NAME:

NO:

Boğaziçi University
Department of Economics
Fall 2013
EC 301 ECONOMICS OF INDUSTRIAL ORGANIZATION
FINAL EXAM - Answer Key
15/01/2014
Wednesday

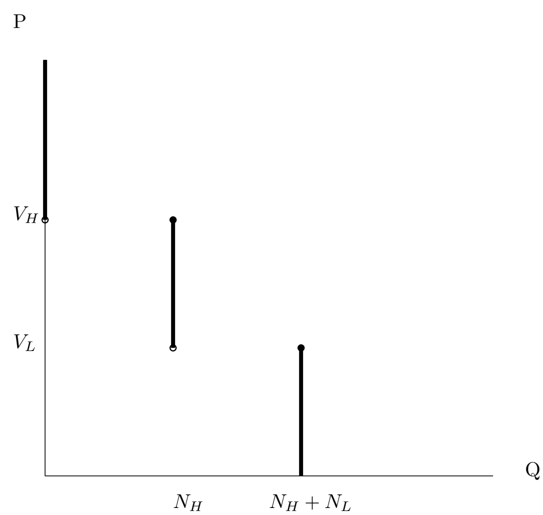
- **Do not forget to write your full name and student number on the top.**
- **Turn off your cell phone and put it away. If you are seen with a cell phone, on or off, 50 points will be taken off immediately.**
- **Put away all your lecture notes, books, etc.**
- **There are 4 questions and 12 pages in the exam. Make sure you have them all.**
- **Please answer all of the questions in the space provided for each question.**
- **Show your work.**
- **You have 140 minutes.**

GOOD LUCK!!

1. **(14 pts)** Consider a monopoly selling its product to a single market where there are N_L many consumers who are willing to pay a maximum of V_L for one unit, and there are N_H many consumers who are willing to pay a maximum of V_H for one unit. Suppose each consumer buys at most one unit of the product. The monopoly has a marginal cost, $c > 0$ and a fixed cost $f > 0$. Suppose $V_H > V_L > c > 0$. If a consumer is indifferent between buying and not buying, assume she buys.

   (a) (6pts) Draw the market aggregate demand curve that the monopoly is facing. Be careful with your labels on your graph.

   Answer:



   (b) (8pts) Find the profit maximizing price set by the monopoly, as a function of the parameters c, f, N_L, N_H, V_L and V_H.

   Answer:

   $p^* > V_H$ if $(V_H - c)N_H < f$.

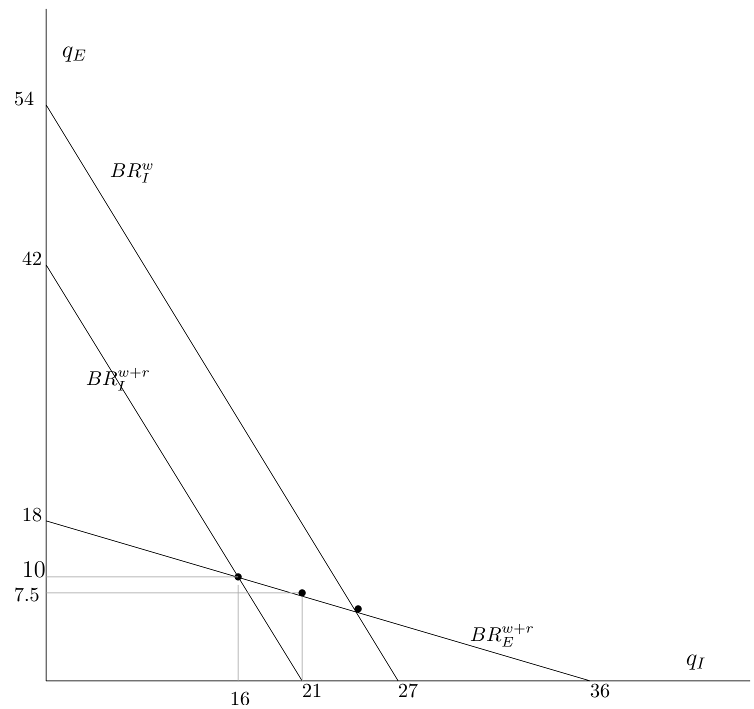
   $p^* = V_H$ if $(V_H - c)N_H \geq f$ and $(V_H - c)N_H > (V_L - c)(N_H + N_L)$.

   $p^* = V_L$ if $(V_L - c)(N_H + N_L) \geq f$ and $(V_H - c)N_H < (V_L - c)(N_H + N_L)$

2. **(30 pts)** In a market for a homogeneous product, there is an incumbent firm, Firm I, and there is a potential entrant, Firm E. The inverse demand function is given by $P = 60 - Q$, where $Q = q_I + q_E$ and q_i is the quantity level of Firm i, $i = I, E$. Firm I invests in capacity, k, and publicly announces it. Then, Firm E, after observing k, decides whether or not to enter. If Firm E enters, firms compete in quantities. Firm E builds its capacity simultaneously with production (that is, $k_E = q_E$), and Firm I, on the other hand can expand its capacity if needed. The cost of capacity is 12 for the incumbent and 18 for the potential entrant. The cost of labor is 6 for both firms. There is an entry cost for the potential entrant, given by $f > 0$.

(a) (7 pts) For $k = 20$, find a range of values of f for which there would be entry.

Answer:



Here $BR_I^w = 27 - \frac{q_E}{2}$, $BR_I^{w+r} = 21 - \frac{q_E}{2}$ and $BR_E^{w+r} = 18 - \frac{q_I}{2}$.
When $k = 20$, $q_E = 18 - (20/2) = 8$. Then, total quantity is $Q = 28$ and price is $P = 60 - 28 = 32$. Then, $\pi_E(k = 20) = (32 - 24)q_E - f = 64 - f$. So, for all $f \leq 64$, there is entry.

(b) (7 pts) Find the minimum capacity that the incumbent firm would ever install in the first stage.

Answer: Minimum possible capacity is $k_{min} = 16$.

(c) (7 pts) Find a range of values of f for which there is no entry regardless of k.

Answer: If π_E at point (16,10) is 0 then there is no entry regardless of the capacity k. At point (16,10), $Q = 26 = 10 + 16$ and $P = 34 = 60 - 26$. So $\pi_E = (34 - 24)10 - f = 100 - f$. So for all $f \geq 100$, there is no entry regardless of capacity k.

(d) (9 pts) Let $f = 49$. Will the incumbent firm choose to deter entry? If so, find the entry deterring capacity. If not, find the equilibrium quantities.

Answer: Note that the Stackelberg leader's output is $q_S = 24$. (If the marginal costs of the two firms are not the same, the Stackelberg leader's output may be different than the monopoly output. In this case the monopoly output of the incumbent with marginal cost $12 + 6 = 18$ is 21). To find this Stackelberg leader's output, q_S, plug best response of the entrant $18 - (q_E/2)$ into incumbent's profit (where the marginal cost is $w + r = 12 + 6 = 18$) and then maximize incumbent's profit with respect to q_I.

When $k_I = q_S = 24$, the entrant picks $q_E = 18 - (24/2) = 6$. The price becomes $60 - 24 - 6 = 30$. Then the entrant's entire profit is then, $(30 - 18 - 6)6 - 49 = -13$. So if the capacity of the incumbent is the Stackelberg leader's output, there will be no entry.

Note, for a given k, the entrant picks $q_E = 18 - (k/2)$. Then, $Q = k + 18 - (k/2) = 18 + (k/2)$ and $P = 60 - Q = 42 - (k/2)$. Then the profit of the entrant is $\pi_E(k) = (42 - (k/2) - 24)(18 - (k/2)) - 49$, where $\pi_E(k) = 0$ implies $k = 22$. So at $k = 22$ entrant's profit is zero. With $k = 22$, there will be no entry, and then incumbent will produce monopoly output, 21, and the incumbent's profit will be $\pi_I^{det.} = (60 - 21 - 18)21 - 12(22 - 21) = 429$.

Thus, there is no need to pick $k = 24$. At $k = 22$, entry is already deterred.

If the incumbent chooses $k = 21$, though, then the entrant's profit will be positive and it will enter: $\pi_E = (60 - 21 - 7.5 - 18 - 6)7.5 - 49 = 7.5^2 - 49 > 0$. Then incumbent will get a profit equal to $(60 - 21 - 7.5 - 12 - 6)21 = 283.5$.

Thus it is optimal to pick $k_I = 22$, deter entry, and then produce $q_M = 21$.

3. **(28 pts)** Suppose that there are two firms producing a homogenous product, and they are engaged in an infinitely repeated game of price competition. The demand function each period is $Q = Q(P)$. Both firms have zero marginal cost and no fixed cost. A firm's action in a period is to choose a price from $[0, \infty)$. The lowest-priced firm gets all the demand, unless there is a tie, in which case they each get half the demand. They both have the same discount factor $\delta \in (0, 1)$.

   (a) (8 pts) Consider the following grim-trigger strategy: play collusive price every period as long as the other firm has played the collusive price so far; if the other firm has deviated to some other price in a period, then from next period on, play marginal cost pricing for 2 periods and then go back to the collusive price for only the next period, and then play marginal cost pricing for the next 2 periods, and then go back to the collusive price for only the next period, and then play marginal cost pricing for the next 2 periods, and so on. Find a condition that guarantees the collusive outcome to be played every period under this particular grim-trigger strategy.

   Answer: Check $\frac{1}{1-\delta}\frac{\pi^M}{2} \geq \pi^M + 0 + 0 + \delta^3\pi^M + 0 + 0 + \delta^6\pi^M + 0 + 0 + \delta^9\pi^M + 0 + ...$.
   That is, $\delta(1+\delta) \geq 1$.

(b) (10 pts) Suppose the demand is stochastic and there are five possible demand realizations: $Q_i(P)$, for $i = 1, 2, 3, 4, 5$, each with equal probability. Suppose each period, firms simultaneously pick their prices, after observing the realization of the demand. Suppose the monopoly profit for a demand realization $Q_i(P)$ is $i \times \pi$, where $\pi > 0$, for instance, $\pi_3^M = 3\pi$. Find a condition that guarantees the collusive outcome to be played every period under the grim trigger strategies with infinite number of punishment periods. How does your condition depend on π? Provide intuition.

Answer: $V = \frac{1}{1-\delta}\frac{1}{2}\frac{1}{5}[\pi + 2\pi + 3\pi + 4\pi + 5\pi] = \frac{1}{1-\delta}\frac{3\pi}{2}$. Now check Q_5 for no deviation. We need $\frac{5\pi}{2} + \delta V \geq 5\pi$, that is, $\frac{5\pi}{2} + \delta[\frac{1}{1-\delta}\frac{3\pi}{2}] \geq 5\pi$, which implies, $\delta \geq 5/8$. For such δs collusion is sustained.

(c) (10 pts) Suppose that the demand is stochastic and it can be either zero or $Q(P)$, with equal probability. Suppose also that at the end of each period the firms observe only their own profit levels, but not the demand realization or the price picked by the other firm. Consider the grim trigger strategy with the following punishment structure: Start the punishment phase only if you see your own profit being zero in three consecutive periods. Once you start the punishment mode, keep it forever. Under this strategy, define V to be the present discount value of a firm's profit, in the entire game starting with the collusive phase. Let the monopoly profit when the demand is $Q(P)$ be 172. Let $\delta = 1/2$. Find V.

Answer:

$$
\begin{aligned}
V &= \frac{1}{2}(\frac{\pi^M}{2} + \delta V) + \frac{1}{2}(\frac{1}{2}[\delta\frac{\pi^M}{2} + \delta^2 V] + \frac{1}{2}[\frac{1}{2}(\delta^2\frac{\pi^M}{2} + \delta^3 V) + \frac{1}{2}\delta^2 0]) \\
&= \frac{1}{2}(\frac{\pi^M}{2} + \delta V) + \frac{1}{4}\delta(\frac{\pi^M}{2} + \delta V) + \frac{1}{8}\delta^2(\frac{\pi^M}{2} + \delta V) \\
&= \frac{1}{2}(\frac{\pi^M}{2} + \delta V)(1 + \frac{\delta}{2} + \frac{\delta^2}{4})
\end{aligned}
$$

Plugging $\pi = 172$ we get $V = 84$.

4. **(28 pts)** Consider a linear city that stretches along a $1km$ distance. Suppose 1000 consumers are distributed uniformly along the city which can be best represented by an interval $[0, 1]$. There are three firms, A, B and Z. Firm A is located at the end point 0. Firm B is located at the other end point 1. Firm Z is located outside of the city, at exactly $1km$ distance from firm A to its north, that is, the line connecting A and Z makes a 90 degree angle with the line connecting A and B. There are no other consumers anywhere outside the $[0, 1]$ interval. A consumer on the $[0, 1]$ interval can travel to any of the three firms on the line connecting the firm's location and the consumer's location. Each consumer buys exactly one unit from the firm which has the smallest overall cost (sum of the firm's price and consumer's cost of traveling to the firm). For each consumer, the traveling cost (for the round trip) is given by the function $t(x) = x^2$, where x is the consumer's distance from the firm she is buying from. Firm Z has a fixed price given by $p_Z = 1$. Let P_A and P_B denote the prices of firm A and firm B, respectively.

(a) (10 pts) Suppose $P_A < 2$. Find the demand each firm gets as functions of prices, P_A and P_B.

Answer: For a consumer at point x, buying from A costs $P_A + x^2$, buying from B costs $P_B + (1-x)^2 = P_B + 1 - 2x + x^2$, and buying from Z costs $P_Z + 1 + x^2 = 1 + 1 + x^2 = 2 + x^2$.

If $P_A < 2$, then $P_A + x^2 < 2 + x^2$ for any x. In this case, the demand for Z is $D_Z = 0$. So, consider the consumer who is indifferent between buying from A and buying from B. Say this consumer is located at x. Then we have $P_A + x^2 = P_B + 1 - 2x + x^2$. Solving this equation we get $x = \frac{P_B - P_A + 1}{2}$. Then the demand A gets is

$$D_A = \begin{cases} \frac{P_B - P_A + 1}{2} & \text{if} \quad -1 < P_B - P_A < 1 \\ 1 & \text{if} \quad P_B - P_A \geq 1 \\ 0 & \text{if} \quad P_B - P_A \leq -1 \end{cases}$$

And $D_B = 1 - D_A$ and $D_Z = 0$.

(b) (10 pts) Suppose $P_A > 2$. Find the demand each firm gets as functions of prices, P_A and P_B.

Answer: If $P_A > 2$, then $P_A + x^2 > 2 + x^2$ for any x. In this case, the demand for Z is $D_A = 0$. Then, if the consumer who is indifferent between buying from Z and buying from B is located at x, then it must be $2 + x^2 = P_B + 1 - 2x + x^2$, which implies $x = \frac{P_B - 1}{2}$. Then, the demands are

$$D_Z = \begin{cases} \frac{P_B-1}{2} & \text{if} \quad 1 < P_B < 3 \\ 0 & \text{if} \quad P_B \leq 1 \\ 1 & \text{if} \quad P_B \geq 3 \end{cases}$$

and

$$D_B = \begin{cases} \frac{3-P_B}{2} & \text{if} \quad 1 < P_B < 3 \\ 1 & \text{if} \quad P_B \leq 1 \\ 0 & \text{if} \quad P_B \geq 3 \end{cases}$$

and $D_A = 0$.

(c) (8 pts) Suppose $P_A = 1/2$. Find the optimal price, P_B, for firm B.

Answer: When $P_A = 1/2$, then $D_Z = 0$ and $D_B = 1 - \frac{P_B - P_A + 1}{2} = \frac{1 - P_B + P_A}{2}$. Plugging $P_A = 1/2$ we get $D_B = \frac{3}{4} - \frac{P_B}{2}$ and the profit is $\pi_B = P_B(\frac{3}{4} - \frac{P_B}{2})$. Maximizing this expression we get $P_B^* = 3/4$. Note that profit is positive at this price.