NAME:

NO:

Boğaziçi University
Department of Economics
Fall 2014

EC 301 ECONOMICS OF INDUSTRIAL ORGANIZATION

FINAL EXAM 1 - Answer Key

30/12/2014
Tuesday 17:00

- **Do not forget to write your full name and student number on the top.**
- **Turn off your cell phone and put it away. If you are seen with a cell phone, on or off, 50 points will be taken off immediately.**
- **Put away all your lecture notes, books, etc.**
- **There are 4 questions and 8 pages in the exam. Make sure you have them all.**
- **Please answer all of the questions in the space provided for each question.**
- **Show your work.**
- **You have 150 minutes.**

GOOD LUCK!!

1. **(22 pts)** There are two firms and two markets: Firm 1 and Firm 2, and Market A and Market B. Firms produce perfect substitutes, that is, homogeneous goods. Each firm has a total cost of producing q units equal to $c(q) = q^2/2$. The inverse demand in Market A is given by $P(Q_A) = 100 - q_1 - q_2$ where $q_1 + q_2 = Q_A$. The inverse demand in Market B is given by $P(Q_B) = b - Q_B$, where $b < 200$. Suppose that Firm 1 has the opportunity to sell on both markets. If Firm 1 sells q_1 in the first market and x_1 on the other market, its total cost is $(q_1 + x_1)^2/2$. Suppose that Firm 2 can sell only on Market A. So, in Market B, there can only be Firm 1. Consider the Cournot game in which Firm 1 chooses q_1 and x_1, and Firm 2 chooses q_2, simultaneously. Find the equilibrium quantities in Market A. How does an increase in b affect Firm 2's equilibrium profit? Provide intuition.

   Answer: The profit of Firm 1 is given by

$$\pi_1 = q_1(100 - q_1 - q_2) + x_1(b - x_1) - \frac{(q_1 + x_1)^2}{2}$$

   The first order condition with respect to q_1 is given by $100 - 3q_1 - q_2 - x_1 = 0$. And with respect to x_1 is given by $b - 3x_1 - q_1 = 0$, which implies $x_1 = \frac{b - q_1}{3}$. Plugging this into $100 - 3q_1 - q_2 - x_1 = 0$, we get $q_1^{BR} = \frac{300 - b - 3q_2}{8}$.

   The profit of Firm 2 is given by

$$\pi_1 = q_2(100 - q_1 - q_2) - \frac{q_2^2}{2}$$

   The first order condition with respect to q_2 is given by $100 - q_1 - 3q_2 = 0$, which implies $q_2^{BR} = \frac{100 - q_1}{3}$.

   Solving the two best responses we get, $q_1^* = \frac{200 - b}{7}$ and $q_2^* = \frac{500 + b}{21}$.

   When b increases Firm 2's quantity increases. The market price is $P = 100 - \frac{200 - b}{7} - \frac{500 + b}{21} = \frac{1000 + 2b}{21}$ which also increases with b. Thus, Firm 2's profit increases in b. This is because as b increases the demand in Market B increases, thus, for Firm 1, it's more profitable in that market, thus, Firm 1 will focus more on Market B and less on Market A, in terms of concentrating its output. Thus, Firm 2 will be relatively stronger in Market A, thus will have a higher profit.

2. **(30 pts)** In a market for a homogeneous product, there is an incumbent firm, Firm I, and there is a potential entrant, Firm E. The inverse demand function is given by $P = 150 - Q$, where $Q = q_I + q_E$ and q_i is the quantity level of Firm i, $i = I, E$. One unit of production requires one unit of labor and one unit of capacity. Potential entrant has already a predetermined capacity, $k_E = 30$, which is publicly observed and cannot be changed by the entrant until entry occurs. Incumbent invests in capacity, k_I, and publicly announces it. Then, Firm E, after observing k_I, decides whether or not to enter. If Firm E enters, firms compete in quantities. During the quantity competition, each firm can expand its capacity if needed. The cost of capacity is 20 and the cost of labor is 30, for both firms. Capacity cost is sunk. There is an entry cost for the potential entrant, given by $f > 0$.

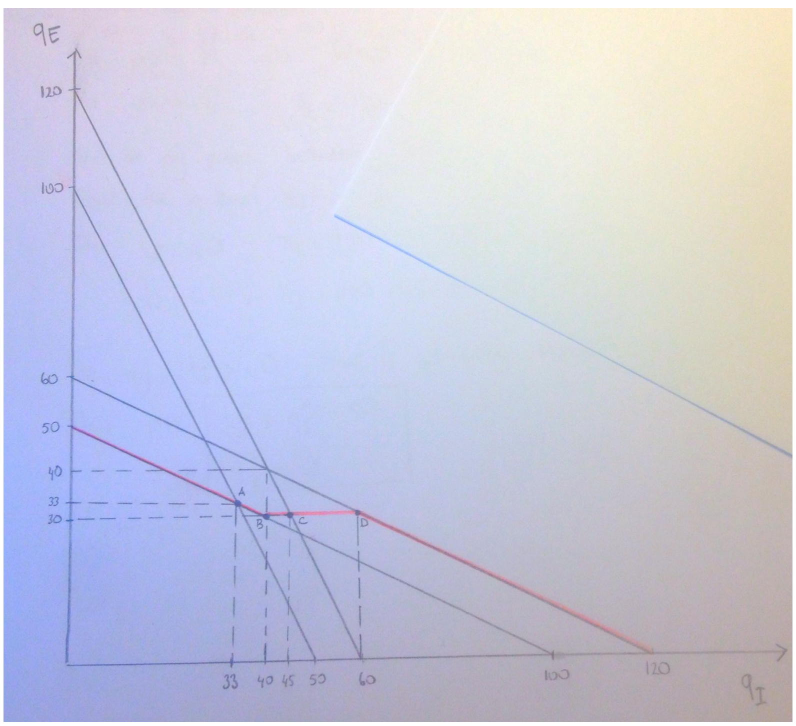
(a) (10 pts) Find the best response function of the potential entrant in the quantity competition stage.

Answer: The profit of potential entrant when marginal cost is only w is $\pi_E^w = (150 - q_I - q_E - 30)q_E$. The first order condition gives $120 - 2q_E - q_I = 0$. Thus, $BR_E^w = 60 - \frac{q_I}{2}$. When the marginal cost is $w + r$, the profit of potential entrant is $\pi_E^{w+r} = (150 - q_I - q_E - 50)q_E$. The first order condition gives $100 - 2q_E - q_I = 0$. Thus, $BR_E^{w+r} = 50 - \frac{q_I}{2}$.

For $k_E = 30 = BR_E^w = 60 - \frac{q_I}{2}$, it must be $q_I = 60$.
For $k_E = 30 = BR_E^{w+r} = 50 - \frac{q_I}{2}$ it must be $q_I = 40$.
Thus,

$$BR_E = \begin{cases} 60 - \frac{q_I}{2} & \text{if} \quad q_I \geq 60 \\ 30 & \text{if} \quad 60 \geq q_I \geq 40 \\ 50 - \frac{q_I}{2} & \text{if} \quad 40 \geq q_I \end{cases}$$

Best response of the entrant is in red in the picture below.



(b) (10 pts) Find a range of values of f for which there is no entry regardless of k_I.

Answer: At point A, the quantities are $\frac{100}{3}$ for both firms. Then the profit of entrant at point A is $\pi_E = (150 - \frac{100}{3} - \frac{100}{3} - 50)\frac{100}{3} - f = (\frac{100}{3})^2 - f$. Thus, if $f \geq (\frac{100}{3})^2$ then there is no entry regardless of k_I.

(c) (10 pts) Let $f = 900$. What capacity level of the incumbent induces zero-profit for the entrant? What will be the equilibrium capacity and the quantities?

Answer: Note that the quantity equilibrium can only occur at a point between A and C, if there is entry.

At point A, when $f = 900$, $\pi_E > 0$. At point B, $\pi_E = (150 - 40 - 30 - 50)30 - 900 = 0$. And at any point between B and C, $\pi_E < 0$, since q_I increases and price decreases although q_E is the same along the [BC] segment. Thus, for any $k_I > 40$, there is no entry. For such capacity choices, the best one is the monopoly quantity, which is 50. Thus, incumbent picks $k_I^* = q_I^* = 50$ and there is no entry and the incumbent gets the monopoly profit.

3. **(24 pts)** Suppose that there are two firms, A and B, producing a homogenous product, and they are engaged in an infinitely repeated game of price competition. The demand function each period is $Q = Q(P)$. Both firms have zero marginal cost and no fixed cost. A firm's action in a period is to choose a price from $[0, \infty)$. The lowest-priced firm gets all the demand, unless there is a tie. Whenever there is a tie in prices, A gets r portion of the demand and B gets the rest of the demand, where $r \in (0, 1)$. They both have the same discount factor $\delta \in [0, 1]$.

(a) (12 pts) Consider the following grim-trigger strategy: play collusive price (monopoly price) every period as long as the other firm has played the collusive price so far; if the other firm has deviated to some other price in a period, then from next period on, play marginal cost pricing for 5 periods and then go back to the collusive price. Letting $r = 1/3$, find a condition on the discount factor which guarantees the collusive outcome to be played every period under this particular grim-trigger strategy.

Answer: The condition for no deviation is

$$r\Pi^M(1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5 + \delta^6 + \delta^7 ...) \geq \Pi^M + \delta 0 + \delta^2 0 + \delta^3 0 + \delta^4 0 + \delta^5 0 + r\Pi^M(\delta^6 + \delta^7 + ...)$$

that is

$$r\Pi^M(1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5) \geq \Pi^M$$

that is

$$r(1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5) \geq 1 \tag{1}$$

When $r = 1/3$, this inequality which boils down to

$$1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5 \geq 3$$

that is

$$\delta(1 + \delta^2 + \delta^3 + \delta^4 + \delta^4) \geq 2$$

that is

$$\delta(\frac{1 - \delta^5}{1 - \delta}) \geq 2$$

that is

$$\delta(3 - \delta^5) \geq 2$$

Note that we also need

$$(1 - r)(1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5) \geq 1$$

But this is implied by the inequality (1) since $1 - r > r$ when $r = 1/3$.

Thus, under the condition $\delta(3 - \delta^5) \geq 2$ we get collusion.

(b) (12 pts) Suppose that the demand is stochastic and it can be either zero or $Q(P)$, with probabilities, 1/3 and 2/3, respectively. Suppose also that at the end of each period each firm observes only its own profit level, but not the demand realization or the price picked by the other firm. Consider the grim trigger strategy with the following punishment structure: Play collusive prices (monopoly price) as long as you have a positive profit. When you get a zero profit in a period, say at t, then in the beginning of the next period, at $t+1$, toss a fair coin to determine whether to start the punishment (at $t+1$) or not. If you start punishment at $t+1$, keep it forever. If you don't start the punishment at $t+1$, start the punishment in $t+2$ if your profit in $t+1$ is also zero. Otherwise, don't start the punishment phase at $t+2$. Find a condition that depends on r and δ only, which guarantees no deviation from the collusive prices, under this strategy. If $\delta = 1$, find a range of r values, for which there is no deviation from the collusive prices, under this strategy. Provide intuition for your range of r values.

Answer: Let's define V^+ as we did in class, to be the expected profit from the entire game starting at a collusive period, and V^- be the expected profit from the entire game starting at a punishment period. Note that $V^- = 0$, since the punishment takes forever.

Now let's write V^+ for firm A who gets the r portion of the demand in case of a tie.

$$V^+ = \frac{2}{3}(r\Pi^M + \delta V^+) + \frac{1}{3}[\frac{1}{2}0 + \frac{1}{2}(\frac{2}{3}(\delta r\Pi^M + \delta^2 V^+) + \frac{1}{3}0)]$$

that is

$$V^+ = \frac{2}{3}(r\Pi^M + \delta V^+) + \frac{1}{9}[\frac{2}{3}(\delta r\Pi^M + \delta^2 V^+]$$

Solving for V^+ we get $V^+ = \frac{6+\delta}{9-6\delta-\delta^2} r\Pi^M$.

The no deviation constraint is given by

$$V^+ \geq \frac{2}{3}\Pi^M + \frac{1}{2}\frac{2}{3}(r\Pi^M + \delta^2 V^+) + \frac{1}{2}0$$

Plugging $V^+ = \frac{6+\delta}{9-6\delta-\delta^2} r\Pi^M$ into the no deviation constraint and arranging we get

$$1 - r \leq \frac{\delta(\delta+3)}{3(3-\delta)}$$

We also need a similar condition for firm B, who gets $1-r$ portion of the demand in case of a tie, which is given by

$$r \leq \frac{\delta(\delta+3)}{3(3-\delta)}$$

Combining these two conditions when $\delta = 1$, we get $\frac{1}{3} \leq r \leq \frac{2}{3}$. For very low r values, firm A will deviate, and for very high r values firm B will deviate. So we need an intermediate range for r for no firm to deviate.

4. **(24 pts)** Consider a linear city that stretches along a $1km$ distance. Suppose consumers are distributed uniformly along the city which can be best represented by an interval $[0, 1]$. There are two firms, A and B. Firm A is located at the end point 0. Firm B is located at the other end point 1. Each firm has zero marginal cost and zero fixed cost. Each consumer buys exactly one unit from the firm which has the smallest overall cost: sum of the firm's (expected) price and consumer's cost of going to and back from the firm. A consumer located at point x has a transportation cost $\frac{tx}{2}$ in the direction towards the 0-end point and tx^2 in the direction towards the 1-end point. So, round trip of going to and coming back from A would be $\frac{tx}{2} + tx^2$. Firms announce their prices simultaneously, P_A and P_B. However, Firm A also announces that each consumer who buys from it, will get either of the three following prices: P_A, $P_A/2$ or 0, each with equal probability. Firm B also announces that each consumer who buys from it, will get either of the two following prices: P_B or $P_B/2$, each with equal probability.

(a) (12 pts) Find the equilibrium prices, P_A and P_B.

Answer: If the indifferent consumer is located at x distance to firm A, then we have

$$\frac{1}{3}(p_A + \frac{p_A}{2} + 0) + \frac{tx}{2} + tx^2 = \frac{1}{2}(p_B + \frac{p_B}{2}) + \frac{t(1-x)}{2} + t(1-x)^2$$

Arranging and solving for x we get

$$x = \frac{p_B}{4t} - \frac{p_A}{6t} + \frac{1}{2}$$

Expected profit for A is

$$\pi_A = \frac{1}{3}(p_A + \frac{p_A}{2} + 0)x = \frac{p_A}{2}(\frac{p_B}{4t} - \frac{p_A}{6t} + \frac{1}{2})$$

Best response is then $p_A = \frac{3p_B}{4} + \frac{3t}{2}$.

Expected profit for B is

$$\pi_B = \frac{1}{2}(p_B + \frac{p_B}{2})(1-x) = \frac{3p_B}{4}(\frac{p_A}{6t} - \frac{p_B}{4t} + \frac{1}{2})$$

Best response is then $p_B = \frac{p_A}{3} + t$.

Solving the best responses together we get $p_A = 3t$ and $p_B = 2t$.

Note that $x = \frac{p_B}{4t} - \frac{p_A}{6t} + \frac{1}{2} = 1/2$.

(b) (12 pts) Now, suppose that, in addition to the above, Firm B also announces the following strategy: Each consumer who buys from it, will be given a lottery ticket which pays $V > 0$ with probability x^*, where x^* is the location of the consumer who is indifferent between the two firms. Find the equilibrium prices, P_A and P_B, in terms of t and V. How does V affect Firm A's price? Provide intuition.

Answer: If the indifferent consumer is located at x distance to firm A, then we have

$$\frac{1}{3}(p_A + \frac{p_A}{2} + 0) + \frac{tx}{2} + tx^2 = \frac{1}{2}(p_B + \frac{p_B}{2}) + \frac{t(1-x)}{2} + t(1-x)^2 - Vx$$

Arranging and solving for x we get

$$x = \frac{1}{3t+V}(\frac{3p_B - 2p_A}{4} + \frac{3t}{2})$$

Expected profit for A is

$$\pi_A = \frac{1}{3}(p_A + \frac{p_A}{2} + 0)x = \frac{p_A}{2}(\frac{1}{3t+V}(\frac{3p_B - 2p_A}{4} + \frac{3t}{2}))$$

Best response is then $p_A = \frac{3p_B}{4} + \frac{3t}{2}$.

Expected profit for B is (if V is paid by firm B)

$$\pi_B = \frac{1}{2}(p_B + \frac{p_B}{2})(1-x) - Vx = \frac{3p_B}{4} - (\frac{3p_B}{4} + V)x = \frac{3p_B}{4} - (\frac{3p_B}{4} + V)\frac{1}{3t+V}(\frac{3p_B - 2p_A}{4} + \frac{3t}{2})$$

Best response is then $p_B = \frac{p_A}{3} + t$.

Note that the best responses did not change!! Thus the prices are the same as in part (a), $p_A = 3t$ and $p_B = 2t$.

(Note: If V is not paid by firm B, then the best response of B would change. Check that in this case it would be $\frac{3t+2V+p_A}{3}$. Then the prices would be $p_A = 3t + \frac{2V}{3}$ and $p_B = 2t + \frac{8V}{9}$.)