Exercise 1 Consider a standard, neoclassical growth, representative agent, infinite horizon economy in which there are only consumption taxes, at rate $\tau_{ct}$ in period $t$. Assume that preferences are given by $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$. Assume that $g_t = 0$ for all $t$ and any revenue generated from this tax is redistributed to the consumer in a lump sum manner.

(a) Define a competitive equilibrium for this economy.
(b) Consider two alternative fiscal policies
   (i) $\tau_{c1} > 0$ and $\tau_{ct} = 0$ for all $t$
   (ii) $\tau_{c2} > 0$ and $\tau_{ct} = 0$ for all $t$

   Are the competitive equilibrium allocations the same for these two fiscal policies?

Exercise 2 Consider a representative consumer, representative firm, infinite horizon dynamic economy with taxation. Define a competitive equilibrium for the case in which the only tax present is the tax $\tau_c > 0$ on consumer purchases. Assume that all revenues are lump sum rebated to consumers. Similarly, define a competitive equilibrium in which the representative firm is required to pay taxes on all sales of consumption goods (but consumers are not) at the same rate $\tau_c > 0$. Again assume that all revenues are lump-sum rebated to the consumers. Formulate and prove a result stating that these two models are equivalent. That is, show that they give rise to the same real, equilibrium, allocations. What is the relationship between the equilibrium prices in the two formulations?

Exercise 3 Consider the neoclassical growth model with taxation. Assume that government uses a constant tax $\tau$ applied to both labor and capital income uniformly. Assume that tax proceeds are used to finance $g_t$ and to balance the budget in every period. Assume that labor supply is inelastic. Make the standard assumptions on utility function and production function.

(a) Define a competitive equilibrium for this economy.
(b) Characterize the steady-state of this economy and describe how capital and consumption changes as $\tau$ changes.

Exercise 4 Consider the neoclassical growth model we have studied in class. Assume that preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t)$$

where $g_t$ is the amount of government expenditures each period. Feasibility constraint reads:

$$c_t + i_t + g_t \leq F(k_t, n_t)$$

Assume that government can use labor and capital taxes to finance its expenditures and freely borrow and lend, i.e. faces a present value budget constraint. Assume that consumer takes $\tau_{nt}$, $\tau_{kt}$ and $g_t$ as given when making decisions.
(a) Define a competitive equilibrium for this economy.

(b) Assume that equilibrium is unique and interior. Characterize the equilibrium given the fiscal policy \( \{g_t, \tau_{nt}, \tau_{kt}\}_{t=0}^{\infty} \).

(c) If the government acts benevolently in choosing \( \{g_t, \tau_{nt}, \tau_{kt}\}_{t=1}^{\infty} \), will it be true that \( \tau_{kt} \to 0 \)? That is, does the Chamley-Judd characterization of the asymptotic behavior of Ramsey tax systems extend to this setting in which \( g_t \) enters the utility function? If your answer is yes, prove it, if your answer is no, prove it.