Boğaziçi University
Department of Economics
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EC 301 ECONOMICS of INDUSTRIAL ORGANIZATION
Problem Set 2 - Answer Key

1. Is the following statement true or false? "Perfect price discrimination involves charging each consumer a different take-it-or-leave-it price. This results in an efficient level of output."

   Answer: True.

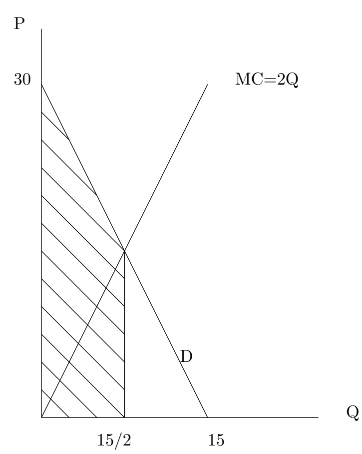
2. Suppose that a monopoly faces a market demand given by $P = 30 - 2Q$. Its total cost is $TC(Q) = 5 + Q^2$.

   (a) Find the profit of this monopoly if it charges a uniform price.

   Answer: The profit is $\Pi(Q) = P(Q)Q - 5 - Q^2 = (30 - 2Q)Q - 5 - Q^2$. Then, the first order condition is $\Pi(Q) = 30 - 6Q = 0$ which implies $Q^M = 5$, $P^M = 20$ and $\Pi^M = 70$.

   (b) If it can perfectly price discriminate, what would be the profit level?

   Answer:



   The quantity in perfect price discrimination, Q^{PD}, solves $30-2Q = 2Q$. Thus, $Q^{PD} = 15/2$. At $Q = 15/2$, the consumer surplus is the shaded area, which is $((30 + 15)/2)(15/2) = 3(15/2)^2$. Each consumer, facing a price equal to her willingness to pay, will be indifferent between buying and not buying. Assuming they buy when indifferent, the whole CS is captured. The cost is $5 + (15/2)^2$. Thus, the profit is $\Pi^{PD} = CS - TC(Q^{PD}) = 3(15/2)^2 - 5 - (15/2)^2 = 2(15/2)^2 - 5 = 15^2/2 - 5$. Thus, $\Pi^{PD} = 107.5$.

3. A discriminating monopoly sells in two markets. Assume that no arbitrage is possible. The demand curve in market 1 is given by $p_1 = 100 - Q_1/2$, and the demand curve in market 2 is

given by $p_2 = 100 - Q_2$. Let $Q = Q_1 + Q_2$ be the total production of the monopoly. The cost function of the monopoly is $TC(Q) = Q^2$.

(a) Find the profit maximizing Q_1 and Q_2, and the total profit.

Answer: The profit is $\Pi(Q_1, Q_2) = (100 - Q_1)Q_1 + (100 - Q_2)Q_2 - (Q_1 + Q_2)^2$. Then, the first order conditions are

$0 = \frac{\partial \Pi}{\partial Q_1} = 100 - Q_1 - 2(Q_1 + Q_2)$ and

$0 = \frac{\partial \Pi}{\partial Q_2} = 100 - 2Q_2 - 2(Q_1 + Q_2)$

We get, $Q_1^M = 25$, $Q_2^M = 12.5$ and $\Pi^M = 1875$.

(b) Suppose that the monopoly decides to decompose the monopoly plant into two plants, where plant 1 sells in market 1 only and plant 2 sells in market 2 only. Now, find the profit maximizing quantities in each plant and the total profit.

Answer:

Each plant sells only in one market. The first order conditions are

$0 = \frac{\partial \Pi}{\partial Q_1} = 100 - Q_1 - 2Q_1$ which implies $Q_1 = 100/3$, and

$0 = \frac{\partial \Pi}{\partial Q_2} = 100 - 2Q_2 - 2Q_2$ which implies $Q_2 = 25$.

Then,

$P_1 = 100 - \frac{100}{6} = \frac{250}{3}$

$P_2 = 100 - 25 = 75.$

Then,

$\Pi = \Pi_1(Q_1) + \Pi_2(Q_2) = \frac{250}{3}\frac{100}{3} - \frac{100}{3}\frac{100}{3} - 75 \cdot 25 - 25^2 = 2917.$

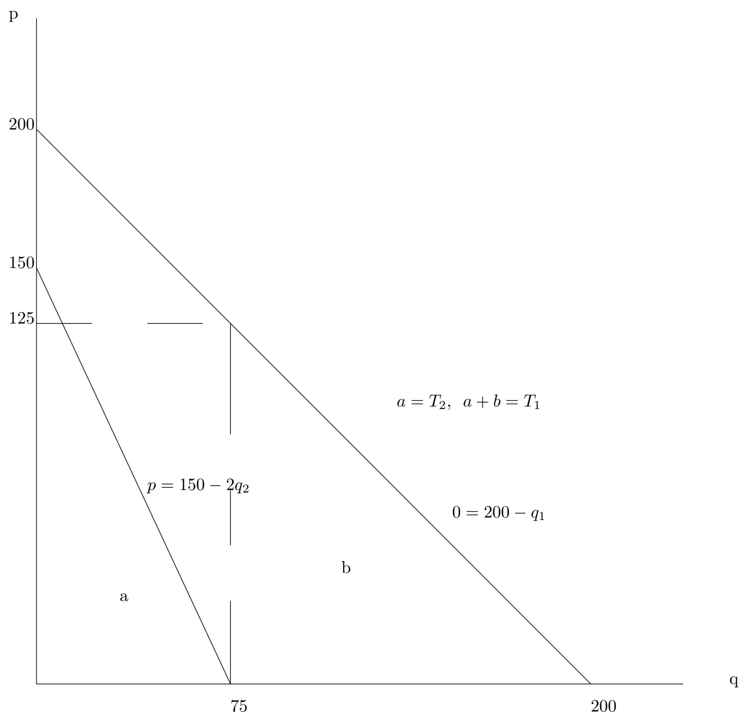
(c) Suppose you become the new CEO of this monopoly and they ask you, with an explanation, whether the monopoly should decompose into two plants or not. What would your answer be?

Answer: This decomposition increases profit since technology has DRTS.

4. Suppose a monopoly is facing two different types of consumers with inverse demand functions $P = 200 - q_1$ and $P = 150 - 2q_2$. Monopoly cannot observe who belongs to which group. Monopoly's marginal cost is constant and zero and there is no fixed cost.

(a) Suppose the monopoly offers two options specifying quantity and total price. The first offer is targeted at consumer 1 with $q_1 = 200$ such that it makes consumer 1 indifferent between the two offers. The second offer is targeted at consumer 2 with $q_2 = 75$ and extracts all of consumer 2's surplus. Find the total prices in each option offered. Find the overall profit of the monopoly.

Answer:



The offers are ($q_2 = 75, T_2 = a$) and ($q_1 = 200, T_1 = a + b$). Then, $T_2 = a = \frac{150 \cdot 75}{2} = 75^2 = 5625$. And, $T_1 = a + b = 5625 + \frac{125 \cdot 125}{2} = 13473.5$. Therefore, the total profit is $\Pi = 5625 + 13473.5 = 19062.5$.

(b) Can the monopoly increase its profit level in part (a) by offering a new pair of options? Show your work.

Answer:

Yes. Consider, for instance ($q_2' = 70, T_2' = \frac{150 \cdot 10}{2} \cdot 70$) and ($q_1' = 200, T_1' = T_2' + \frac{130 \cdot 130}{2} \cdot 70$). That is, (70,5600) and (200,14050) are the offers. Then, $\Pi' = 5600 + 14050 = 19650 > \Pi$.

5. Consider a market for a product with two buyers: one has inverse demand curve of $P_H = 30 - Q_H$ and the other has $P_L = 20 - 2Q_L$. The firm uses nonlinear pricing schemes, offering two bundles (Q_1, A_1) and (Q_2, A_2), where Q_i is the quantity and A_i is the total price of the quantity Q_i, for $i = 1, 2$. Suppose the firm does not know who is of which type.

(a) Suppose that the firm offers $(Q_1, A_1) = (10, 100\text{TL})$ and $(Q_2, A_2) = (30, 450\text{TL})$. How much total revenue does the firm make?

Answer:

With these offers, the both type prefer bundle $(Q_1, A_1) = (10, 100\text{TL})$. This is because the H-type gets a strictly positive consumer surplus from the bundle $(Q_1, A_1) = (10, 100\text{TL})$

whereas the bundle $(Q_2, A_2) = (30, 450\text{TL})$ leaves him zero consumer surplus. Hence the revenue is 200TL.

(b) Suppose that the firm offers $(Q_1, A_1) = (8, 96\text{TL})$ and $(Q_2, A_2) = (30, 338\text{TL})$. How much total revenue does the firm make?

Answer:

With these offers, the L-type prefers bundle $(Q_1, A_1) = (8, 96\text{TL})$ because this one gives him zero consumer surplus whereas the other bundle gives him negative surplus. The H-type prefers the other bundle $(Q_2, A_2) = (30, 338\text{TL})$, this is because the bundle $(Q_1, A_1) = (8, 96\text{TL})$ gives the H-type a consumer surplus of 112(=208-96) where 208 is the area below its demand up to quantity 8, and 96 is the total price. And the other bundle gives him exactly the same surplus which is $((30 \times 30)/2) - 338 = 112$, hence the revenue is $96 + 338 = 434$TL.

(c) Show that the total revenue in part (b) can be increased by changing exactly one of the quantities by 1 unit and adjusting the fees A_1 and A_2.

Answer:

The following two bundles achieve a higher revenue: $(Q_1, A_1) = (7, 91\text{TL})$ and $(Q_2, A_2) = (30, 355.5\text{TL})$.

6. Consider a market for a product with one seller. Suppose the seller has no fixed cost and the marginal cost is zero. Suppose the seller faces only two buyers one with an inverse demand curve $P_1 = 30 - Q_1$ and the other buyer with $P_2 = 25 - (5/4)Q_2$. Suppose the only information the seller has is that there are these two buyers with the two different demand curves given above. Suppose the seller offers three quantity and total payment options: (10 units, 160TL), (20 units, 300TL) and (30 units, 450TL). Each buyer observes the offers and chooses exactly one of them. How much total profit does the firm make?

Answer:

For the buyer with the $P_1 = 30 - Q_1$:

• choosing (10 units, 160TL) brings a net benefit of 90: Her consumer surplus at this offer is the are below her demand curve up to $Q_1 = 10$ is 250. She pays 160. Thus, net benefit is 90.

• choosing (20 units, 300TL) brings a net benefit of 100: Her consumer surplus at this offer is the are below her demand curve up to $Q_1 = 20$ is 400. She pays 300. Thus, net benefit is 100.

• choosing (30 units, 450TL) brings a net benefit of 90: Her consumer surplus at this offer is the are below her demand curve up to $Q_1 = 30$ is 450. She pays 450. Thus, net benefit is 0.

Thus this buyer chooses (20 units, 300TL).

For the buyer with the $P_2 = 25 - (5/4)Q_2$:

• choosing (10 units, 160TL) brings a net benefit of 27.5: Her consumer surplus at this offer is the are below her demand curve up to $Q_2 = 10$ is 187.5. She pays 160. Thus, net benefit is 27.5.

• choosing (20 units, 300TL) brings a net benefit of -50 : Her consumer surplus at this offer is the are below her demand curve up to $Q_2 = 20$ is 250. She pays 300. Thus, net benefit is -50.

• choosing (30 units, 450TL) brings a net benefit of -200: Her consumer surplus at this offer is the are below her demand curve up to $Q_2 = 30$ is 250. She pays 450. Thus, net benefit is -200.

Thus this buyer chooses (10 units, 160TL).

Therefore the seller obtains a total profit of 460.

7. Suppose a monopoly is facing two consumers, one with an inverse demand function $P = 100 - 2q_1$ and the other with $P = 60 - 3q_2$. Monopoly cannot observe which consumer has which demand. Monopoly's marginal cost is constant and zero and there is no fixed cost. Suppose the monopoly offers two options each one specifying a quantity and a total price. The first offer is (q_1, T_1) and it is targeted at consumer with demand $P = 100 - 2q_1$, such that it makes this consumer indifferent between the two offers. The second offer is (q_2, T_2) and it is targeted at the other consumer and extracts all of his consumer surplus. In an offer, q denotes the quantity and T denotes the total price.

   (a) Let $q_1 = 50$ and $q_2 = 20$. Find the profit maximizing T_1 and T_2.

   Answer: The consumer surplus of type 2 consumer at $q_2 = 20$ is $(60 \times 20)/2 = 600$. Thus, $T_2 = 600$. Then, at q_1, the payment T_1 that makes type 1 consumer indifferent between the offers is given by $600 + [(100 - (2 \times 20)) \times (50 - 20)]/2 = 600 + 900 = 1500$. Thus, $T_1 = 1500$.

   (b) Let $q_1 = 50$. Find the profit maximizing q_2, T_1 and T_2.

   Answer: First we find q_2. The optimal q_2 must be such that $100 - 2q_2 = 2 \times (60 - 3q_2)$, that is, the height of the first demand at q_2 must be twice the height of the second demand at q_2. Thus, $q_2 = 5$. Then, the consumer surplus for type 2 consumer is the area below the second demand up to q_2: $((60 + 45)/2) \times 5 = 262.5$. Thus, $T_2 = 262.5$. Then, at $q_1 = 50$, T_1 that makes type 1 consumer indifferent between the offers is $262.5 + (90 \times 45)/2 = 262.5 + 2025 = 2287.5$. Thus, $T_1 = 2287.5$.

8. There are two types of consumers, type $\theta_1 = 1$ in proportion 1/2 and type $\theta_2 = 2$ in proportion 1/2. Each consumer has a net utility given as $\theta_i u(q_i) - T(q_i)$ where q_i is the quantity bought, $u(q_i)$ is the utility from consuming q_i and $T(q_i)$ is the total payment made for q_i. Let $u(q_i) = 2\sqrt{q_i}$ and $T(q_i) = A + pq_i$. Assume that the monopoly has a constant marginal cost $MC = c = 0.5$ and there are no fixed costs.

   (a) Find the demand function for each type of consumer, $D_1(p)$ and $D_2(p)$.
   Answer:

$$\max_x \theta_i 2\sqrt{q_i} - pq_i - A$$

FOC(q_i): $\theta_i 2\frac{1}{2\sqrt{q_i}} - p = 0 \Longrightarrow \frac{\theta_i^2}{p^2} = q_i = D_i(p)$

So $D_1(p) = \frac{1}{p^2}, D_2(p) = \frac{4}{p^2}$

(b) Find the consumer surplus for each type of consumer, $CS_1(p)$ and $CS_2(p)$. (Hint: $CS_i(p) = \theta_i u(D_i(p)) - pD_i(p)$)

Answer: $CS_i(p) = \theta_i u(D_i(p)) - pD_i(p) = \theta_i 2\sqrt{\frac{\theta_i^2}{p^2}} - p\frac{\theta_i^2}{=}\theta_i 2\frac{\theta_i p}{-}\frac{\theta_i^2}{p} = \frac{\theta_i^2}{p}$

$\Longrightarrow CS_1(p) = \frac{1}{p}, CS_2(p) = \frac{4}{p}$ since $\theta_1 = 1, \theta_2 = 2$

(c) Find the profit level of the monopoly if it can perfectly price discriminate.

Answer: $CS_1(p = c) = \frac{1}{c}$, $CS_2(p = c) = \frac{4}{c}$

Then $\pi = \frac{1}{2}CS_1(c) + \frac{1}{2}CS_2(c) = \frac{1}{2}\left(\frac{1}{c} + \frac{4}{c}\right) = \frac{5}{2c} = \frac{5}{2 \times 0.5} = 5$

(d) Suppose that the monopoly cannot observe which consumer is which type and decides to use a two part tariff, $T(q_i) = A + pq_i$.

i. Find the optimal two part tariff, that is, find the optimal A^* and p^*.

Answer:

$$\max_p \frac{1}{2}\left[CS_1(p) + (p-c)\frac{1}{p^2}\right] + \frac{1}{2}\left[CS_1(p) + (p-c)\frac{4}{p^2}\right] = CS_1(p) + (p-c)\frac{5}{2p^2}$$

$$\max_p \frac{1}{p} + \frac{5}{2p^2}(p - \frac{1}{2}) = \frac{1}{p} + \frac{5}{2p} - \frac{5}{4p^2} = \frac{7}{2p} - \frac{5}{4p^2}$$

FOC(p): $-\frac{7}{2}\frac{1}{p^2} + 2\frac{5}{4}\frac{1}{p^3} = 0 \Longrightarrow \frac{7}{2}\frac{1}{p^2} = \frac{5}{2}\frac{1}{p^3}$

$p^* = \frac{5}{7}, A^* = \frac{1}{p^*} = \frac{7}{5}$.

ii. Suppose that the monopoly could learn each consumer's type at a total price of P. Find the highest P the monopoly would be willing to pay to learn each consumer's type?

Answer: $\pi(p = \frac{5}{7}) = \frac{1}{p} + \frac{5}{2p^2}(p - \frac{1}{2}) = 2.45$ and $\pi(p = c) = 5$

Then, $P = \pi(p = c) - \pi(p = \frac{5}{7}) = 5 - 2.45 = 2.55$

(e) Now, suppose the monopoly decides to offer two quantity and total payment combinations, (q_1, T_1) and (q_2, T_2), still not observing which consumer is of which type, and hopes that each θ_i type chooses (q_i, T_i).

i. Find the set of optimal bundles that maximize the monopoly profit.

Answer: Note that at (q_1, T_1) the IR_1 should bind. Thus, $T_1 = \theta_1 2\sqrt{q_1} = 2\sqrt{q_1}$.

Also note that the IC_2 binds as well. Thus, $\theta_2 2\sqrt{q_2} - T_2 = \theta_2 2\sqrt{q_1} - T_1$.
Plugging T_1 and θ values we get, $4\sqrt{q_2} - T_2 = 4\sqrt{q_1} - 2\sqrt{q}_1$.
That is, $T_2 = 4\sqrt{q_2} - 2\sqrt{q}_1$.
Then the profit of the seller becomes,
$(1/2)[T_1 - cq_1 + T_2 - cq_2] = (1/2)[2\sqrt{q}_1 - (q_1/2) + 4\sqrt{q_2} - 2\sqrt{q}_1 - (q_2/2)]$.
That is, $(1/2)[4\sqrt{q_2} - (q_1/2) - (q_2/2)]$. Note that, this profit function is decreasing in q_1. Thus, $q_1^* = 0$ and T_1^*. The FOC with respect to q_2 implies $q_2^* = 16$. Thus, $T_2 = 4\sqrt{16} = 16$. Thus, the seller sells only to the high types.

ii. Suppose that the monopoly could learn each consumer's type at a total price of P'. Find the highest P' the monopoly would be willing to pay to learn each consumer's type?

Answer: The profit in part $e(i)$ above is $\pi = 4$. Thus, the seller is willing to pay $5 - 4 = 1$, where 5 is the perfect price discrimination profit.

(f) Compare P and P' and provide intuition.

Answer: Note that part $e(i)$ has a higher profit than the optimal two-part tariff in part $d(i)$. This is because the seller can extract more surplus from the high types under part $e(i)$ than under $d(i)$. In fact under $e(i)$, the seller chooses not to sell to the low types, which is not the case in $d(i)$. Thus, price the seller is willing to pay in $d(ii)$ is larger than the price in $e(ii)$.

(g) Now suppose that the monopoly is facing three types of consumers with $\theta_1 = 1$, $\theta_2 = 2$ and $\theta_3 = 3$, in equal proportions. The monopoly cannot observe which consumer is of which type and decides to offer three options of quantity and total payment, each one targeted to one of the three types of consumers. That is, the monopoly offers $(q_1, T_1), (q_2, T_2)$ and (q_3, T_3), inducing self-selection. Construct the profit maximization problem of the monopoly together with the appropriate constraints. Are there any redundant constraints? If yes, explain why.

Answer: $max_{(q_i,T_i)_{i=\{1,2,3\}}} \ \frac{1}{3}(T_1 - cq_1) + \frac{1}{3}(T_2 - cq_2) + \frac{1}{3}(T_3 - cq_3)$
subject to
$U(q_1) - T_1 \geq 0$ $\quad IR_1$
$2U(q_2) - T_2 \geq 0$ $\quad IR_2$
$3U(q_3) - T_3 \geq 0$ $\quad IR_3$
$U(q_1) - T_1 > U(q_2) - T_2$ $\quad IC_{1>2}$
$U(q_1) - T_1 > U(q_3) - T_3$ $\quad IC_{1>3}$
$2U(q_2) - T_2 > 2U(q_1) - T_1$ $\quad IC_{2>1}$
$2U(q_2) - T_2 > 2U(q_3) - T_3$ $\quad IC_{2>3}$
$3U(q_3) - T_3 > 3U(q_1) - T_1$ $\quad IC_{3>1}$
$3U(q_3) - T_3 > 3U(q_2) - T_2$ $\quad IC_{3>2}$
IR_2 is implied by $IC_{2>1}$ and IR_1. And IR_3 is implied by $IC_{3>1}$ and IR_1.

9. Suppose a monopoly is facing two consumers, consumer 1 has a net benefit function $u_1(q, T) = q^{1/2} - (T/3)$ and consumer 2 has a net benefit function $u_2(q, T) = q^{1/2} - (T/2)$, where q is the quantity consumed and T is the total payment made. Monopoly cannot observe which consumer has which benefit function, but knows that they are equally likely. Monopoly's marginal cost is constant and equal to $c = 1/4$ and there is no fixed cost. Suppose the monopoly offers two options each one specifying a quantity and a total price: (q_1, T_1) targeted at consumer 1 and (q_2, T_2) targeted at consumer 2. Suppose monopoly picks these two options to maximize its expected profit, by making sure that each consumer gets the option targeted at him, and monopoly sells to both types.

(a) Construct the profit maximization problem of the monopoly with all the conditions.

Answer: $\max_{q_1,T_1,q_2,T_2} \quad (1/2)[T_1 - (q_1/4)] + (1/2)[T_2 - (q_2/4)]$ subject to

$(IR_1) : q_1^{1/2} - (T_1/3) \geq 0$
$(IR_2) : q_2^{1/2} - (T_2/2) \geq 0$
$(IC_1) : q_1^{1/2} - (T_1/3) \geq q_2^{1/2} - (T_2/3)$
$(IC_2) : q_2^{1/2} - (T_2/2) \geq q_1^{1/2} - (T_1/2)$

(b) Let $q_2 = 9$. Find the profit maximizing q_1, T_1 and T_2.

Answer: Note that, the low type here is the consumer 2. Thus, when $q_2 = 9$, the profit maximizing set of options will leave consumer 2 zero surplus: $q_2^{1/2} - (T_2/2) = 0$, that is, $3 - (T_2/2) = 0$, that is, $T_2 = 6$. Now, IC_1 will hold with equality, that is, $q_1^{1/2} - (T_1/3) = 3 - (6/3) = 1$, thus, $T_1 = 3q_1^{1/2} - 3$. Plugging these into the objective function we get,

$\max_{q_1} \quad (1/2)[3q_1^{1/2} - 3 - (q_1/4)] + (1/2)[6 - (9/4)]$

which has a first order condition given by $\frac{3}{2q_1^{1/2}} = \frac{1}{4}$, which implies $q_1 = 36$, which in turn implies $T_1 = 15$.

Check that at these quantity and payment pairs, the IR_1 and IC_2 also holds:

$(IR_1) : q_1^{1/2} - (T_1/3) = 6 - (15/3))1 > 0$

$(IC_2) : q_2^{1/2} - (T_2/2) \geq q_1^{1/2} - (T_1/2)$, which is $3 - (6/2) = 0 > 6 - (15/2) = -1.5$.

Thus, if $q_2 = 9$, then the optimal values are $q_1 = 36, T_1 = 15$ and $T_2 = 6$.