Exercise 1  Consider a job search model where an infinitely-lived risk neutral job seeker receives a job offer at wage $w$, which is a draw from the known wage distribution $F(w)$ and decides whether to accept or reject the wage offer. We assume that jobs are destroyed at rate $\delta$. Assume that job finding rate is exogenous and given by $\lambda$. 

(a) Denote the interest rate by $r$. Write down the discounted lifetime utility $V_E$ of an employed worker as a dynamic programming problem.

(b) Denote the value of an unemployed worker as $V_U$. What is the optimal stopping rule? Give the equation which define the reservation wage $w_r$.

(c) Define unemployment benefit as $b < w$. Derive the lifetime utility of an unemployed worker. $(V_U)$ Use the discrete time approximation we have covered in class and write

$$V_U = \frac{1}{1 + rd} \left[ bdt + (1 - \lambda dt)V_U + \lambda dt \left[ \int_0^{W_r} V_U dH(w) + \int_{W_r}^{\infty} V_E dH(w) \right] \right]$$

(d) Rewrite the equation that defines the reservation wage in term so $V_U$ and $V_E$.

(e) Assume that wage distribution is uniform with support $[1, 5]$ i.e. $H(w) = \frac{w - 1}{4}$, with density function $h(w) = 1/4$ for $w \in [1, 5]$ Calculate the reservation wage $w_r$ and argue that it is unique.

(f) How will the reservation wage changes with $b, r, \lambda, \delta$?

(g) Assume that a unit time period is three months. Unemployment benefit is $b = 2$, interest rate is $r = 0.015$, $\delta = 0.04$ and $\lambda = 1$. Calculate the unique reservation wage, average employment duration and average unemployment duration. Do the same for $b = 1$.

Exercise 2  Consider the model we have studied in class. We consider a continuum of workers and size of the population is normalized to one. The aggregate matching function is given by

$$m = AM(U, V) = AV^\alpha U^\beta$$

where $A > 0$, $0 < \alpha, \beta < 1$.

(a) Show that the matching function is strictly increasing and concave in its argument. What is the economic interpretation of these results? What is the returns to scale of this matching function? What $A$ supposed to capture? Define $\theta = V/U$. What is the economic interpretation of this variable?
(b) Suppose that $\alpha = \beta = 1/2$. By using the matching function, define the rate at which a firm meets a worker as function of $\theta$, denoted by $q(\theta)$. Also define the rate at which a worker meets a firm as a function of $\theta$, denote this function by $p(\theta)$. Determine the signs of $q'(\theta)$ and $p'(\theta)$. Interpret. Calculate the elasticity of $q(\theta)$ with respect to $\theta$ and show that it is negative and less than one in absolute value.

(c) Now suppose that $\alpha = \beta = 3/4$. What does this imply for the matching function. Calculate the rates at which workers meet firms and forms meet workers? Are these a function of only $\theta$? Explain.

(d) Write the values of an employed and unemployed worker, denote them by $V_E$ and $V_U$ as we did in class. Assume that jobs are destroyed at an exogenous rate $\lambda$, and denote unemployment benefit by $b$. Write the value of a matched firm, denoted by $J$ and the value of a vacancy, denoted by $V$. Assume that posting a vacancy cost $c$ to the firm and productivity of a match given by $p$.

(e) When does a firm find it not profitable to enter in the labor market? Using your answer, write the free-entry condition. From this free-entry condition and the Bellman equations of firms, write the firms labor demand that gives a relationship between $q(\theta)$ and the net production $p - w$. This will be the job creation condition.

(f) Wages are determined through Nash bargaining. Worker’s bargaining power is given by $\beta$. Write the Nash maximization program and solve it. Give the exact value of the negotiated wage $w$ as a function of $\theta$. This will be the wage equation.

(g) Write the equation for the steady state unemployment, i.e. flows in an out unemployment are equal. By combining the job creation and wage equation obtain and equation that relates $\theta$ to the parameters of the model. Use the steady-state unemployment condition to calcualte the unique steady-state. Show how steady-state $\theta$, $u$ and $w$ vary with $b$. 

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