DONOR-ORIENTED FOREIGN AID, TRADE and GROWTH

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Abstract

Departing from explanations that involve either altruistic or political motivations of donors, we link a donor’s willingness to give aid to economic incentives and returns. Since targeted aid might increase the returns from trade and FDI linkages for donors, there exists a non-altruistic basis for helping poor countries. We explore this idea by developing a two-country dynamic Stackelberg interaction model of foreign aid and trade. We endogenize aid-tying process by incorporating it into the donor’s optimization problem. We analyze the dynamics of aid composition and show how different components of foreign aid, such as education, investment, commodity and general budget support are optimally supplied over time. We report on a variety of macroeconomic outcomes such as savings, tax, infrastructure investments and debt accumulation rates under such a scenario. We show even under donor-ideal conditions aid might fail to generate growth for the recipient, whereas donor optimally provides aid at all times.

JEL classification: F35, O11, P16, C61, F12, O41

Keywords: Official Development Assistance, Composite Aid, Trade, Economic Growth, Donor-oriented Foreign Aid, Debt Relief, Repeated Stage Game

Accompanying computer code can be found online here: http://www.econ.boun.edu.tr/hatipoglu/Aiditor.m.zip
Full appendix can be found online here: http://www.econ.boun.edu.tr/hatipoglu/aidappendix.pdf

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1 Introduction

Foreign aid has several indirect benefits for the donor country when recipient and donor are linked by trade and direct investment\(^1\). For example, by increasing the return on foreign investment, education aid might increase the income growth rate of the recipient country, which in turn can build up its capacity to repay debt and increase its demand for donor country’s goods and services. Alternatively, infrastructure aid when coupled with direct investment by the donor country, might increase the speed of specialization in the recipient country augmenting the benefits of trade conducive to growth for both countries. In both scenarios, it might be beneficial for the donor to provide aid geared towards increasing overall productivity and returns from direct investment. An altruistic motive for giving aid in such a scenario might not be necessary.

In this paper, we take into account trade and direct investment links between a donor and a recipient and develop a two-country strategic and dynamic interaction model to highlight economic returns of foreign aid both for the recipient and the donor. Our model has two novelties: It considers the optimal allocations of both the recipient and the donor country agents when they are connected through trade and direct investment, and it derives the optimal dynamic composition of foreign aid as an outcome of a strategic interaction rather than its amount in ad hoc fashion.

The economic benefits of a foreign aid interaction for the donor are not fully explored by previous theoretical work, which mostly motivates the existence of foreign aid either by ascribing altruistic or political preferences to the donor\(^2\). While altruism is one of the rational approaches in justifying aid as a means for improving donor’s welfare, it leaves questions on how the amount and the composition of aid are determined by the donor, what portion of aid should be tied by the donor and especially why the donor should give tied aid in the first place, not fully answered\(^3\). Pursuing the donor-oriented approach might help us to understand, firstly, how and why the composition of aid has evolved over the span of the last century the way it did. Secondly, it might shed light on the effect of aid on intermediate outcomes such as recipients’ investment spending in social infrastructure or its tax policy in addition to the effect on final outcome, which is mostly taken to be recipient’s growth rate. A newly emerging literature has been emphasizing the role of aid not only in determining the growth rate but rather in its effect on factors conducive to growth. This shift of focus from the final outcomes of the aid process

\(^1\)A number of studies document foreign direct investment and trade links between donor and recipients, see e.g. Temple and de Sijpe (2014), Bandyopadhyay et al. (2013), Selaya and Sunesen (2012), Nowak-Lehmann D. et al. (2009) and Younas (2008)

\(^2\)Studies on foreign aid can be classified according to the emphasis put on each end of the transfer. Roughly, these studies can be grouped under three categories: Recipient-oriented donor-oriented and interaction-oriented.

\(^3\)As a technical consequence, the deep parameters of these general equilibrium models are more difficult to forecast.
into intermediate goals is taken to forefront by Channing Arndt (2015) and Bazzi and Clemens (2013). This study contributes to this debate by taking into account endogenous effects of aid on productivity of intermediate goods sector and human capital formation.

There are two types of aid that fall outside the scope of traditional foreign aid and growth models but have become important during the last decade. The first one is direct investment, given in kind and undertaken and paid for by a donor. China’s aid to African countries is such a case (Zhang (2017)). The second one is aid, given in a harmonized fiscal context towards a political and developmental goal such as aid given towards European Union accession and harmonization. In those cases a political threat to cut-off aid is implied, and therefore whether aid is beneficial or not is not necessarily dependent on recipient’s ability to use aid as intended, but rather on additional growth mechanisms that arise when direct investment and trade link donor and recipient. Our model aims to explain these additional cases.

A prominent feature of the model is that it enables us to break down foreign aid into its components and solve for its dynamically optimal composition from donor’s point of view. How to compose aid disbursements over a time span is one of the most contentious areas in the optimal provision of aid. Disregarding heterogeneity of components implies that each aid type contributes equally to growth and welfare. Such an omission may cause a bias in measuring aid’s impact on the recipient economy growth rate by suppressing the magnitudes of estimated coefficients (Easterly, 2001). Bazzi and Clemens (2013) suggest basing growth regressions in ‘generalized theoretical models’ as one of the key steps in avoiding pitfalls especially prominent in aid to growth regressions. Our model allows us to draw conclusions on the effects of a variety of endogenized aid combinations on both the donor and the recipient’s economic performance.

In our model, donor interacts with the recipient strategically through trade, foreign direct investment and aid. All aid components except commodity aid have indirect effects on the intermediate good sectors in both countries. Moreover, since countries run their own tax policies based on aid components among other factors, aggregate outcomes such as welfare and growth are also partially determined by

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4Despite the voluminous empirical literature on the subject, there has been no broad consensus on foreign aid’s contribution to recipient countries’ economic growth and long run welfare. Finding the right instruments that satisfy the exclusion restriction in growth regressions has been elusive (Brückner, 2013). Dreher and Langlotz (2015), Rajan and Subramanian (2008) and Boone (1996) find no impact of aid on growth whereas Hansen and Tarp (2001) maintains that aid increases the growth rate. A positive impact has also been found recently by Channing Arndt (2015) and earlier by Burnside and Dollar (2000), Lensink and White (2001) and Economides et al. (2008). Dalgaard and Hansen (2001) and Easterly et al. (2004) condition this positive correlation on good policy. See Hansen and Tarp (2001) for a summary of the earlier results found by empirical literature and Easterly (2009) for a discussion. See S. Galiani (2016) and Channing Arndt (2015) for an overview of the recent literature. Some of the discrepancy in these findings can also be attributed to the variety of empirical specifications employed in these studies, but they might also be due to a “lack of clear guidance by theory” (Easterly, 2003).
aid policies. We set up the interaction between the donor and the recipient governments as a repeated stage game, where the donor acts as a Stackelberg leader in choosing the optimal policy response given the recipients strategy space. Specifically, donor determines the amount and the composition of aid and domestic tax policy optimally by taking the best response decisions of the recipient government as well as its own domestic households and firms as given. Initially, the recipient government has no means of committing to any action other than that of a Stackelberg follower. Once it observes donor’s actions, it runs domestic fiscal policy freely and decides on how much to invest in infrastructure \(^5\).

Our results indicate that aid is supplied in positive amounts at all times even when donor’s motives are completely non-altruistic. Donor-optimal composite aid helps the recipient to bridge the labor productivity gap between itself and the donor, and improvements in labor productivity increases returns to foreign direct investment in intermediate goods sector\(^6\). Aid also increases global demand for goods from the more productive sector in the donor country and thereby leads to a more efficient allocation of resources in the donor country. Thus, it acts as an indirect subsidy to donor country’s intermediate goods sector. By channeling donor resources to where the productivity gains are higher a composite aid creates a positive externality for donor intermediate goods producers and on investment returns from low productivity goods in the recipient country. We find infrastructure aid is supplied in relatively higher amounts over time whereas debt relief is insignificant when compared to other forms of aid such as education and manufactured commodity aid. Specifically, a donor-preferred aid package consists of a high level of infrastructure aid coupled with manufactured commodity aid, education aid, debt relief and budget aid which are respectively decreasing in significance. We find no role for basic commodity aid under our scenario.

We further analyse the effects of a foreign aid composite on a variety of macroeconomic outcomes such as growth rates, tax rates, debt to output ratio, capital to output ratio and debt-financing. We compare our results both to autarky and to a situation where there are trade and investment links but no aid transfers. With the introduction of a composite aid transfer, we find increased growth rates are possible but not necessarily sustainable. On the other hand, welfare is improved for both the donor and the recipient. We find optimal tax rates are decreasing for the recipient. However, we also find that the recipient savings rates are lower, which derives our result that growth rates are not sustainable under certain scenarios. While the share of productive government spending such as infrastructure investment is lower with the simultaneous introduction of foreign aid, trade and foreign

\(^5\) We use the term infrastructure interchangeably with public capital

\(^6\) Selaya and Sunesen (2012) estimate the relationship between aid and foreign direct investment and argue that the composition of aid is important for its efficiency. Aid raises the marginal productivity of capital when it is used to finance complementary inputs such as public infrastructure and human capital
direct investment, its rate decreases over time. In contrast, donor’s infrastructure investment level and its rate are higher when compared to autarky. We further report on the optimal time path of a variety of aid components. In terms of optimal compositional aid dynamics, we find infrastructure aid significantly increases over time but commodity and education aid decrease in transition. On the other hand, budget aid and debt relief slightly increase, although they remain at low levels.

This paper is organized as follows. In the next section, we motivate our paper with a brief overview of the relevant literature. In section 3, we introduce how different components of aid have behaved over the last century. We present our main assumptions, describe the nature and timing of the strategic interaction among agents and present a summary of our solution methodology in section 4. In section 5, we provide simulation results of our model and discuss findings. Section 6 concludes.

2 Relation to the literature

To our best knowledge, this study is the first attempt to incorporate trade and direct investment links between a donor and a recipient in a strategic framework. It is also the first attempt in which a non-altruistic donor dynamically optimizes the composition of aid to maximize its own welfare.

In an early attempt to model the effect that donor preferences have on the allocation of aid, Dudley and Montmarquette (1976) interpret the impact of aid as a public good consumed by the donor country residents. In their study, the optimal amount of aid is determined by the first order condition which equates the marginal utilities of the consumable good to that of impact of foreign aid. Dudley and Montmarquette further argue that higher levels of economic interdependence between the two parties increase efficacy of aid. Later studies have lent empirical evidence to this argument. Our paper is also loosely related to Younas (2008) which extends Dudley and Montmarquette (1976) to a multilateral setup and argues that more aid is allocated to those countries who import goods from the donor country in which the donor country has a comparative advantage. Further support for aid and trade links is provided by Martínez-Zarzoso et al. (2014) which finds that infrastructure aid promotes export growth and Nowak et al (2009), who document that in the long run aid causes exports from donors to recipient countries. By performing a sectorial decomposition analysis Selaya and Thieleb (2010) find aid has positive growth effects in both the tradable and non-tradable sectors. Temple and de Sijpe (2014) find that aid raises net imports and total consumption of the recipient as a share of GDP.

While treating donor or recipient behavior as exogenous render valuable insights to the behavior of the other party, the donors and recipients perhaps strategically interact in their allocation decisions.

7See (Martínez-Zarzoso et al., 2014; Faye and Niehaus, 2012; Selaya and Sunesen, 2012; Nowak-Lehmann D. et al., 2009; Alesina and Weder, 2002; Alesina and Dollar, 2000)
An important strand within this group of models have focused on the effect institutional environments of the participants have on their policy decisions (Svensson, 2000; Azam and Laffont, 2003). For example, Bandyopadhyay et al. (2013) examine the effect of relaxing a binding borrowing constraint for a recipient country on the amount of foreign aid, in a two-country, two-period, trade-theoretic framework with an altruistic donor. The relaxation unambiguously reduces the flow of foreign aid. Scholl (2009) analyzes optimal aid policy in a dynamic contract framework in which there are incentive compatibility problems, that arise from this conflict. Such a contract can be written on conditions that can be sustained with a threat of permanent aid cutoff at any point in time. In an overlapping generations framework, Dalgaard (2008) employs an endogenous aid allocation rule that depends on the recipient’s economic performance as well as the donors’ political interests and their aversion to risk. Different donor policies result in different “trajectories for GDP growth” in the recipient countries. Similar to the above literature, we let donors’ policy choices to influence aid outcomes, but endogenize policy responses for both parties.

Our paper differs from the aforementioned literature in several more ways. First we let the donor act similar to a Stackelberg leader, in that at each stage it makes its decision after observing the optimal allocation choices of the recipient. Secondly, we decompose aid into several dynamic components each of which has a different role in shaping the economic exchange between the donor and the recipient and determining equilibrium outcomes. By taking the general equilibrium approach our aim is to tackle the causation problem inherent in empirical aid studies.

Other donor-oriented studies focus on how donors design aid schemes. Chatterjee and Turnovsky (2007), for instance, analyze the impact of transfers tied to investments in public infrastructure. They find that tied transfers has a higher impact than pure transfers but the nature of the impact depends on the initial stock of public capital. Chatterjee and Turnovsky (2005), deals with an international transfer problem where the degree to which transfers made by an altruistic donor are tied to infrastructure investments and borrowing costs are determinants of the long-run effectiveness of aid.

3 Empirical relevance

To motivate our approach we provide a look at the OECD classification, which distinguishes between eleven types of aid. Figure 1 shows the change of sectors' shares in total aid flows using OECD defi-
There have been substantial changes in the composition of total aid since 1960’s. Specifically, production aid and commodity aid have decreased after 1980’s whereas total infrastructure aid, which consists of both social and economic infrastructure aid, has relatively increased since 1960’s and holds the highest share in total aid as of 2017. The figure indicates that taking aid as a homogenous input, from which growth can be generated, might be misleading, and why the composition of aid has evolved the way it did during the last century requires an explanation.

4 Analytical framework

4.1 Main assumptions, a summary of the model and description of the strategic interaction

Assume there are two countries: the donor and the recipient, denoted by superscripts $D$ and $R$, respectively. The donor country is characterized by its developed economy and high-skilled labor force relating to debt, Humanitarian Aid, Administrative Costs of Donors, support to NGOs, aid to refugees in donor countries and unallocated or unspecified.
whereas the recipient country is characterized by its developing small economy and low-skilled labor force. In each country, there are three types of agents: firms, households and the government. Production by firms takes place in two sectors: a final goods sector and an intermediate goods sector. The donor and recipient countries are connected via trade in final goods, via direct investment in intermediate goods sector and via various forms of aid flows from donor government to recipient government and households. We assume there are two goods produced exclusively in each country. For quick reference, we arbitrarily name the good produced in the donor country as the “manufactured good” and the one produced in the recipient country as the “basic commodity.” To construct a foreign direct investment link between the two countries, we assume that the aggregate capital in each country is the sum of domestic and foreign capital supplied by the households.

In both countries, household dynasties live forever, earn wage and rental income, pay taxes and consume a bundle of basic commodity and manufactured goods. In addition, the recipient country households receive aid in form of manufactured goods and basic commodities from the donor country government. Both governments collect taxes, invest in infrastructure and decide on debt policy. The recipient country receives general budget support and debt relief from donor country government. The donor country government gives donations in form of manufactured goods, basic commodities, infrastructure and education aid, debt relief and general budget support. We assume that there are no informational asymmetries among agents. The governments are fully informed of the firms’ and households’ strategies and determine their optimal allocation strategies in light of these best response functions. In each period, governments take action against each other in a turn-based fashion following a "within-period" sequential game. We will describe the nature and timing of the interaction later.

We solve the allocation problem in each period by backward induction. Once the firms’ and households’ problems are introduced, the competitive equilibrium is derived as a set of best response functions whose arguments reflect the governments’ decision vector and competitive prices.

4.2 Composition of aid

We assume in each period \( t \) the donor country transfers the total amount \( A_t \) to the recipient country. This amount, is distributed to different sectors of the recipient economy. The following equation holds in each period:

\[
A_t = a_{It} + a_{Gt} + a_{Dt} + L_t^R (a_{Et} + a_{ct} + a_{mt})
\]

(1)

Where \( A_t \) is the total amount of aid given, \( a_{It} \) is the aid invested in economic infrastructure, \( a_{Gt} \)
is the general budget support, \( a_{Dt} \) is debt relief, \( L^R_t \) is the population of the recipient country, \( a_{Et} \) is per capita aid spent on education, \( a_{ct} \) is per capita basic commodity aid and \( a_{mt} \) is per capita manufactured good aid.

4.3 Production

In each country, the final goods (FG) sector, operating under perfect competition, produces consumables and the intermediate goods (IG) sector, operating under perfect monopoly, produces a variety of capital inputs for final goods production.

4.3.1 Final good firms

We assume there are two types of final good firms, each producing either a manufactured good or a basic commodity exclusively. Manufactured good producing firms are located only in the donor country whereas the basic commodity producing firms are located in the recipient country.

Both the manufactured good, \( m \), and the basic commodity, \( c \) are produced by using labor, intermediate goods and available economic infrastructure. Final good technologies in the recipient, \( R \), and the donor, \( D \), are respectively given as

\[
Y^R_t = \left( I^R_t + a_{It} \right)^{\varphi^R} \left( Z^R_t L^R_t \right)^{1-\alpha^R} \sum_{j \in J^R} (q_{jt})^{1-\alpha^R} \left( X^R_{jt} \right)^{\alpha^R}
\]

\[
Y^D_t = \left( I^D_t \right)^{\varphi^D} \left( Z^D_t L^D_t \right)^{1-\alpha^D} \sum_{j \in J^D} (q_{jt})^{1-\alpha^D} \left( X^D_{jt} \right)^{\alpha^D}
\]

Here \( Y_t \) is the level of final, output \( I_t \) denotes the level of infrastructure expenditures of the government, \( Z_t \) denotes the labor productivity, \( L_t \) denotes the size of the labor force, \( X_t \) denotes the amount of capital, \( q_j \) is the quality of intermediate good and the set of intermediate capital good qualities are given by \( J^D \) and \( J^R \). The infrastructure in the recipient country is financed partly by the government and partly by foreign aid. This specification allows us to identify foreign direct investment by firm origin and determine net factor-income flows between countries. Recipient and donor firms solve the following problems.

\[
\max_{\{L^R_t, X^R_t\}} p^R_{Y^R_t} - w^R_t L^R_t - p^R_t X^R_t
\]

\[
\max_{\{L^D_t, X^D_t\}} p^D_{Y^D_t} - w^D_t L^D_t - p^D_t X^D_t
\]
Prices \( w_t^R, w_t^D, p_t^R \) and \( p_t^D \) are found by taking the usual first order conditions. A part of the total production, \( \alpha \), is paid to the producers of intermediate capital goods. The remaining \( 1 - \alpha \) is paid to the labor. Labor is supplied inelastically and evolves exogenously characterized by the equations (6) and (7).

\[
L_{t+1}^R = (1 + n^R)L_t^R \tag{6}
\]

\[
L_{t+1}^D = (1 + n^D)L_t^D \tag{7}
\]

where \( n^R \) and \( n^D \) denote the population growth rate in recipient and donor countries, respectively.

### 4.3.2 Intermediate goods producers

There are \( J \) number of intermediate good (IG) producing firms which use capital to produce output. These firms operate on an international scale, but are each owned completely by the consumers of either country. Specifically, if \( j \in J^R \) the firm is owned by the consumers in the recipient country and \( j \in J^D \) the firm is owned by the consumers in the donor country. There are no other possibilities, therefore \( J^R \cup J^D = J \). All profits flow to the owners. The maximization problems of IG producing firms are, thus, given as:

\[
\text{if } j \in J^R : \max_{X_{jt}^D, X_{jt}^R} \left( p_{jt}^D X_{jt}^D - r_t^R f_{jt}^R \right) + \left( p_{jt}^R X_{jt}^R - r_t^R k_{jt}^R \right) \tag{8}
\]

\[
\text{if } j \in J^D : \max_{X_{jt}^D, X_{jt}^R} \left( p_{jt}^D X_{jt}^D - r_t^D f_{jt}^D \right) + \left( p_{jt}^R X_{jt}^R - r_t^D k_{jt}^D \right) \tag{9}
\]

where \( X_{jt}^i \) is amount of IG supplied to country \( i \) by firm \( j \), \( f_{jt}^i \) is the capital amount transferred to country \( i \) by firm \( j \) and \( k_{jt}^i \) is the capital amount left in country \( i \) by firm \( j \) for \( i = R, D \). In this setup, production of IG can be carried out in each country by using capital borrowed from home country. Substituting \( p_{jt} \) obtained by solving the final good production problem, the level of IG production can be found as

\[
X_{jt}^R = \left( \frac{\left( \alpha^R \right)^2 p_{jt}^R (I_t^R + a_t^R) e_t^R}{r_t^R} \right)^{\frac{1}{1-\alpha^R}} \left( Z_t^R L_t^R \right) (q_{jt}) \tag{10}
\]

\[
X_{jt}^D = \left( \frac{\left( \alpha^D \right)^2 p_{jt}^D (I_t^D + a_t^D) e_t^D}{r_t^D} \right)^{\frac{1}{1-\alpha^D}} \left( Z_t^D L_t^D \right) (q_{jt}) \tag{11}
\]
now we can state

**Proposition 1.** Foreign Aid indirectly increases income and welfare for the consumers in the donor country.

**Proof** See appendix

This result follows from the fact that intermediate goods production occur in both countries and an increase in infrastructure aid increases the income of the recipient and results in a higher revenue stream for donor IG producers in both countries through both increased demand for their exports and through increased productive capacity in the recipient country.

### 4.4 Households

There are infinitely-lived household dynasties in both countries which maximize the sum of their discounted time-separable utility functions subject to their budget constraints, given competitive price- and policy-vectors. Let $\beta$ denote the discount factor, $m$ and $c$ denote the amount of manufactured good and basic commodity consumed. Furthermore, let $i = R, D$ denote the households country of residence, then $x_{itD}$ and $x_{itR}$ denote the amount of per capita capital rented to the production sector in the donor and recipient country by country $i$. The problem of the recipient country households is defined by

$$\max_{\{m_{it}^R, c_{it}^R, x_{it+1}^R, \tilde{\omega}_{it}\}} \sum_{t=0}^{\infty} \left( \beta^t \right) \left( m_{it}^R \right)^{\sigma^R} \left( c_{it}^R \right)^{1-\sigma^R}$$

subject to the budget constraint

$$\frac{p_{it}^m}{p_{it}^c} (m_{it}^R - a_{it}) + c_{it}^R - a_{ct} = \left[ \frac{\tilde{\omega}_{it}^R}{P_t} + (1 - \delta) \left( \sum_{j \in J^{RR}} x_{jt}^R + \frac{p_{it}^m}{P_t} \sum_{j \in J^{RD}} x_{jt}^D \right) - (1 + n^{R}) \left( \sum_{j \in J^{RR}} x_{jt+1}^R + \frac{p_{it}^c}{P_t} \sum_{j \in J^{RD}} x_{jt+1}^D \right) - \tau^R \right]$$

where $j \in (J^{RR} \cup J^{RD})$ and $x_{jt}^R = \frac{X_{jt}^R}{P_{it}}$ for $j \in J^{RR}$, $x_{jt}^R = \frac{X_{jt}^R}{P_{it}}$ for $j \in J^{RD}$

Above, $\tilde{\omega}$ denotes the per capita income (sum of labor and capital income) and $\delta$ denotes the depreciation rate. Furthermore, $n$ denotes the population growth rate, $\tau$ denotes the per capita lump sum taxes.
Additionally there are forced choice constraints to guarantee the commodity aid supplied is at most the amount demanded for each consumable.

$$c_t^R \geq a_{ct}$$  \hspace{1cm} (14)

$$m_t^R \geq a_{mt}$$  \hspace{1cm} (15)

The problem of the donor country households is defined as:

$$\max \{m_t^D, c_t^D, x_{t+1}^D, x_{t+1}^D \} \sum_{t=0}^{\infty} (\beta^D)^t (m_t^D)^{1-a^D} (c_t^D)^{a^D}$$  \hspace{1cm} (16)

subject to the budget constraint

$$m_t^D + \frac{p_t^m}{p_t} c_t^D = \left[ \frac{z_t^R}{p_t} + (1 - \delta) \left( \sum_{j \in JD} x_{jt}^D + \frac{p_t^m}{p_t} \sum_{j \in JDR} x_{jt}^R \right) - (1 + n^D) \left( \sum_{j \in JDD} x_{jt+1}^D + \frac{p_t^m}{p_t} \sum_{j \in JDR} x_{jt+1}^R \right) - \tau_t^D \right]$$  \hspace{1cm} (17)

Proposition 2 follows.

**Proposition 2.** Foreign Aid indirectly increases demand for tradable consumables in the recipient country.

*Proof* See Appendix

### 4.5 Labor productivity

We assume education aid contributes towards closing the labor productivity gap between the two countries. The evolution of labor productivity levels is determined by the following equations.

$$Z_t^R = (1 + z^R) Z_{t-1}^R + \left( \frac{Z_t^D - Z_{t-1}^R}{Z_{t-1}^R} \right)^{\chi} (1 - e^{-\gamma a_{Et}})$$  \hspace{1cm} (18)

$$Z_t^D = (1 + z^D) Z_{t-1}^D$$  \hspace{1cm} (19)

The term $$\left( \frac{Z_{t-1}^D - Z_{t-1}^R}{Z_{t-1}^R} \right)^{\chi}$$ captures the natural catch-up effect between the productivity levels due
to diminishing returns on capital, but the speed of convergence depends also on the education aid through the term $1 - e^{-\gamma a_{Et}}$. Here $\gamma$ represents the effectiveness of education aid and $\chi$ determines the speed of convergence. When $\gamma = 0$ the labor productivity in the recipient country evolves exogenously. The right hand side of the plus sign in equation (18) goes to zero as the productivity gap closes or education aid goes to zero. In the benchmark case, we let $\gamma = 0$ and compare the results to the model with aid which we obtain by calibrating both $\chi$ and $\gamma$.

### 4.6 Governments

#### 4.6.1 Payoff functions

The recipient country government has three revenue sources: taxes collected form its own citizens, borrowings and general budget support from the donor country government. It can invest these funds as infrastructure or pay some of its debt. It maximizes the welfare of its citizens by determining the per capita lump-sum taxes, debt and contribution made to the infrastructure subject to the budget constraint (21).

$$\max \left\{ I^{R}_t, D^{R}_{t+1}, \tau^{R}_t \right\}_{t=0}^{\infty} \sum_{t=0}^{\infty} \left( \beta^{R} \right)^t u^{R} \left( \hat{c}^{R}_t, \hat{m}^{R}_t \right)$$  \hspace{1cm} (20)

subject to

$$I^{R}_t + (1 + i_t) \left( D^{R}_t - a_{Dt} \right) = L^{R}_t \tau^{R}_t + D^{R}_{t+1} + a_{Gt} + a_{Dt}$$  \hspace{1cm} (21)

The donor country government has two revenue sources: taxes collected from its own citizens and debt repayments received from the poor country government. It can invest these funds as infrastructure or lend some money or supply foreign aid to the recipient country government.

$$\max \left\{ I^{D}, a_{ct}, a_{Du}, a_{Et}, a_{Gt}, a_{mt}, \tau^{D}_t \right\}_{t=0}^{\infty} \sum_{t=0}^{\infty} \left( \beta^{D} \right)^t u^{D} \left( \hat{c}^{D}_t, \hat{m}^{D}_t \right)$$  \hspace{1cm} (22)

subject to

$$I^{D}_t + D^{R}_{t+1} + L^{R}_t a_{Et} + a_{It} + a_{Gt} + L^{R}_t a_{ct} + L^{R}_t a_{mt} = L^{D}_t \tau^{D}_t + (1 + i_t) \left( D^{R}_t - a_{Dt} \right)$$  \hspace{1cm} (23)

#### 4.7 Competitive equilibrium

For the above two-economy setup we define a world competitive equilibrium which is described in the Appendix.
4.7.1 Timing of events and actions

Each period consists of two stages. At the beginning of each period, firms and households in each country maximize their objective functions given their governments’ policy parameters. At the second stage, the recipient country’s government solves a Ramsey problem by choosing infrastructure investment, debt and tax level to maximize social welfare of its own citizens, taking competitive equilibrium at home and the foreign government’s action parameters as given. At the final stage, the donor country’s government acts as the leader in a sequential game versus the recipient country’s government. Specifically, it solves a Ramsey problem by determining infrastructure investments, tax levels and the amount and composition of foreign aid to maximize social welfare of its own citizens, given the competitive equilibrium at home and foreign government’s decision rules that are based on its own actions.

Governments are fully informed about the firms’ and households’ best response functions that determine the competitive equilibrium and determine their policies with the strategic objective of maximizing the welfare of their own citizens given others’ actions. Specifically, they play a repeated game, where the donor government acts as a leader and the recipient government acts as a follower. Each period in the game consists of three stages. At the beginning of each period, donor households and firms maximize their objective functions as described in sections 4.3-4.4. At the second stage both governments take the set of competitive equilibrium decision rules by firms and households in each country as given and individually solve Ramsey problems. In addition, the recipient government takes the composition of aid determined by the donor government also given. At the second stage, the donor government acts as the leader in a sequential game versus the recipient government. Specifi-
cally, it solves a Ramsey problem by determining tax levels, $\{\tau^D_t\}_{t=0}^{\infty}$, debt, $\{D^D_{t+1}\}_{t=0}^{\infty}$, infrastructure investments, $\{f^D_t\}_{t=0}^{\infty}$, and the amount and composition of foreign aid, $\{a_{ct}, a_{Dt}, a_{Et}, a_{It}, a_{Gt}, a_{mt}\}_{t=0}^{\infty}$, to maximize social welfare of its own citizens, given the competitive equilibrium at home and foreign government’s decision rules that are based on its own contingent actions. At the last stage the recipient government chooses per capita lump-sum taxes, $\{\tau^R_t\}_{t=0}^{\infty}$, debt, $\{D^R_{t+1}\}_{t=0}^{\infty}$, and infrastructure investments, $\{f^R_t\}_{t=0}^{\infty}$ to maximize welfare. In our setup, there are some further constraints to sustain a classical Stackelberg equilibrium. We assume donor must knows ex-ante that the recipient observes his action. The recipient has no means of committing to a future non-Stackelberg follower action and the donor has full information on this.

5 Numerical Analysis

5.1 Data and parameters

For this study we utilize OECD (2016) official development assistance (ODA) and trade data. The set consists of 28 development assistance committee (DAC) member countries and 19 non-DAC members that act as donors as well as 212 recipient countries for the years 1972-2016. It includes total amount of aid defined as official development assistance excluding military transfers as well as categories of aid based on OECD classification.

Observed import shares act also as preference parameters due to the assumption that donor and recipient specialize in a specific type of good. Import shares are found by calculating the fraction of total import flows to countries that receive greater than 3% of their GDP as aid. This exercise pins down the import shares for the recipient at 0.3 and for the donor at 0.7.

For the labor share in the recipient country we follow Gourinchas and Jeanne (2008) and use $\alpha = 0.3$. We set the discount rate at 0.95 for both the recipient and the donor, but later relax it for the recipient for sensitivity analysis. The population growth rates are obtained from United Nations Population Division (2017) using World Bank and productivity growth rates are taken from Van Biesebroeck (2005). Using data on 48 OECD and Non-OECD countries for the period 1960-2011 and by doing robustness check on time intervals and varying depreciation rates Arslanalp et al. (2010) estimate the elasticity of public capital for OECD and Non-OECD countries as 0.042 and 0.021, respectively. We adopt both figures in our estimation. We later run a sensitivity analysis for the externality of public capital in the recipient country by increasing its elasticity to 0.30. We first calibrate the parameters of the system under autarky and without aid. The elasticity parameter of intermediate inputs in the donor country to output is chosen so that the benchmark economy reproduces the intermediate input
Table 1: The benchmark economy

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Import Shares:</td>
<td>$\sigma^R = 0.3, \sigma^D = 0.7$</td>
</tr>
<tr>
<td>Production parameters:</td>
<td>$\varphi^R = 0.021, \varphi^D = 0.042, \alpha^R = 0.3, \alpha^D = 0.3$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = 0.0635$</td>
</tr>
<tr>
<td>Discount rates</td>
<td>$\beta^R = 0.95, \beta^D = 0.95$</td>
</tr>
<tr>
<td>Population growth</td>
<td>$n^R = 0.02, n^D = 0.01$</td>
</tr>
<tr>
<td>Labor productivity growth rate</td>
<td>$z^R = 0.01, z^D = 0.01$</td>
</tr>
<tr>
<td>Knowledge transfer parameter</td>
<td>$\gamma = 0.049$</td>
</tr>
<tr>
<td>Speed of convergence</td>
<td>$\chi = 10.1$</td>
</tr>
</tbody>
</table>

to output ratio in the OECD set of recipients and donors.

There are two unknown parameters, $\gamma$ and $\chi$ in our model. To obtain $\gamma$, we first collect the sample of countries who received no education aid, and using growth accounting we estimate the evolution of labor productivity to produce the series $Z^R_t$. Next we estimate a log-linearized version of (18) for the set of countries who received education aid during the same period. This gives us the intercept, $\mu$, as $\ln((1 - e^{-\gamma a_E})$ where $a_E$ is the average education aid received. Simple algebra yields $\gamma = \frac{\ln(1 - e^{-\mu})}{a_E}$. Finally $\chi$ can be found by calibrating the model to this value.

The rest of the parameters are standard for both developed and developing countries and also used by Scholl(2009), Gourinchas and Jeanne (2008) and Gollin(2002) among others. Table 1 contains the parameters used to calibrate the benchmark economy as well as calibrated parameters.

5.2 Algorithm

Since further solutions cannot be carried out analytically, we resort to numerical methods to illustrate the results obtained from the model. Our two-tiered simulation algorithm is as follows:

i) **Employ a grid-search method to locate the steady states.**

ii) **Compute transition paths using decision rules of agents.**

iii) **Use backwards induction based on Figure 2 starting from the last step to solve for the optimal reaction functions of each government numerically at previous stage, alternating between donor and the recipient .**

iv) **Restart at i) until convergence is achieved.**

---

10A supplement describing the full set of steady state equilibria, the solution of the model as well as the accompanying Matlab code are available from authors
Table 2: Steady State Properties

<table>
<thead>
<tr>
<th>Catch-up parameter</th>
<th>$\gamma = 0$</th>
<th>$\gamma = 0.049$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>$\bar{g}_R = 0.0221, \bar{g}_D = 0.0161$</td>
<td>$\bar{g}_R = 0.0244, \bar{g}_D = 0.0226$</td>
</tr>
<tr>
<td>With Aid</td>
<td>$\bar{g}_R = 0.0155, \bar{g}_D = 0.0232$</td>
<td></td>
</tr>
<tr>
<td>Tax rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>$\bar{t}_R = 23.16%, \bar{t}_D = 15.29%$</td>
<td>$\bar{t}_R = 14.06%, \bar{t}_D = 29.59%$</td>
</tr>
<tr>
<td>With Aid</td>
<td>$\bar{t}_R = 26.66%, \bar{t}_D = 17.32%$</td>
<td>$\bar{t}_R = 14.06%, \bar{t}_D = 29.59%$</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>$(\bar{K}/\bar{Y})_R = 0.29, (\bar{K}/\bar{Y})_D = 0.59$</td>
<td>$\bar{K}_R = 0.39, (\bar{K}/\bar{Y})_D = 0.61$</td>
</tr>
<tr>
<td>With Aid</td>
<td>$(\bar{K}/\bar{Y})_R = 0.41, (\bar{K}/\bar{Y})_D = 0.68$</td>
<td>$\bar{K}_R = 0.39, (\bar{K}/\bar{Y})_D = 0.61$</td>
</tr>
<tr>
<td>Recipient debt-output ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>$(\bar{D}/\bar{Y}) = 0.45$</td>
<td>$\bar{D}_R = 0.45$</td>
</tr>
<tr>
<td>With Aid</td>
<td>$(\bar{D}/\bar{Y}) = 0.69$</td>
<td>$\bar{D}_R = 0.45$</td>
</tr>
<tr>
<td>Aid Components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>$\bar{a}_I = \bar{a}_G = \bar{a}_D = \bar{a}_E = \bar{a}_c = \bar{a}_m = 0$</td>
<td>$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$</td>
</tr>
<tr>
<td>With Aid</td>
<td>$\bar{a}_I = 0.86, \bar{a}_G = 0.04, \bar{a}_D = 0.01$</td>
<td>$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}_E = 0, \bar{a}_m = 0.09, \bar{a}_c = 0$</td>
<td>$\bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0$</td>
</tr>
</tbody>
</table>

Notes: $\gamma = 0$ : all aid flows except education is non-negative and education aid is exactly zero in equilibrium.

v) Use optimal decision rules of agents, who respond to government fiscal and aid policy adjustments at the previous stage, by adjusting consumption and investment at the current stage as well as government reaction functions at the current stage to solve for optimal levels of consumption, investment, capital and fiscal and aid policies.

To understand the effects of aid, we first simulate a benchmark model with trade and investment links but without any catch up effect, i.e. $\gamma = 0$ and compute its steady state values. Next, we simulate the model with positive aid flows and a positive catchup effect as well as with positive aid flows and a zero catch up effect. In this case, $\gamma = 0$, describes effectively the situation when all aid flows except education is non-negative and education aid is exactly zero in equilibrium.
5.3 Simulation results
5.3.1 Growth and welfare comparisons

In Table 2, we report the steady state properties of the model and compare results that are obtained by setting the effect of education aid to zero to results obtained from the calibrated model. The first case ($\gamma = 0$) refers to a situation where individual country labor productivities evolve exogenously, whereas the second case ($\gamma = 0.049$) represents a situation where recipients’ productivity level is positively affected by education aid. In addition, we also report steady state values under two different scenarios. The benchmark scenario refers to the situation when there are trade and investment links but no aid flows. Here, both governments run only fiscal policies and decide on infrastructure investments. The scenario with aid refers to the situation when there are positive aid, investment and trade flows between countries. In this scenario, the donor government decides also on aid composition.

When labor productivities evolve exogenously, we find that foreign aid reduces steady state growth rate of the recipient by 0.66 percentage points, while it increases the donor’s growth rate by 0.007. When there are positive productivity returns to education aid, foreign aid increases recipient’s growth rate by 0.23 and donor’s growth rate by 0.006. In the first case, recipient tax rates increase by 3.50 percentage points and donor tax rates increase by 2.03. With the introduction of education aid, donor tax rates jump by 14.30 percentage points, whereas recipient tax rates drop by 9.10. Steady state physical capital-output ratios are higher for the recipient and the donor in both cases. The recipient’s capital-output ratio drops from 0.41 to 0.39 and the donor’s capital-output ratio drops from 0.68 to 0.61 when we allow for education aid. The steady state debt-output ratio of the recipient mimics its tax rates. With the introduction of budget support and debt relief, the steady state debt-output ratio of the recipient reaches to 0.69. After the introduction of education aid, however, it drops to 0.61. Finally, infrastructure aid constitutes the biggest part of an optimally combined aid package irrespective of whether there are positive returns to education aid. The introduction of education aid leads to simultaneous increases in debt relief and manufactured commodity aid but it decreases infrastructure aid and budget support.

Figure 2 shows growth rate comparisons between the benchmark model and two models with aid along the transition path. It also shows welfare comparisons between the benchmark model and the model with education aid. When the optimal aid package does not contain education aid, recipient growth rate increases temporarily fourfold, but drops below the benchmark level during transition. Inclusion of education aid increases the growth rate even more, by about six times initially, but the

\[11^\text{Note that, in this case education aid is optimally zero.}\]
growth rate steadily drops until just above the benchmark level. For the donor, transitions are markedly different. Whether the aid package includes education aid or not growth rates are permanently higher than the benchmark level. Recipient welfare is improved with aid whereas donor’s welfare first drops but then eventually reaches above its equilibrium path that it would follow without aid.

5.3.2 Savings rates, tax rates, infrastructure investments and debt accumulation

Figure 3 shows the evolution of savings, tax and investment rates as well as debt to output ratio both at the benchmark steady state and after the introduction of aid flows in both countries. We find that, in transition, recipient households’ savings rate first increases and then drops below the benchmark level. Savings rate in the donor country first drops significantly but recovers to 28%, roughly 4% below its benchmark level. Recipient country’s equilibrium tax rate initially jumps by 1.2% with aid flows but eventually drops 9% below the benchmark level whereas donor’s tax rate initially jumps by 3.2% and continues to increase towards steady state level of 29.59%.

Since savings rates are related to consumption smoothing behavior in both countries, they respond to both movements in tax rates as well as movements in labor productivity that are in turn partly induced by infrastructure and education aid flows. Once the aid flows start, both countries increase tax rates which cause the recipient households to save significantly less but the effect is dampened over time due to benefits from increased aid, trade and investment flows. The donor households, on the other hand, increase their savings rate that, however, also drops below its benchmark level toward steady state.

In our setup, how the infrastructure investment and the debt of the recipient country is financed by each party can be determined optimally by the governments of two countries. In figure 3, we report the infrastructure investment rates, defined as the total infrastructure expenditures as a share of total output in the recipient country and the percentage contribution of each government to finance the debt of the recipient country. The investment expenditures by the donor country in the recipient country increases to 22.0% of the recipient output once we introduce aid flows. Recipient investment rates almost double with the introduction of aid but they eventually drop to 22.4% the steady state. This is hardly surprising, as infrastructure aid turns out to be the biggest component of the optimal aid package, a result we introduce in the next section. Once aid flows start the debt-output ratio of the recipient country increases steadily to 0.61. While this ratio is below the level that is attained when there is no education aid it is still significantly higher than the benchmark level. The recipients contribution drops to 70.1% after aid flows and further drops to 66.1% whereas the donor finances 33.9% of the recipient debt at the steady state.
Figure 3: Growth and welfare in certainty equivalence consumption units
Figure 4: Savings, taxes and financing
5.3.3 Composition of aid and dynamics of aid components

One of the main arguments of this paper is that the optimal aid shares are not constant and evolve over time. As we presented in Figure 1, actual evidence points to an increasing infrastructure aid, relatively low levels of commodity aid and budget aid including debt relief. Figure 4 shows the transition of aid shares toward steady state. We find that all types of aid except basic commodity aid are supplied optimally in positive amounts at all times. Broadly consistent with the actual evidence we also find that infrastructure aid makes up the bulk of total aid. Manufactured commodity aid has a larger share than all other types combined except infrastructure and education aid. Inclusion of education aid increases optimal levels of infrastructure aid because it increases returns to infrastructure investments and leads to efficiency gains. The aid process generates a convergence in labor productivity levels, which eventually increases the overall productivity and increases returns to infrastructure investments. We find little role for budget aid and debt relief.

5.3.4 Sensitivity analysis

Easterly (2002) points out that higher discount rate for the recipient can account for political economy features that cause overspending. We run a sensitivity analysis where the recipient discounts the future...
Table 3: Sensitivity of aid effectiveness: Discount rates

<table>
<thead>
<tr>
<th>Discount Rates</th>
<th>( \beta_R = 0.95, \beta_D = 0.95 )</th>
<th>( \beta_R = 0.90, \beta_D = 0.95 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rates</td>
<td>( \bar{g}_R = 0.0244, \bar{g}_D = 0.0226 )</td>
<td>( \bar{g}_R = 0.0192, \bar{g}_D = 0.0211 )</td>
</tr>
<tr>
<td>Tax rates</td>
<td>( \bar{t}_R = 14.06, \bar{t}_D = 29.59 )</td>
<td>( \bar{t}_R = 15.69, \bar{t}_D = 29.06 )</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>( (\bar{K}/\bar{Y})_R = 0.39, (\bar{K}/\bar{Y})_D = 0.61 )</td>
<td>( (\bar{K}/\bar{Y})_R = 0.44, (\bar{K}/\bar{Y})_D = 0.57 )</td>
</tr>
<tr>
<td>Recipient debt-output ratio</td>
<td>( (\bar{D}/\bar{Y}) = 0.65 )</td>
<td>( (\bar{D}/\bar{Y}) = 0.73 )</td>
</tr>
<tr>
<td>Aid Components</td>
<td>( \bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03 )</td>
<td>( \bar{a}_I = 0.60, \bar{a}_G = 0.09, \bar{a}_D = 0.14 )</td>
</tr>
<tr>
<td></td>
<td>( \bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0 )</td>
<td>( \bar{a}_E = 0.08, \bar{a}_m = 0.09, \bar{a}_c = 0 )</td>
</tr>
</tbody>
</table>

At a higher rate than the donor. Table 3 summarizes the results obtained from this exercise. In our setup, a higher discount rate leads to lower growth, higher tax rates and a lower capital-output ratio for the recipient. A higher discount rate also increases the magnitude of the donors response to debt relief and budget aid. It causes the recipient country to increase taxes by 1% whereas the recipient debt output ratio increases by 8%. The degree of externality of public capital is an important source of dynamic adjustment in our model. We run a further sensitivity analysis where we increase the externality of public capital in the recipient country to 0.3 from 0.2. Results are reported in Table 4. When the externality of public capital in the recipient country is increased to 0.3, growth rates increase considerably. With a higher public capital externality, private capital accumulation speeds up and welfare improves significantly. The donor responds by increasing infrastructure and education aid. Our simulations highlight the complementarity between foreign aid and the externality of public capital. The results suggest that aid policies where the recipient government is required to maintain its commitment to public capital yields the highest level effects to capital accumulation, debt and output.

6 Conclusion

In a world, where the donor has trade and investment links with the recipient, there are economic returns to giving certain forms of aid. These returns flow mainly through two channels. Firstly, increased production in the recipient economy yields higher returns to foreign direct investments made by the donor. Secondly, a higher income for the donor and increased foreign aid together imply a higher income for the recipient economy, which translates to increased demand for donor exports.
Table 4: Sensitivity of aid effectiveness: Externality of public capital

<table>
<thead>
<tr>
<th>Degree of Externality</th>
<th>$\varphi^R = .1, \varphi^D = .1$</th>
<th>$\varphi^R = .15, \varphi^D = .1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth rates</td>
<td>$\bar{g}_R = 0.0244, \bar{g}_D = 0.0226$</td>
<td>$\bar{g}_R = 0.0301, \bar{g}_D = 0.0244$</td>
</tr>
<tr>
<td>Tax rates</td>
<td>$\bar{t}_R = %14.06, \bar{t}_D = %29.59$</td>
<td>$\bar{t}_R = %15.87, \bar{t}_D = %28.55$</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>$(\bar{K}/\bar{Y})_R = 0.39, (\bar{K}/\bar{Y})_D = 0.61$</td>
<td>$(\bar{K}/\bar{Y})_R = 0.52, (\bar{K}/\bar{Y})_D = 0.60$</td>
</tr>
<tr>
<td>Recipient debt-output ratio</td>
<td>$(\bar{D}/\bar{Y}) = 0.65$</td>
<td>$(\bar{D}/\bar{Y}) = 0.44$</td>
</tr>
<tr>
<td>Aid Components</td>
<td>$\bar{a}_I = 0.74, \bar{a}_G = 0.02, \bar{a}_D = 0.03$</td>
<td>$\bar{a}_I = 0.77, \bar{a}_G = 0.01, \bar{a}_D = 0.001$</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}_E = 0.06, \bar{a}_m = 0.15, \bar{a}_c = 0$</td>
<td>$\bar{a}_E = 0.07, \bar{a}_m = 0.14, \bar{a}_c = 0$</td>
</tr>
</tbody>
</table>

This study investigates the effects of aid flows on both the recipient and the donor country. We focus on the optimal of the composition of aid from donors point of view and show that under ideal conditions aid might fail to generate sustainable growth. The model we construct ignores altruistic political motivations of the donor or the recipient in order to understand the effects of economic returns of the aid process. Not all forms of aid contributes to increasing growth. The model predicts no economic role for humanitarian or commodity aid as well as any kind of untied transfer made to the recipient government such as general budget support. In our setup, the governments of two countries can optimally determine the share of infrastructure investments in the recipient economy, that are to be financed by each party.

Our study suggests, when combined, all except one form of aid contribute to growth. When combined, three categories of aid, education aid, capital aid and infrastructure aid can generate even substantial growth. It is assumed that there is an initial gap between the donor and the recipient countries’ labor productivity levels. It is also assumed that making the necessary investments in education, which can be financed by education aid, can close this gap. This process generates a convergence in labor productivity levels, which eventually increases the overall productivity and increases returns to infrastructure investments. This is one of the key propositions of the model constructed in this paper. Based on our results, assumptions on regarding how education aid affects labor productivity in a dynamic setting in the recipient country turns out to be crucial in determining aid outcomes. If we allow labor productivities evolve exogenously, foreign aid does not lead to higher growth rates that are sustainable in the long run.
Our paper differs from previous literature in several ways. First we let the donor act similar to a Stackelberg leader, in that at each stage it makes its decision after observing the optimal allocation choices of the recipient. Secondly, we decompose aid into several dynamic components each of which has a different role in shaping the economic exchange between the donor and the recipient and determining equilibrium outcomes. In our model, all realized transfers are optimal from the donors point of view. Finally, we allow for trade and investment links between the donor and the recipient to account for nested interests between the parties.

Scholars have seldom endogenized foreign aid and decomposed it into its constituent parts. However as some practitioners argue, and as we demonstrate above, specific combinations of foreign aid can stimulate the recipient’s economy in varying ways. Furthermore, few studies that decompose foreign aid as a stimulant of economic growth, do not employ a strategic interaction framework that also feature trade and investment links between the parties. Our model contributes to the existing debate by simultaneously decomposing foreign aid into its constituent parts and examining these parts’ effects on economic growth over time in a strategic framework.

Trade and investment links between a donor and a recipient opens up a new framework, in which returns to aid for the donor and the recipient can be analysed simultaneously. This model can be extended to allow the recipient country to have a dynamic contract with the donor while maintaining such links. Although there has been an impetus in recent research to analyze such dynamic contracts, a more realistic analysis of the economic exchange between the donor and the recipient, who are interconnected in more than one way, is still missing.

References


Appendix

A1 Proof of Proposition 1 Using (10) and (11) prices of intermediate goods in respective countries $j$ can be found as:

$$p_{jt}^R = \frac{r_{kt}^R}{\alpha^R}$$  \hspace{1cm} (24)

$$p_{jt}^D = \frac{r_{kt}^D}{\alpha^D}$$  \hspace{1cm} (25)

Assuming equilibrium in the capital markets:

$$L_t^R k_t^R + L_t^R f_t^R = \sum_{j=1}^J X_{jt}^R = \left( \frac{(\alpha^R)^2 p_t^R (I_t^R + a_{lt})}{r_{kt}^R} \right)^{\frac{1}{1-\alpha^R}} (Z_t^R L_t^R) (\tilde{q}_t)$$  \hspace{1cm} (26)

$$L_t^D k_t^D + L_t^D f_t^R = \sum_{j=1}^J X_{jt}^D = \left( \frac{(\alpha^D)^2 p_t^D (I_t^D) p_{mD}}{r_{kt}^D} \right)^{\frac{1}{1-\alpha^D}} (Z_t^D L_t^D) (\tilde{q}_t)$$  \hspace{1cm} (27)

where $\tilde{q}_t$ is a total quality measure. Then $r$ can be found as:

$$r_{kt}^R = (\alpha^R)^2 p_t^R (I_t^R + a_{lt})^{\alpha^R} \left( \frac{Z_t^R L_t^R}{L_t^R k_t^R + L_t^D f_t^R} \right)^{1-\alpha^R}$$  \hspace{1cm} (28)

$$r_{kt}^D = (\alpha^D)^2 p_t^D (I_t^D)^{\alpha^D} \left( \frac{Z_t^D L_t^D}{L_t^D k_t^D + L_t^R f_t^R} \right)^{1-\alpha^D}$$  \hspace{1cm} (29)

It is possible to determine how much capital is allocated among sectors by using previous results.

$$x_{jt}^R = \left( \frac{\tilde{q}_t}{\tilde{q}_t} \right) \left( L_t^R k_t^R + L_t^D f_t^R \right)$$  \hspace{1cm} (30)

$$x_{jt}^D = \left( \frac{\tilde{q}_t}{\tilde{q}_t} \right) \left( L_t^D k_t^D + L_t^R f_t^R \right)$$  \hspace{1cm} (31)

Using all the results, we can now write per capita incomes as functions of the gross domestic products of respective countries:

$$\tilde{w}_t^R = \frac{1}{L_t^R} \left[ (1 - \alpha^R) + (\alpha^R)^2 \left( \frac{\tilde{q}_t}{\tilde{q}_t} \right) \alpha^R (1 - \alpha^R) \right] p_c [Y_t^R]$$

$$+ \frac{1}{L_t^R} \left[ (\alpha^D)^2 \left( \frac{\tilde{q}_t}{\tilde{q}_t} \right) \alpha^D (1 - \alpha^D) \right] p_m [Y_t^D]$$  \hspace{1cm} (32)
\[ \hat{w}_t = \frac{1}{L_t^m} \left[ (1 - \alpha^D) + (\alpha^D)^2 \left( \frac{\hat{q}^D}{q} \right) + \alpha^D (1 - \alpha^D) \frac{\hat{q}^D}{q} \right] p_t^m [Y_t^D] \]
\[ + \frac{1}{L_t^m} \left[ (\alpha^R)^2 \left( \frac{\hat{q}^D}{q} \right) + \alpha^R (1 - \alpha^R) \frac{\hat{q}^D}{q} + (\alpha^R)^2 \left( \frac{1}{L_t^m k_t^R + L_t^r t'_{J}} \right) \right] p_t^R [Y_t^R] \]
q.e.d.

**A2 Proof of Proposition 2**

The first order conditions are derived from the Lagrangian problem where \( \lambda_t^R \) is the multiplier on the budget constraint (17), \( \mu_t^{R1} \) is the multiplier on the forced choice constraint (14) and \( \mu_t^{R2} \) is the multiplier on the forced choice constraint (15).

\[ m_t^R : \sigma^R (\beta_t^R) \left( m_t^R \right)^{\sigma^R-1} \left( c_t^R \right)^{1-\sigma^R} - \lambda_t^R \frac{p_t^m}{p_t^m} + \mu_t^{R1} = 0 \]
(34)

\[ c_t^R : \left( 1 - \sigma^R \right) \left( \beta_t^R \right) \left( m_t^R \right)^{\sigma^R} \left( c_t^R \right)^{-\sigma^R} - \lambda_t^R + \mu_t^{R2} = 0 \]
(35)

\[ x_{jt+1}^{RR} : - \lambda_t^R \left( 1 + n_t^R \right) + \lambda_t^{R1} \frac{\partial \tilde{w}_{t+1}^P}{\partial x_j^R} + 1 - \delta = 0 \]
(36)

\[ x_{jt+1}^{RD} : - \lambda_t^R \frac{p_t^m}{p_t^m} \left( 1 + n_t^R \right) + \lambda_t^{R1} \frac{p_t^m}{p_t^m} \frac{\partial \tilde{w}_{t+1}^P}{\partial x_j^{RD}} + 1 - \delta = 0 \]
(37)

Substituting the intra-temporal Euler conditions into the budgets gives demand levels as functions of capital allocations and competitive prices. For simplicity, assume demand for consumables in the Recipient country always exceeds commodity aid, therefore setting \( \mu_t^{R1} = \mu_t^{R2} = 0 \).

\[ c_t^R = \left( 1 - \sigma^R \right) \left[ \frac{z_t^R}{p_t^R} + (1 - \delta) \left( \sum_{j \in J^{RR}} x_{jt}^R + \frac{p_t^m}{p_t^R} \sum_{j \in J^{RD}} x_{jt}^D \right) \right] \]
(38)

\[ m_t^R = \sigma^R \frac{p_t^m}{p_t^m} \left[ \frac{z_t^R}{p_t^R} + (1 - \delta) \left( \sum_{j \in J^{RR}} x_{jt+1}^R + \frac{p_t^m}{p_t^R} \sum_{j \in J^{RD}} x_{jt+1}^D \right) \right] \]
(39)
q.e.d.

**A3 World Competitive Equilibrium**

i) Consumers maximize their lifetime utility by choosing the amounts of basic commodity, manufactured good, and foreign and domestic investment for the next period, given prices.

ii) Intermediate good producing firm \( j \) maximizes its profits by choosing how much intermediate good \( j \) will be produced in respective countries. The level of production also equals to its demand for
iii) Final goods producing firms maximize their profits by choosing the amount of labor and intermediate good used, given prices.

iv) There exists a \((2J + 6) \times 1\) price vector consisting of equilibrium prices of the intermediate goods, basic commodity, manufactured good, labor and capital in both countries such that:

v) World demand for the basic commodity equals world supply, i.e. poor country’s gross domestic product equals per capita demands for basic commodity multiplied by the populations.

\[ Y_t^R = L_t^D c_t^D + L_t^R c_t^R \]

vi) Each government maximizes the welfare of its citizens as given in section 3.6.1

vii) World demand for the manufactured good equals world supply, i.e. rich country’s gross domestic product equals per capita demands for manufactured good multiplied by the populations.

\[ Y_t^D = L_t^D m_t^D + L_t^R m_t^R \]

viii) Aggregate ordinary capital demand of intermediate good production in each country equals the sum of domestic and foreign capital inflows.

\[ \sum_{j=1}^{J} x_{jt}^D = K_t^D = L_t^D k_t^D + L_t^R f_t^R \]

\[ \sum_{j=1}^{J} x_{jt}^R = K_t^R = L_t^R k_t^R + L_t^D f_t^D \]

ix) The dynamic resource constraint is given by

\[ K_{t+1}^D + K_{t+1}^R = Y_t^D + Y_t^R + (1 - \delta) \left( K_t^D + K_t^R \right) - L_t^D \left( c_t^D + m_t^R + \tau_t^D \right) - L_t^R \left( c_t^R + m_t^R + \tau_t^R \right) \]

x) World resource constraint holds, i.e. sum of total production and depreciated capital stock equals the sum of consumption, investment and taxes.

\[ \hat{Y}_t^R + \hat{Y}_t^D + (1 - \delta) (L_t^R (\hat{x}_{t+1}^{RR} + \hat{x}_{t+1}^{RD}) + L_t^D (\hat{x}_{t+1}^{DD} + \hat{x}_{t+1}^{DR})) = L_t^D (\hat{c}_t^D + \hat{m}_t^D + (1 + n^D) (\hat{x}_{t+1}^{DD} + \hat{x}_{t+1}^{DR})) + L_t^R (\hat{c}_t^R + \hat{m}_t^R + (1 + n^R) (\hat{x}_{t+1}^{RR} + \hat{x}_{t+1}^{RD})) + \tau_t^R + \tau_t^D \]

Competitive equilibrium consists of quantities that solve the households’ and firms’ problem and a competitive price vector for each period. The quantities are determined by response functions, that
have competitive prices and government decisions, $G$, as arguments. The competitive equilibrium is denoted by the following response functions.

\[ \hat{c}_t^R = c^R (G; \hat{p}, t), \hat{c}_t^D = c^D (G; \hat{p}, t), \]
\[ \hat{\bar{m}}_t^R = \bar{m}^R (G; \hat{p}, t), \hat{\bar{m}}_t^D = \bar{m}^D (G; \hat{p}, t), \]
\[ \hat{x}_{i+1}^{RR} = x^{RR} (G; \hat{p}, t), \hat{x}_{jt+1}^{DD} = x^{DD} (G; \hat{p}, t) \]
\[ \hat{x}_{i+1}^{RD} = x^{RD} (G; \hat{p}, t), \hat{x}_{j+1}^{DR} = x^{DR} (G; \hat{p}, t) \]

In the competitive equilibrium, the world total output is distributed according to (41) and (42).

\[ \tilde{\omega}_t^R = \pi_t^{RR} \hat{Y}_t + \pi_t^{RD} \hat{Y}_t \]

(41)

\[ \tilde{\omega}_t^D = \pi_t^{DD} \hat{Y}_t + \pi_t^{DR} \hat{Y}_t \]

(42)

where $\pi_{i}^{ij}$ represent country $i$'s share of claims in the gross domestic product of country $j$. 

A4