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Department of Economics
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EC 301 ECONOMICS OF INDUSTRIAL ORGANIZATION
MIDTERM - Answer Key
12/11/2015
Thursday, 17:30

- **Do not forget to write your full name and student number on the top.**
- **Turn off your cell phone and put it away.**
- **Put away all your lecture notes, books, etc.**
- **There are 4 questions and 6 pages in the exam. Make sure you have them all.**
- **You are provided some scrap paper. Please, first work on your solutions on scrap paper, and then put your solutions in your exam.**
- **Please answer all of the questions in the space provided for each question.**
- **Show your work.**
- **You have 110 minutes.**

GOOD LUCK!!

1. (20 pts.) Suppose there is a dominant firm which faces a competitive fringe in a market. The demand in the market is given by $Q(P) = 200 - kP$, where $k > 0$. The marginal cost of the dominant firm is given by $c = 10$. The fringe has a supply given by $Q^f(P) = P$. The dominant firm chooses the price to maximize its profit, and the fringe takes the price as given. Using the following Lerner index formula, $L = \frac{s^d}{\epsilon_f s^f + |\epsilon_D|}$, find a range for k for which $L \geq 1/2$. Interpret the effect of k on L.

Answer: The dominant firm solves

$$max_p \ (p - 10)(200 - kP - P)$$

which yields $P = \frac{105+5k}{1+k}$.

Note that $s^d = \frac{200-kP-P}{Q} = \frac{95-5k}{Q}$. And $s^f = P/Q$. The elasticities are $\epsilon_f = 1 \cdot (P/Q^f) = P/P = 1$ and $|\epsilon_D| = k\frac{P}{Q}$. Plugging these into L, we get $L = \frac{(95-5k)/Q}{(p/Q)+(kP/Q)} = \frac{95-5k}{p(1+k)} = \frac{95-5k}{105+5k}$. So, $L \geq 1/2$ if $\frac{95-5k}{105+5k} \geq 1/2$, which is equivalent to $k \leq 17/3$.

As k increases the demand is more elastic, thus the market power of the dominant firm goes down, hence L goes down. So for L to be larger than a certain level, k should be sufficiently low, that is, lower than a threshold, which is 17/3 in this case.

2. (20 pts.) Suppose there is a single firm, producing a durable good, which is durable for only two periods. The marginal cost is zero and there are no fixed costs. The firm does not discount, that is, the firm's discount factor is $\delta_f = 1$. There are N many potential buyers in the first period, who live for two periods. For one period of usage of the good, $\alpha \in (0, 6/7)$ proportion of them are willing to pay a maximum price of 120 and they have a discount factor $\delta = 1/2$. And, the rest of the buyers are willing to pay a maximum price of 90 for one period of usage of the good, and they have a discount factor of $\delta = 2/3$. Also, assume that if a buyer is indifferent between buying in the first period and buying in the second period, she buys in the first period. And if a buyer is indifferent between buying and not buying she prefers to buy. Assume that the buyers are strategic and each one demands at most one unit of the good. Suppose, at the beginning of the first period, the firm announces prices for each period, P_1 and P_2, and credibly commits to these prices. For which values of α, does the firm choose to set prices that induce α proportion of buyers buy in the first period and the rest buy in the second period, and what are those prices?

Answer:

Note that the ones in α proportion, have an aggregate willingness to pay for two period consumption equal to $120+120(1/2) = 180$. And the others have $90 + 90(2/3) = 150$. I will call those in α proportion 180-types, and the others 150-types.

Case 1: If the firm chooses to sell both in the first period, then $P_1 = 150$. Note that if firm sells only to 180-types, then it would earn $\alpha 180$. Note $150 \geq \alpha 180$ whenever $\alpha \leq 5/6$.

Case 2: If the firm sells to 180-types in the first period and to 150-types in the second period, P_2 must be 90. Then, P_1 is such that $180 - P_1 = (1/2)(120 - 90) = 15$, that is, $P_1 = 165$. Then the profit would be $\alpha 165 + (1 - \alpha)90 = 90 + 75\alpha$, with $P_1 = 165$ and $P_2 = 90$.

Case 3: If the firm sells to both types in the second period, then $P_1 > 180$ and P_2 is either 90 or 120. If $\alpha 120 \geq 90$ then $P_2 = 120$, otherwise $P_2 = 90$. That is, if $\alpha \geq 3/4$, profit is $\alpha 120$ with $P_2 = 120$, and if $\alpha \leq 3/4$, profit is 90 with $P_2 = 90$. (Here I also considered the option of not selling to low types as well.)

Note that, $90 + 75\alpha \geq \alpha 180$ if $\alpha \leq 6/7$. Thus, selling to only 180-types is ruled out since $\alpha \in (0, 6/7)$.

Also, since $150 > \alpha 120$ and $150 > 90$, case 3 is also ruled out.

Thus, we only need to compare $90 + 75\alpha$ to 150. Now, $90 + 75\alpha \leq 150$ if and only if $\alpha \leq 4/5$.

Therefore, we get the following:

If $6/7 > \alpha \geq 4/5$, optimal prices are $P_1 = 165$ and $P_2 = 90$, and 180-type buyers buy in the first period and 150-type buyers buy in the second period, and the firm's profit is $90 + 75\alpha$.

If $4/5 \geq \alpha > 0$, optimal prices are $P_1 = 150$ and $P_2 > 120$, and all buyers buy in the first period, and the firm's profit is 150.

3. (26 pts.) Suppose a firm can produce two goods, x and y. Marginal cost of good x is $c_x = 20$ and the marginal cost of good y is $c_y = 30$. If the firm bundles one unit of x and one unit of y, the bundle has a marginal cost $c_b = c_x + c_y + c = 50 + c$, where $c > 0$, that is, bundling process itself is also costly. There are no fixed costs. The firm can produce and sell any amount demanded of each good. There are 900 potential buyers: 300 of them value one unit of x at 80 and one unit of y at 20. 400 of them value one unit of x at 50 and one unit of y at 70. The rest value one unit of x at 10 and one unit of y at 60. Suppose each buyer demands *at most* one unit of each good.

(a) (6 pts.) Suppose the firm only sells bundles, which consist of one unit of each good. Find the profit maximizing bundle price, P_b, for $c = 10$.

Answer:

The bundle price can be either 100, 120 or 70.

If $P_b = 100$, the profit is 700(100-60)=28000

If $P_b = 120$, the profit is 400(120-60)=24000

If $P_b = 100$, the profit is 900(70-60)=9000

Thus, with pure bundling the optimal price for the firm is $P_b = 100$

(b) (20 pts.) Suppose the firm sells bundles at price P_b, and also sells x and y separately at prices P_x and P_y.

i. (10 pts.) Find the profit maximizing P_b, P_x and P_y for $c = 10$.

Answer:

Consider prices $P_b = 110$, $P_x = 80$ and $P_y = 60$. Then profit is $400(110-60)+300(80-20)+200(60-30) = 44000$.

Consider prices $P_b = 120$, $P_x = 80$ and $P_y = 70$. Then profit is $400(120-60)+300(80-20)+0 = 42000$.

Consider prices $P_b > 120$, $P_x = 50$ and $P_y = 60$. Then profit is $0+700(50-20)+600(60-30) = 39000$.

Consider prices $P_b > 120$, $P_x = 50$ and $P_y = 70$. Then profit is $0+700(50-20)+400(70-30) = 37000$.

Thus, optimal prices are $P_b = 110$, $P_x = 80$ and $P_y = 60$.

ii. (10 pts.) Find the profit maximizing P_b, P_x and P_y for $c = 25$.

Answer:

Now the bundle has a marginal cost $c_b = 75$.

Consider prices $P_b = 110$, $P_x = 80$ and $P_y = 60$. Then profit is $400(110-75)+300(80-20)+200(60-30) = 38000$.

Consider prices $P_b = 120$, $P_x = 80$ and $P_y = 70$. Then profit is $400(120-75)+300(80-20)+0 = 36000$.

Consider prices $P_b > 120$, $P_x = 50$ and $P_y = 60$. Then profit is $0+700(50-20)+600(60-30) = 39000$.

Consider prices $P_b > 120$, $P_x = 50$ and $P_y = 70$. Then profit is $0+700(50-20)+400(70-30) = 37000$.

Thus, optimal prices are $P_b > 120$, $P_x = 50$ and $P_y = 60$.

4. (34 pts.) Suppose a firm faces one buyer, who has a demand function either $Q_L(p) = 10 - p$ or $Q_H(p) = 12 - p$, where p is the price. The firm cannot tell the buyer's demand but knows that her demand is Q_L with probability λ, and Q_H with probability $1 - \lambda$. Suppose the marginal cost is zero and there are no fixed costs. Suppose the firm offers a pricing scheme (A, p) where A is a fixed fee to be paid by the buyer and p is the unit price for the good. The buyer observes (A, p) and decides how many units to buy.

(a) (10 pts) Assuming the firm is forced to sell to both types, find the profit maximizing p^* in terms of λ.

Answer:

The firm solves

$$max_p \ \frac{1}{2}(10-p)^2 + p[\lambda(10-p) + (1-\lambda)(12-p)]$$

First order condition yields $p^* = 2(1 - \lambda)$.

(b) (12 pts) Now, assume $\lambda = 1/4$ and assume that firm chooses to serve either both types or only H-type. Find the profit maximizing (A^*, p^*).

Answer:

Case 1: Serving all.

Note that $p = 2(1 - \lambda) = 2(1 - (1/4)) = 3/2$.

Then, the expected profit is $\pi(p) = \frac{1}{2}(10 - (3/2))^2 + (3/2)[(1/4)(10 - (3/2)) + (3/4)(12 - (3/2))] = \frac{17^2}{8} + 15$.

Case 2: Serving only to H-type.

Firm sets $p = 0$ and $A = (12 \cdot 12)/2 = 72$. Thus the expected profit in this case is $\pi(p = 0) = (1 - \lambda)72 = (3/4)72$.

Since $(3/4)72 > \frac{17^2}{8} + 15$, the optimal two part tariff consists of $p^* = 0$ and $A^* = 72$.

(c) (12 pts) Now, suppose instead of a two-part tariff, (A, p), the firm offers two bundles (q_L, T_L) and (q_H, T_H), which specify total quantity and total payments. However, there is also another bundle available, provided by the government, which is $q_0 = 6$ and $T_0 = 20$. Find the profit maximizing T_L and T_H, if $q_L = 8$ and $q_H = 12$.

Answer:

For L-type,

$6, 20$ generates a net benefit equal to 22.

$8, T_L$ generates a net benefit equal to $48 - T_L$.

$12, T_H$ generates a net benefit equal to $50 - T_H$.

For H-type,

$6, 20$ generates a net benefit equal to 34.

$8, T_L$ generates a net benefit equal to $64 - T_L$.

$12, T_H$ generates a net benefit equal to $72 - T_H$.

Then, for L-type to choose $(8, T_L)$ we need

$48 - T_L \geq 22$,

$48 - T_L \geq 50 - T_H$.

(Note $48 - T_L \geq 0$ is implied by the first condition.)

Then, for H-type to choose $(12, T_H)$ we need

$72 - T_H \geq 34$,

$72 - T_H \geq 64 - T_L$.

(Note $72 - T_H \geq 0$ is implied by the first condition)

Ignore $48 - T_L \geq 50 - T_H$ and $72 - T_H \geq 34$ for the moment. The other two constraints binding implies $48 - T_L = 22$ and $72 - T_H = 64 - T_L$.

Thus, we get $T_L^* = 26$ and $T_H^* = 34$.

(You can check that at these payments both $48 - T_L \geq 50 - T_H$ and $72 - T_H \geq 34$ hold.)