Boğaziçi University
Department of Economics
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EC 301 ECONOMICS of INDUSTRIAL ORGANIZATION
Problem Set 2

1. Is the following statement true or false? "Perfect price discrimination involves charging each consumer a different take-it-or-leave-it price. This results in an efficient level of output."

2. Suppose that a monopoly faces a market demand given by $P = 30 - 2Q$. Its total cost is $TC(Q) = 5 + Q^2$.

   (a) Find the profit of this monopoly if it charges a uniform price.

   (b) If it can perfectly price discriminate, what would be the profit level?

3. A discriminating monopoly sells in two markets. Assume that no arbitrage is possible. The demand curve in market 1 is given by $p_1 = 100 - Q_1/2$, and the demand curve in market 2 is given by $p_2 = 100 - Q_2$. Let $Q = Q_1 + Q_2$ be the total production of the monopoly. The cost function of the monopoly is $TC(Q) = Q^2$.

   (a) Find the profit maximizing Q_1 and Q_2, and the total profit.

   (b) Suppose that the monopoly decides to decompose the monopoly plant into two plants, where plant 1 sells in market 1 only and plant 2 sells in market 2 only. Now, find the profit maximizing quantities in each plant and the total profit.

   (c) Suppose you become the new CEO of this monopoly and they ask you, with an explanation, whether the monopoly should decompose into two plants or not. What would your answer be?

4. Suppose a monopoly is facing two different types of consumers with inverse demand functions $P = 200 - q_1$ and $P = 150 - 2q_2$. Monopoly cannot observe who belongs to which group. Monopoly's marginal cost is constant and zero and there is no fixed cost.

   (a) Suppose the monopoly offers two options specifying quantity and total price. The first offer is targeted at consumer 1 with $q_1 = 200$ such that it makes consumer 1 indifferent between the two offers. The second offer is targeted at consumer 2 with $q_2 = 75$ and extracts all of consumer 2's surplus. Find the total prices in each option offered. Find the overall profit of the monopoly.

   (b) Can the monopoly increase its profit level in part (a) by offering a new pair of options? Show your work.

5. Consider a market for a product with two buyers: one has inverse demand curve of $P_H = 30 - Q_H$ and the other has $P_L = 20 - 2Q_L$. The firm uses nonlinear pricing schemes, offering two bundles (Q_1, A_1) and (Q_2, A_2), where Q_i is the quantity and A_i is the total price of the quantity Q_i, for $i = 1, 2$. Suppose the firm does not know who is of which type.

   (a) Suppose that the firm offers $(Q_1, A_1) = (10, 100\text{TL})$ and $(Q_2, A_2) = (30, 450\text{TL})$. How much total revenue does the firm make?

   (b) Suppose that the firm offers $(Q_1, A_1) = (8, 96\text{TL})$ and $(Q_2, A_2) = (30, 338\text{TL})$. How much total revenue does the firm make?

   (c) Show that the total revenue in part (b) can be increased by changing exactly one of the quantities by 1 unit and adjusting the fees A_1 and A_2.

6. Consider a market for a product with one seller. Suppose the seller has no fixed cost and the marginal cost is zero. Suppose the seller faces only two buyers one with an inverse demand curve $P_1 = 30 - Q_1$ and the other buyer with $P_2 = 25 - (5/4)Q_2$. Suppose the only information the seller has is that there are these two buyers with the two different demand curves given above. Suppose the seller offers three quantity and total payment options: (10 units, 160TL), (20 units, 300TL) and (30 units, 450TL). Each buyer observes the offers and chooses exactly one of them. How much total profit does the firm make?

7. Suppose a monopoly is facing two consumers, one with an inverse demand function $P = 100 - 2q_1$ and the other with $P = 60 - 3q_2$. Monopoly cannot observe which consumer has which demand. Monopoly's marginal cost is constant and zero and there is no fixed cost. Suppose the monopoly offers two options each one specifying a quantity and a total price. The first offer is (q_1, T_1) and it is targeted at consumer with demand $P = 100 - 2q_1$, such that it makes this consumer indifferent between the two offers. The second offer is (q_2, T_2) and it is targeted at the other consumer and extracts all of his consumer surplus. In an offer, q denotes the quantity and T denotes the total price.

   (a) Let $q_1 = 50$ and $q_2 = 20$. Find the profit maximizing T_1 and T_2.

   (b) Let $q_1 = 50$. Find the profit maximizing q_2, T_1 and T_2.

8. There are two types of consumers, type $\theta_1 = 1$ in proportion 1/2 and type $\theta_2 = 2$ in proportion 1/2. Each consumer has a net utility given as $\theta_i u(q_i) - T(q_i)$ where q_i is the quantity bought, $\theta_i u(q_i)$ is the utility from consuming q_i and $T(q_i)$ is the total payment made for q_i. Let $u(q_i) = 2\sqrt{q_i}$. Assume that the monopoly has a constant marginal cost $MC = c = 0.5$ and there are no fixed costs.

   (a) Find the demand function for each type of consumer, $D_1(p)$ and $D_2(p)$.

   (b) Find the consumer surplus for each type of consumer, $CS_1(p)$ and $CS_2(p)$. (Hint: $CS_i(p) = \theta_i u(D_i(p)) - pD_i(p)$)

(c) Find the profit level of the monopoly if it can perfectly price discriminate.

(d) Suppose that the monopoly cannot observe which consumer is which type and decides to use a two part tariff, $T(q_i) = A + pq_i$.

i. Find the optimal two part tariff, that is, find the optimal A^* and p^*.

ii. Suppose that the monopoly could learn each consumer's type at a total price of P. Find the highest P the monopoly would be willing to pay to learn each consumer's type?

(e) Now, suppose the monopoly decides to offer two quantity and total payment combinations, (q_1, T_1) and (q_2, T_2), still not observing which consumer is of which type, and hopes that each θ_i type chooses (q_i, T_i).

i. Find the set of optimal bundles that maximize the monopoly's profit.

ii. Suppose that the monopoly could learn each consumer's type at a total price of P'. Find the highest P' the monopoly would be willing to pay to learn each consumer's type?

(f) Compare P and P' and provide intuition.

(g) Now suppose that the monopoly is facing three types of consumers with $\theta_1 = 1$, $\theta_2 = 2$ and $\theta_3 = 3$, in equal proportions. The monopoly cannot observe which consumer is of which type and decides to offer three options of quantity and total payment, each one targeted to one of the three types of consumers. That is, the monopoly offers $(q_1, T_1), (q_2, T_2)$ and (q_3, T_3), inducing self-selection. Construct the profit maximization problem of the monopoly together with the appropriate constraints. Are there any redundant constraints? If yes, explain why.

9. Suppose a monopoly is facing two consumers, consumer 1 has a net benefit function $u_1(q, T) = q^{1/2} - (T/3)$ and consumer 2 has a net benefit function $u_2(q, T) = q^{1/2} - (T/2)$, where q is the quantity consumed and T is the total payment made. Monopoly cannot observe which consumer has which benefit function, but knows that they are equally likely. Monopoly's marginal cost is constant and equal to $c = 1/4$ and there is no fixed cost. Suppose the monopoly offers two options each one specifying a quantity and a total price: (q_1, T_1) targeted at consumer 1 and (q_2, T_2) targeted at consumer 2. Suppose monopoly picks these two options to maximize its expected profit, by making sure that each consumer gets the option targeted at him, and monopoly sells to both types.

(a) Construct the profit maximization problem of the monopoly with all the conditions.

(b) Let $q_2 = 9$. Find the profit maximizing q_1, T_1 and T_2.