac{a-w-r}{2b} - \frac{k_1}{2}\right) - w - r\right)\left(\frac{a-w-r}{2b} - \frac{k_1}{2}\right) - f$$
$$= \left(a - bk_1 - \frac{a-w-r}{2} + \frac{k_1}{2} - w - r\right)\left(\frac{a-w-r-bk_1}{2b}\right) - f$$

For k_1 to deter entry, we need $\pi_2 = 0$. So k_1 is such that

$$\left(a - bk_1 - \frac{a-w-r}{2} + \frac{k_1}{2} - w - r\right)\left(\frac{a-w-r-bk_1}{2b}\right) = f$$
$$\left(\frac{a-bk_1-w-r}{2}\right)\left(\frac{a-w-r-bk_1}{2}\right) = bf$$
$$\frac{a-bk_1-w-r}{2} = \sqrt{bf} \Longrightarrow k_1^* = \frac{a-w-r-2\sqrt{bf}}{b}$$

Note that as $w\uparrow$ or $r\uparrow$, $k_1^*\downarrow$ and that as $f\uparrow$, $k_1^*\downarrow$.

6. Consider Dixit's entry deterrence model we discussed in class. Let the demand function be given by $p(Q) = 120 - Q = 120 - (q_1 + q_2)$, where q_1 is the incumbent firm's output and q_2 is the potential entrant's output. Let $w = r = 30TL$ and $f_1 = f_2 = 200TL$. One unit of production requires one unit of labor and one unit of capacity and capacity cost is sunk.

   (a) Suppose that the incumbent firm invests in capacity $\overline{k_1}$. Find the best response function of each firm in the second stage where they compete in quantities.

   Answer: If $q_1 < \bar{k_1}$ then $MC = 30(= w)$
   Then $MR(q_1, q_2) = 30$. That is; $120 - 2q_1 - q_2 = 30$
   So $BR_1(q_2) = 45 - \dfrac{q_2}{2}$.
   If $q_1 \geqslant \bar{k_1}$ then $MC = 60(= w + r)$
   Then $MR(q_1, q_2) = 60$. That is; $120 - 2q_1 - q_2 = 60$
   So $BR_1(q_2) = 30 - \dfrac{q_2}{2}$.
   For firm 2 $MC = 60$ regardless of $\bar{k_1}$. So $BR_2(q_1) = 30 - \dfrac{q_1}{2}$

   (b) Find the Stackelberg leader's output level.

   Answer:
   $\pi_2(q_1, q_2) = (120 - q_1 - q_2 - 60)q_2 - 200 = 60q_2 - q_2^2 - q_1q_2 - 200$
   FOC: $60 - 2q_2 - q_1 = 0 \Longrightarrow q_2 = 30 - \dfrac{q_1}{2}$.
   $= \pi_1(q_1, q_2, k_1 = q_1) = (120 - q_1 - q_2 - 30)q_1 - 30k_1 - 200$
   $= \left[90 - q_1 - \left(30 - \dfrac{q_1}{2}\right)\right] q_1 - 30q_1 - 200 = 60q_1 - \dfrac{q_1^2}{2} - 30q_1 - 200$

FOC: $60 - 2q_1 - 30 = 0 \Longrightarrow q_1^{St} = 30$

(c) Suppose that $\overline{k_1} = 30$. Will the potential entrant enter? If so, find q_2.

Answer: If $\overline{k_1} = 30$, then $q_1 = \overline{k_1} = 30$

Then $q_2 = 30 - \dfrac{30}{2} = 15$ and $Q = 45$ $P = 75$

$\pi_2 = (75 - 60).15 - 200 = 25 > 0 \Rightarrow$ Entrant will enter!! and picks $q_2 = 15$

(d) Suppose that $\overline{k_1} = 32$. Will the potential entrant enter? If so, find q_2.

Answer: If $\overline{k_1} = 32$ then q_1 is at least 32. Since $q_1^{St} = 30$, $q_1 = k_1 = 32$.

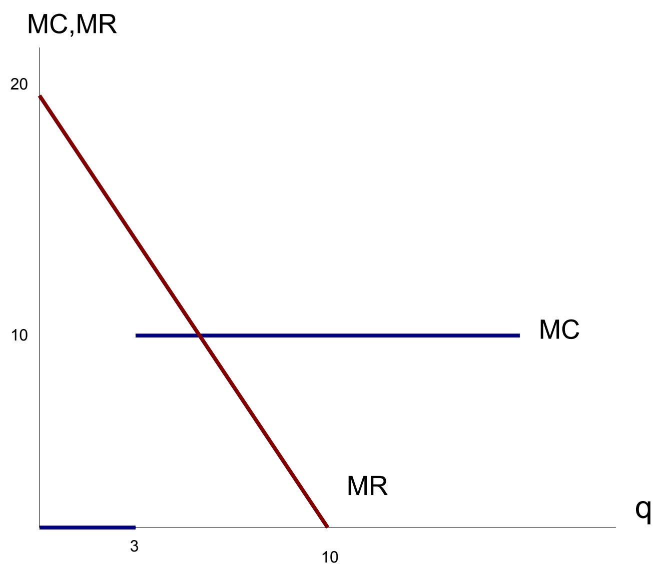
So $q_2 = 30 - \dfrac{32}{2} = 14$ and $Q = 32 + 14 = 46$, $P = 120 - 46 = 74$

$\pi_2 = (74 - 60).14 - 200 = -4 < 0 \Rightarrow$ Entry is blockaded by picking $k_1 = 32$

7. In a market for a homogeneous product let the inverse demand function be given by $p = 50 - q_1 - q_2$. The incumbent firm's unit cost of production is zero up to its capacity k_1. Beyond k_1, the unit cost of production is $10TL$. A potential entrant produces the same good with a cost function $C(q_2) = 10q_2 + 256$.

(a) Draw the incumbents marginal cost function for $k_1 = 3$. On the same graph, draw the incumbent's marginal revenue for $q_2 = 30$.

Answer:



(b) Find the level of capacity $\bar{k}$, that induces the entrant firm to break even if it enters. Will the incumbent firm choose to deter entry?

Answer: For a given k_1, the entrant's best response is $q_2 = 20 - (k_1/2)$. To see this maximize entrant's profit: $\pi_2 = (p - 10)q_2 - 256 = (40 - k_1 - q_2)q_2 - 256$. The first order condition yields $40 - k_1 - 2q_2 = 0$, which gives $q_2 = 20 - (k_1/2)$. So the $\bar{k}_1$ that leaves the entrant zero profit is found by plugging the entrant's best response function into its profit function and solve for k_1: $(40 - k_1 - 20 + k_1/2)(20 - k_1/2) - 256 = 0$, which gives $\bar{k}_1 = 8$.

Note that the Stackelberg leader's output is $q_S^L = 20$. Since $\bar{k}_1 = 8 < 20$, there will be no entry when $k_1 = 20$. Thus, with $k_1 = q_1 = q_S^L = 20$, the incumbent will be on its own in the market receiving monopoly profits. In fact, the intersection of $BR_1^{w+r} = 20 - (q_2/2)$ and $BR_2^{w+r} = 20 - (q_1/2)$ occurs at $q_1 = 40/3$ which is also larger than $\bar{k}_1 = 8$. Thus, in fact, there is no entry regardless of k_1.

8. In a market for a homogeneous product let the inverse demand function be given by $p = 100 - q_1 - q_2$, where q_1 is the incumbent firm's output and q_2 is the potential entrant's output. One unit of production requires one unit of labor and one unit of capacity where capacity cost is sunk. The wage rate for one unit of labor is $w = 20$. One unit of capacity costs $r = 10$. There is an entry cost for the potential entrant, $f = 144$. Incumbent picks it capacity. Potential entrant observes incumbent's capacity and decides whether to enter or not. If there is no entry, potential entrant gets 0. If there is entry, they compete in quantities. What capacity will the incumbent choose? Will the potential entrant enter?

Answer:

Suppose that the incumbent firm invests in capacity $\overline{k_1}$.

If $q_1 < \overline{k_1}$ then $MC = 20 (= w)$. Then $MR(q_1, q_2) = 20$ implies $100 - 2q_1 - q_2 = 20$. So $BR_1^w(q_2) = 40 - \frac{q_2}{2}$.

If $q_1 \geqslant \overline{k_1}$ then $MC = 30 (= w + r)$. Then $MR(q_1, q_2) = 30$ implies $100 - 2q_1 - q_2 = 30$. So $BR_1^{w+r}(q_2) = 35 - \frac{q_2}{2}$.

For firm 2, $MC = 30$ regardless of $\overline{k_1}$. So $BR_2(q_1) = 35 - \frac{q_1}{2}$.

The profit level of the potential entrant firm from entering is

$$\pi_2(q_1, q_2) = (100 - q_1 - q_2 - 30)q_2 - 144 \tag{1}$$

Inserting the best response of entrant firm, $q_2 = 35 - \frac{q_1}{2}$, we get

$$\pi_2(q_1, q_2) = (100 - q_1 - (35 - \frac{q_1}{2}) - 30)(35 - \frac{q_1}{2}) - 144 \tag{2}$$

That is, $\pi_2(q_1, q_2) = (35 - \frac{q_1}{2})^2 - 144$. Note that $\pi_2(q_1, q_2) = 0$ when $q_1 = 46$.

Note that the intersection of $BR_1^w = 40 - \frac{q_2}{2}$ and $BR_2^{w+r} = 35 - \frac{q_1}{2}$ occurs at $q_1 = 30$. This is the highest possible q_1 that can come out in the quantity competition. Since $46 > 30$, the entry is inevitable.

In the Stackelberg game the leader's output for firm 1 is $q_L^S = 35$. However, since $30 < 35$, the Stackelberg leader's output is not within the range of possible quantity competition outputs. Thus, the incumbent would choose $q_1 = k_1 = 30$, and the entry would occur.