Growth and Income Inequality in an Endogenous Growth Model with Public Capital under Progressive Taxation

Murat Koyuncu       Alper Ünsal

Bogazici University

CEE Annual Conference - May 17, 2016
Exploiting the Public Capital-Inequality Link

2 The recent literature on the economic growth effects of public capital suggests various ways of solving this problem. Some of the earlier studies have also been criticized for not taking the stationarity of the data properly into account (see, for instance, Sturm and de Haan 1995). Unit root tests often suggest that output and public capital contain a unit root. However, it is well known that unit root tests have low power to discriminate between unit root and near unit root processes. This problem is especially pronounced for small samples. One way to alleviate the small-sample problem that has become popular in recent research is to make use of the cross-sectional dimension of the data and to apply panel data techniques.

Figure 1. Government investment in 22 OECD countries, 1961-2001, average (% GDP) and standard deviation

In some of the earlier studies unit roots in GDP and capital stock were removed by taking first differences. But this may ignore evidence of a long-run relationship in the data if the series are cointegrated (Munnell 1992). Indeed, various recent studies report evidence for such a cointegrating relationship between public capital (or infrastructure) and output. By exploiting this cointegrating relationship, these studies estimate the long-run effect of public capital (or infrastructure) on GDP per capita. However, the existence of a cointegrating relationship in itself does not necessarily imply that causality runs from infrastructure to long-run growth (Canning and Pedroni 1999).

In their survey of the earlier literature, Sturm et al. (1998) show that the literature contained a relatively wide range of estimates, with a marginal product of public capital that is much higher than that of private capital (e.g., Aschauer 1989), roughly equal to that of private capital (e.g., Munnell 1990b). Early estimates for the impact of public capital on economic growth cover a wide range—making them almost useless from a policy perspective. The problem not only occurs in studies like that of Aschauer (1989), but also in studies based on panel data, like Munnell (1990b), who found positive elasticities of output to public capital using panel data at the US state level. According to Holtz-Eakin (1994, p. 13), “[b]ecause more prosperous states are likely to spend more on public capital, there will be a positive correlation between the state-specific effects and public sector capital. This should not be confused, however, with the notion that greater public capital leads a state to be more productive.”
Motivation

Exploring the Public Capital-Inequality Link

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Figure: Changes in Gini coefficients in OECD countries, 1985 and 2008. Source: OECD (2011)
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- Models with progressive taxation:
  - Sarle (1997), Sorger (2002) and Li and Sarle (2004): Distribution of wealth (and income) would be non-degenerate under progressive taxation (with exogenous labor).
  - Carroll and Young (2009): Additional heterogeneity in skills, focus on labor income-asset income correlation.
  - Bosi and Seegmuller (2010): Elastic labor. Borrowing constraints. Focus on an SS where only the most patient HH holds capital and they may not supply labor.
  - Koyuncu and Turnovsky (2016): Elastic labor. Shows how the labor supply responses of different classes may differ under progressive taxes.
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- **Chatterjee and Turnovsky (2012):** Only heterogeneous agent model with public capital. Shows higher public investment worsens inequality. Considers flat taxation.
What this paper adds to the literature

- Higher public investments \( \Rightarrow \) Higher inequality and growth rates.
- Higher tax progressivity \( \Rightarrow \) Lower inequality and growth rates.

Combination of the two to discuss policy options to reduce inequality without harming growth.

Different levels of progressivity on capital and labor income to make a deeper analysis and to make comparisons with other cases.

Simultaneous determination of aggregate and distributional variables enables a more realistic analysis.

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Simultaneous determination of aggregate and distributional variables enables a more realistic analysis.
Chatterjee and Turnovsky (2012): Identical firms produce output according to

\[ Y_j = A\left[\alpha(X_P L_j)^{-\rho} + (1 - \alpha)K_j^{-\rho}\right]^{-\frac{1}{\rho}} \]

\[ X_P = K^\varepsilon K_G^{1-\varepsilon}, \ 0 \leq \varepsilon \leq 1 \]

\( s \equiv 1/(1 + \rho) \) represents the elasticity of substitution in production between capital and effective units of labor.

Equations per aggregate private capital:

\[ y \equiv y(z, \ell) = A\left[(1 - \alpha) + \alpha\{(1 - \ell)z^{1-\varepsilon}\}^{-\rho}\right]^{-\frac{1}{\rho}}; \ \ell = 1 - L \]

\[ r \equiv r(z, \ell) = (1 - \alpha)A^{-\rho}y(z, \ell)^{1+\rho} \]

\[ w = \omega(z, \ell)K; \ \omega(z, \ell) \equiv \alpha A^{-\rho}y(z, \ell)^{1+\rho}z^{-\rho(1-\varepsilon)}(1 - \ell)^{-(1+\rho)} \]

The ratio of public to private capital: \( z = K_G/K \)
Preferences

- A unit mass of a continuum of infinitely-lived consumers, indexed by $i$, heterogenous in their rates of time preference, $\beta_i$.

\[ U_i = \int_0^\infty \frac{1}{\gamma} [C_i^{-\nu} + \theta(X_U \ell_i)^{-\nu}]^{-\frac{\gamma}{\nu}} e^{-\beta_i t} \, dt \]

\[ X_U = K^{\varphi} K^{1-\varphi}_G, \quad 0 \leq \varphi \leq 1 \]

- $q \equiv 1/(1 + \nu)$: the intra-temporal elasticity of substitution between consumption and leisure.

- $e \equiv 1/(1 - \gamma)$: the inter-temporal elasticity of substitution.
Tax Schedule

- Following Guo and Lansing (1998); progressive tax rates on capital income $\tau_{k,i}$ where $\phi_k$ measures progressivity

\[ \tau_{k,i} = \zeta_k \left( \frac{rK_i}{rK} \right)^{\phi_k} = \zeta_k \left( \frac{K_i}{K} \right)^{\phi_k} \]

- Progressive tax rates on labor income: $\tau_{w,i}$ where $\phi_w$ measures progressivity

\[ \tau_{w,i} = \zeta_w \left( \frac{w(1 - \ell_i)}{w(1 - \ell)} \right)^{\phi_w} = \zeta_w \left( \frac{1 - \ell_i}{1 - \ell} \right)^{\phi_w} \]

- Flat tax rate on consumption: $\tau_c$

- Endogenously determined lump-sum tax rates (or subsidies) to equalize the target government investment rate: $\tau$
Households’ Utility Maximization

- The budget constraint of the agent $i$:

$$\dot{K}_i = (1 - \tau_{k,i})rK_i + (1 - \tau_{w,i})w(1 - \ell_i) - (1 + \tau_c)C_i - T$$

- Individual $i$’s consumption per aggregate capital:

$$c_i \equiv \frac{C_i}{K} = \ell_i \Omega_i(\ell_i, \ell, z)$$

$$\Omega_i(\ell_i, \ell, z) \equiv \left[ \frac{\omega z^{\nu(1-\varphi)}}{\theta (1 + \tau_c)} \right]^{\frac{1}{1+\nu}} \left( 1 - \tau_{w,i}^m \right)^{\frac{1}{1+\nu}}$$

- TVC $\Rightarrow$ The upper limit for the private capital accumulation:

$$\frac{\dot{K}_i}{K_i} < r(1 - \tau_{k,i})$$
The flow equation for financing public investments, $G$:

$$\dot{K}_G = gY = G$$

$$G = r \int [\tau_k, iK_i] di + w \int [\tau_w, i(1 - \ell_i)] di + \tau_c C + \tau Y$$

$$gy = \frac{\dot{K}_G}{K} = r \bar{\tau}_k + \omega \bar{\tau}_w (1 - \ell) + \tau_c \bar{\Omega} \ell + \tau y$$
Equations for the evolution of the economy

- Growth rate of the aggregate wealth:

\[
\frac{\dot{K}}{K} = (1 - g) y(z, l) - \bar{\Omega}(z, \ell, \ell_1, \ldots, \ell_N) \ell
\]

- Evolution of relative wealth:

\[
\dot{k}_i = k_i \left( \frac{\dot{K}_i}{K_i} - \frac{\dot{K}}{K} \right) = \left[ (1 - \tau_{w,i}) \omega(1 - \ell_i) - (1 + \tau_c) \ell_i \Omega_i - \tau y \right]
\]
\[
- \left[ (\tau_{k,i} - \bar{\tau}_k) r + (1 - \bar{\tau}_w) \omega(1 - \ell) - (1 + \tau_c) \bar{\Omega} \ell - \tau y \right] k_i ; \quad k_i \equiv \frac{K_i}{K}
\]

- The flow equation for public investment:

\[
\frac{\dot{z}}{z} = \frac{\dot{K}_G}{K_G} - \frac{\dot{K}}{K} = \frac{gy(z, \ell)}{z} - [(1 - g)y(z, \ell) - \bar{\Omega} \ell]
\]
Equations for the evolution of the economy

- The evolution of time devoted to leisure:

\[
\frac{\dot{\ell}_i}{\ell_i} = \frac{\beta_i - r(1 - \tau_{k,i}) - (\gamma - 1) \frac{\dot{K}}{K} - \Gamma_{N,i}(\ell_i, \ell, z) \frac{\dot{\ell}}{\ell} - \Gamma_{O,i}(\ell_i, \ell, z) \frac{\dot{z}}{z}}{\Gamma_{M,i}(\ell_i, \ell, z)}
\]

\[
\frac{\dot{\ell}}{\ell} = \frac{\int \frac{\ell_i E_i}{\Gamma_{M,i}} \, di}{\ell + \int \frac{\ell_i \Gamma_{N,i}}{\Gamma_{M,i}} \, di}
\]

where

\[
E_i = \beta_i - r(1 - \tau_{k,i}) - (\gamma - 1) \frac{\dot{K}}{K} - \Gamma_{O,i} \frac{\dot{z}}{z}
\]
Steady state equations

- The balanced growth path of the economy: \( \ell_i = \ell = \dot{z} = \dot{k}_i = 0 \)
- The growth rate of aggregate and individual-specific wealth:
  \[
  \tilde{\psi}_i = \tilde{\psi} = \frac{\beta_i - r(\tilde{z}, \tilde{\ell})(1 - \tau_{k,i})}{\gamma - 1} = \frac{gy(\tilde{z}, \tilde{\ell})}{\tilde{z}}
  \]
- Steady state relative wealth of the individual \( i \):
  \[
  \tilde{k}_i = \left[ \frac{(\gamma - 1)gy(\tilde{z}, \tilde{\ell})/\tilde{z} + r(\tilde{z}, \tilde{\ell}) - \beta_i}{r(\tilde{z}, \tilde{\ell})(1 + \phi_k)\zeta_k} \right]^{\frac{1}{\phi_k}}
  \]
Steady state equations

- The relationship between individual’s wealth and labor supply decision:

\[
\left[ \frac{gy(z, \ell)}{z} - r(1 - \tau_{k,i}) \right] (k_i - 1) + r(\tau_{k,i} - \bar{\tau}_k) = \omega \left[ (\ell - \ell_i) + (1 - \ell)\bar{\tau}_w - (1 - \ell_i)\tau_{w,i} \right] + (1 + \tau_c)(\bar{\Omega}\ell - \Omega_i\ell_i)
\]

- \( k_i > 1 \iff \ell_i > \ell \)
Numerical analysis: General properties

- Numerical analyses for the 5-agent case.
- Comparison of the results with the base model in Chatterjee-Turnovsky (2012).
- Introducing government investment shocks using different tax policies.
- Repeating the shock scenarios for different levels of tax progressivity.
- Transition path analyses to investigate the saddle-path stability of the economy and the SR/LR responses of the economy to the shocks.
Benchmark calibration

Table: Parameter values for the benchmark economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = 1/(1 - \gamma)$</td>
<td>Inter-temporal elasticity of substitution</td>
<td>0.4</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The relative weight of leisure in utility</td>
<td>1.75</td>
</tr>
<tr>
<td>$q = 1/(1 + \nu)$</td>
<td>Intra-temporal elasticity of substitution between consumption and leisure in the utility function</td>
<td>1</td>
</tr>
<tr>
<td>$A$</td>
<td>Technology shift parameter</td>
<td>0.6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of efficiency units of labor</td>
<td>0.6</td>
</tr>
<tr>
<td>$s = 1/(1 + \rho)$</td>
<td>Elasticity of substitution in production between capital and effective units of labor</td>
<td>1</td>
</tr>
<tr>
<td>$\varepsilon, \varphi$</td>
<td>Geometric weight of the aggregate private capital in the aggregate composite externalities</td>
<td>0.6</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Rates of time preference</td>
<td>0.036, 0.038, 0.040, 0.042, 0.044</td>
</tr>
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</table>
Benchmark calibration

**Table**: Benchmark calibrations

<table>
<thead>
<tr>
<th></th>
<th>g</th>
<th>Level of tax schedule</th>
<th>Tax progressivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper</td>
<td>0.05</td>
<td>$\zeta_k = 0.05; \zeta_w = 0.05$</td>
<td>$\phi_k = 0.75; \phi_w = -0.75$</td>
</tr>
<tr>
<td>CT2012</td>
<td>0.05</td>
<td>$\tau = 0.05$</td>
<td>n/a</td>
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**Table**: Steady state values for the benchmark economies

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<tr>
<th>Policy</th>
<th>$\tilde{z}$</th>
<th>$\tilde{\ell}$</th>
<th>$\tilde{y}$</th>
<th>$\tilde{\psi}$ (%)</th>
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<tr>
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<td>0.611</td>
<td>0.719</td>
<td>0.249</td>
<td>2.04</td>
</tr>
<tr>
<td>CT2012</td>
<td>0.531</td>
<td>0.714</td>
<td>0.243</td>
<td>2.29</td>
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Long run responses to various fiscal policy shocks - Growth effects

Table: Steady state growth rates (%) after each shock

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<tr>
<th>Benchmark case</th>
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<th>Consumption tax financed</th>
<th>Lump-sum tax financed</th>
<th>Capital income tax financed</th>
<th>Labor income tax financed</th>
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<td>$(0.5,-0.5)$</td>
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<td>2.33</td>
<td>2.44</td>
<td>2.50</td>
<td>2.13</td>
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<td>$(0.75,-0.75)$</td>
<td>2.04</td>
<td>2.29</td>
<td>2.41</td>
<td>2.48</td>
<td>2.04</td>
</tr>
<tr>
<td>$(1.125,-1.125)$</td>
<td>2.00</td>
<td>2.23</td>
<td>2.38</td>
<td>2.43</td>
<td>1.92</td>
</tr>
<tr>
<td>$(0.5,0.5)$</td>
<td>2.00</td>
<td>2.20</td>
<td>2.36</td>
<td>2.42</td>
<td>2.06</td>
</tr>
<tr>
<td>$(0.75,0.75)$</td>
<td>1.93</td>
<td>2.10</td>
<td>2.29</td>
<td>2.35</td>
<td>1.94</td>
</tr>
<tr>
<td>$(1.125,1.125)$</td>
<td>1.85</td>
<td>1.96</td>
<td>2.20</td>
<td>2.25</td>
<td>1.77</td>
</tr>
</tbody>
</table>
Long run responses to various fiscal policy shocks - Size of the public capital to private capital ratios

Table: Steady state size of the public-private capital ratio, $z$

<table>
<thead>
<tr>
<th>$(\phi_k, \phi_w)$</th>
<th>benchmark case</th>
<th>combined income tax financed</th>
<th>consumption tax financed</th>
<th>lump-sum tax financed</th>
<th>capital income tax financed</th>
<th>labor income tax financed</th>
</tr>
</thead>
<tbody>
<tr>
<td>g = 0.05</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
</tr>
<tr>
<td>$\tau_c = 0$</td>
<td>$\tau_c = 0$</td>
<td>$\tau_c = 0.0363$</td>
<td>$\tau_c = 0$</td>
<td>$\tau_c = 0$</td>
<td>$\tau_c = 0$</td>
<td>$\tau_c = 0$</td>
</tr>
<tr>
<td>$\zeta_k = 0.05$</td>
<td>$\zeta_k = 0.08$</td>
<td>$\zeta_k = 0.05$</td>
<td>$\zeta_k = 0.05$</td>
<td>$\zeta_k = 0.05$</td>
<td>$\zeta_k = 0.125$</td>
<td>$\zeta_k = 0.05$</td>
</tr>
<tr>
<td>$\zeta_w = 0.05$</td>
<td>$\zeta_w = 0.08$</td>
<td>$\zeta_w = 0.05$</td>
<td>$\zeta_w = 0.05$</td>
<td>$\zeta_w = 0.05$</td>
<td>$\zeta_w = 0.05$</td>
<td>$\zeta_w = 0.1$</td>
</tr>
<tr>
<td>(0.5,-0.5)</td>
<td>0.596</td>
<td>0.956</td>
<td>0.890</td>
<td>0.883</td>
<td>1.075</td>
<td>0.888</td>
</tr>
<tr>
<td>(0.75,-0.75)</td>
<td>0.611</td>
<td>0.987</td>
<td>0.910</td>
<td>0.901</td>
<td>1.141</td>
<td>0.905</td>
</tr>
<tr>
<td>(1.125,-1.125)</td>
<td>0.629</td>
<td>1.036</td>
<td>0.937</td>
<td>0.928</td>
<td>1.251</td>
<td>0.926</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>0.605</td>
<td>0.978</td>
<td>0.902</td>
<td>0.894</td>
<td>1.091</td>
<td>0.912</td>
</tr>
<tr>
<td>(0.75,0.75)</td>
<td>0.625</td>
<td>1.024</td>
<td>0.929</td>
<td>0.920</td>
<td>1.166</td>
<td>0.945</td>
</tr>
<tr>
<td>(1.125,1.125)</td>
<td>0.653</td>
<td>1.094</td>
<td>0.966</td>
<td>0.957</td>
<td>1.295</td>
<td>0.992</td>
</tr>
</tbody>
</table>
Long run responses to various fiscal policy shocks - Income distribution

**Table:** Steady state income distributions after each shock (in terms of coefficients of variation)

<table>
<thead>
<tr>
<th>(φ_k, φ_w)</th>
<th>benchmark case</th>
<th>combined income tax financed</th>
<th>consumption tax financed</th>
<th>lump-sum tax financed</th>
<th>capital income tax financed</th>
<th>labor income tax financed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.5,0.5)</td>
<td>16.79</td>
<td>10.00</td>
<td>15.65</td>
<td>15.58</td>
<td>5.54</td>
<td>17.13</td>
</tr>
<tr>
<td>(0.75,0.75)</td>
<td>10.19</td>
<td>5.89</td>
<td>9.42</td>
<td>9.36</td>
<td>3.10</td>
<td>10.64</td>
</tr>
<tr>
<td>(1.125,1.125)</td>
<td>5.81</td>
<td>3.37</td>
<td>5.37</td>
<td>5.31</td>
<td>1.65</td>
<td>6.37</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>15.70</td>
<td>8.99</td>
<td>14.69</td>
<td>14.64</td>
<td>5.20</td>
<td>14.91</td>
</tr>
<tr>
<td>(0.75,0.75)</td>
<td>9.23</td>
<td>5.05</td>
<td>8.58</td>
<td>8.52</td>
<td>2.81</td>
<td>8.70</td>
</tr>
<tr>
<td>(1.125,1.125)</td>
<td>5.02</td>
<td>2.67</td>
<td>4.65</td>
<td>4.64</td>
<td>1.42</td>
<td>4.78</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>16.79</td>
<td>10.00</td>
<td>15.65</td>
<td>15.58</td>
<td>5.54</td>
<td>17.13</td>
</tr>
<tr>
<td>(0.75,0.75)</td>
<td>10.19</td>
<td>5.89</td>
<td>9.42</td>
<td>9.36</td>
<td>3.10</td>
<td>10.64</td>
</tr>
<tr>
<td>(1.125,1.125)</td>
<td>5.81</td>
<td>3.37</td>
<td>5.37</td>
<td>5.31</td>
<td>1.65</td>
<td>6.37</td>
</tr>
</tbody>
</table>
### Long run responses to various fiscal policy shocks - Welfare distribution

**Table:** Steady state welfare distributions after each shock (in terms of coefficients of variation)

<table>
<thead>
<tr>
<th>(φ_κ, φ_ω)</th>
<th>benchmark case</th>
<th>combined income tax financed</th>
<th>consumption tax financed</th>
<th>lump-sum tax financed</th>
<th>capital income tax financed</th>
<th>labor income tax financed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g = 0.05</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
<td>g = 0.08</td>
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<td></td>
<td>τ_κ = 0</td>
<td>τ_κ = 0</td>
<td>τ_κ = 0.0363</td>
<td>τ_κ = 0</td>
<td>τ_κ = 0</td>
<td>τ_κ = 0</td>
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<tr>
<td></td>
<td>ζ_κ = 0.05</td>
<td>ζ_κ = 0.08</td>
<td>ζ_κ = 0.05</td>
<td>ζ_κ = 0.05</td>
<td>ζ_κ = 0.125</td>
<td>ζ_κ = 0.05</td>
</tr>
<tr>
<td></td>
<td>ζ_ω = 0.05</td>
<td>ζ_ω = 0.08</td>
<td>ζ_ω = 0.05</td>
<td>ζ_ω = 0.05</td>
<td>ζ_ω = 0.05</td>
<td>ζ_ω = 0.1</td>
</tr>
<tr>
<td>(0.5,-0.5)</td>
<td>26.85</td>
<td>14.94</td>
<td>24.21</td>
<td>24.72</td>
<td>8.66</td>
<td>24.97</td>
</tr>
<tr>
<td>(0.75,-0.75)</td>
<td>15.58</td>
<td>8.32</td>
<td>13.93</td>
<td>14.27</td>
<td>4.70</td>
<td>14.32</td>
</tr>
<tr>
<td>(1.125,-1.125)</td>
<td>8.37</td>
<td>4.32</td>
<td>7.47</td>
<td>7.63</td>
<td>2.37</td>
<td>7.61</td>
</tr>
<tr>
<td>(0.5,0.5)</td>
<td>27.32</td>
<td>15.50</td>
<td>24.67</td>
<td>25.24</td>
<td>8.86</td>
<td>26.01</td>
</tr>
<tr>
<td>(0.75,0.75)</td>
<td>16.08</td>
<td>8.81</td>
<td>14.42</td>
<td>14.70</td>
<td>4.87</td>
<td>15.35</td>
</tr>
<tr>
<td>(1.125,1.125)</td>
<td>8.81</td>
<td>4.72</td>
<td>7.88</td>
<td>8.03</td>
<td>2.50</td>
<td>8.49</td>
</tr>
</tbody>
</table>
Linearization

To check for saddle-path stability
To document the evolution of the economy from a before-shock steady state to a post-shock one.
Once the $z(t)$ and $\ell_i(t)$’s are obtained, $\ell(t)$, $\psi(t)$ and $k_i(t)$’s can be obtained. Following $(N + 1)\times(N + 1)$ system is sufficient to obtain the transition path dynamics:

$$
\begin{bmatrix}
\dot{z} \\
\dot{\ell}_1 \\
\vdots \\
\dot{\ell}_N
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial \ell_1} & \cdots & \frac{\partial \dot{z}}{\partial \ell_N} \\
\frac{\partial \dot{\ell}_1}{\partial z} & \frac{\partial \dot{\ell}_1}{\partial \ell_1} & \cdots & \frac{\partial \dot{\ell}_1}{\partial \ell_N} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \dot{\ell}_N}{\partial z} & \frac{\partial \dot{\ell}_N}{\partial \ell_1} & \cdots & \frac{\partial \dot{\ell}_N}{\partial \ell_N}
\end{bmatrix}
\begin{bmatrix}
z(t) - \tilde{z} \\
\ell_1(t) - \tilde{\ell}_1 \\
\vdots \\
\ell_N(t) - \tilde{\ell}_N
\end{bmatrix}
$$
Transition path characterization

- In order to have a stable transition path, one needs to have a number of negative eigenvalues equal to the number of variables persistent to shocks.

- The 6x6 linearized system for $i = 5$:

$$z(t) = \tilde{z} + [z(0) - \tilde{z}] e^{\mu t}$$

$$\ell_i(t) = \tilde{\ell}_i + \nu_i [z(t) - \tilde{z}] \quad \forall i$$

- Where $\mu$ is the stable root and the $\nu_i$'s are the components of the normalized eigenvector corresponding $\ell_i$'s.

- Having a combination of high levels of labor income tax progressivity, $\phi_w$, and labor income tax schedule, $\zeta_w$, may destabilize the linearized system.
An example of the transition of labor supply

- $g$ increases from 0.05 to 0.08 through a $\zeta_k$ shock.
An example of the transition of government size and leisure

- $g$ increases from 0.05 to 0.08 through a combined $\zeta_k$, $\zeta_w$ shock.
Responses of welfare distribution under different progressivity levels

- $g$ increases from 0.05 to 0.08 through increased lump-sum taxation.
Conclusion

- Chaterjee and Turnovsky (2012) using flat taxes, increases in public services are accompanied by worsened income distribution. By using progressive taxes to finance government services, this effect can be overcome. Still need to study the parameter space this result holds.
- Capital income tax has the strongest effects on both growth and income inequality.
- It is possible to decrease income and wealth inequality without harming GDP growth by setting a balanced progressivity level. This is accompanied by a high increase in the size of government capital.
- Possible extensions:
  - Heterogenizing the wage rates, i.e. by adding skill heterogeneity or human capital.
  - Dividing government expenditure into its components and heterogenizing in terms of regions or business groups.
  - Adding a political economy framework with disproportionately influential wealthy groups.
Growth and Income Inequality in an Endogenous Growth Model with Public Capital under Progressive Taxation

Murat Koyuncu  Alper Ünsal

Bogazici University

CEE Annual Conference - May 17, 2016