

**Boğaziçi University**  
**Department of Economics**  
**Fall 2016**  
**EC 301 ECONOMICS of INDUSTRIAL ORGANIZATION**  
**Problem Set 3**

1. Consider a market with the demand function  $p(Q) = 1100 - 2Q$ . Suppose entry/exit is free, and that every firm has a marginal cost  $c = 2TL$  and a fixed cost of  $f = 1800TL$ 
  - (a) Find the Cournot (Nash) equilibrium number of firms.
  - (b) Find the socially optimal number of firms, and compare it to the Cournot (Nash) equilibrium number of firms.
2. Suppose that there are two firms,  $A$  and  $B$ ; serving a market, each firm producing the same product. Each firm has a marginal cost given by  $MC(q_i) = 10 + q_i$ ,  $i = A, B$ , and there are no fixed costs. The market demand is  $p(Q) = 100 - 2Q$  where  $Q = q_A + q_B$ . Find the Cournot (Nash) equilibrium quantities and the price.
3. Two firms are competing in quantities. Each firm chooses its own quantity level independently and simultaneously. The inverse demand function in the market is given by  $P = 25 - Q$  where  $Q = q_1 + q_2$  is the total quantity produced by the two firms,  $q_1$  is firm 1's quantity level and  $q_2$  is firm 2's quantity level. Each firm has a constant marginal cost equal to 10 and a fixed cost of 32. If a firm chooses not to produce at all, it does not have to pay its fixed cost. Find the set of pure-strategy Nash equilibria? Are there any **symmetric** equilibrium?
4. Consider the Cournot duopoly model in which two firms, 1 and 2, simultaneously choose the quantities they supply,  $q_1$  and  $q_2$ . The price each will face is determined by the market demand function  $p(q_1, q_2) = a - b(q_1 + q_2)$ . Each firm has a probability  $\mu$  of having a marginal unit cost of  $c_L$  and a probability  $1 - \mu$  of having a marginal unit cost of  $c_H$ , where  $c_H > c_L$ . These probabilities are common knowledge, but the true type is revealed only to each firm individually. Solve for the equilibrium quantities as a function of  $a, b, c_L, c_H$  and  $\mu$ . (Note that for each firm there are two quantities to decide!)
5. In the simplest version of the Bertrand model we first considered in class, draw the best-response function for each firm.
6. Suppose two firms,  $A$  and  $B$ , are producing differentiated products. Each firm has a constant marginal cost equal to \$1, and has no fixed cost. Each firm picks its own price without observing the price picked by the other firm. That is, they

simultaneously pick their prices. Denoting the prices with  $p_A$  and  $p_B$ , firm  $A$  faces a demand given by  $q_A(p_A, p_B) = 2 - 2p_A + p_B$ , and firm  $B$  faces a demand given by  $q_B(p_A, p_B) = 1 - 3p_B + 2p_A$ .

- (a) Find the Bertrand (Nash) equilibrium prices, and profits for each firm.
  - (b) Suppose that they collude and set prices jointly to maximize the sum of their profits. Find the joint-profit maximizing prices, and the total joint profit. Compare this total profit to the sum of the profits you found in part (a).
7. Suppose two firms,  $A$  and  $B$ , are producing differentiated products. Firm  $A$  has a constant marginal cost equal to  $c = 1$  and firm  $B$  has zero marginal cost. Firm  $A$  has a fixed cost  $f_A$  and firm  $B$  has a fixed cost  $f_B$ . Each firm picks its own price without observing the price picked by the other firm. Denoting the prices with  $p_A$  and  $p_B$ , firm  $A$  faces a demand given by  $q_A(p_A, p_B) = 15 - 5p_A + 2p_B$ , and firm  $B$  faces a demand given by  $q_B(p_A, p_B) = 8 - 2p_B + p_A$ .
- (a) Suppose  $f_A = f_B = 0$ . Find the Nash equilibrium prices.
  - (b) Suppose  $f_A = 0$  and  $f_B > 0$ . For what values of  $f_B$ , the prices you find in (a) still constitute a Nash equilibrium?