Why do we need to test for non-stationarity?

- The stationarity or otherwise of a series can strongly influence its behaviour and properties - e.g. persistence of shocks will be infinite for nonstationary series.

- Spurious regressions: If two variables are trending over time, a regression of one on the other could have a high $R^2$ even if the two are totally unrelated.

- If the variables in the regression model are not stationary, then it can be proved that the standard assumptions for asymptotic analysis will not be valid. In other words, the usual “$t$-ratios” will not follow a $t$-distribution, so we cannot validly undertake hypothesis tests about the regression parameters.
STATIONARITY AND UNIT-ROOT TESTING

Value of $R^2$ for 1000 Sets of Regressions of a Non-stationary Variable on another Independent Non-stationary Variable
STATIONARITY AND UNIT-ROOT TESTING

Value of $t$-ratio on Slope Coefficient for 1000 Sets of Regressions of a Non-stationary Variable on another Independent Non-stationary Variable
A WHITE NOISE PROCESS
A RANDOM WALK AND A RANDOM WALK WITH DRIFT
A DETERMINISTIC TREND PROCESS
AUTOREGRESSIVE PROCESSES WITH DIFFERING VALUES OF $\phi$ (0, 0.8, 1)
TYPES OF NON-STATIONARITY

- Various definitions of non-stationarity exist

- We will use the weak form or covariance stationarity

- There are two models which have been frequently used to characterize non-stationarity: the random walk model with drift:
  \[ y_t = \mu + y_{t-1} + u_t \]  
  (1)
  
  and the deterministic trend process:
  \[ y_t = \alpha + \beta t + u_t \]  
  (2)
  
  where \( u_t \) is iid in both cases.
STOCHASTIC NON-STATIONARITY

- Note that model (1) could be generalized to the case where $y_t$ is an explosive process:

  $$y_t = \mu + \phi y_{t-1} + u_t$$

  where $\phi > 1$.

- Typically, the explosive case is ignored and we use $\phi = 1$ to characterize the non-stationarity because
  - $\phi > 1$ does not describe many data series in economics and finance.
  - $\phi > 1$ has an intuitively unappealing property: shocks to the system are not only persistent through time, they are propagated so that a given shock will have an increasingly large influence.
STOCHASTIC NON-STATIONARITY

- To see this, consider the general case of an AR(1) with no drift:

\[ y_t = \phi y_{t-1} + u_t \]  \hspace{1cm} (3)

Let \( \phi \) take any value for now.

- We can write:

\[ y_{t-1} = \phi y_{t-2} + u_{t-1} \]
\[ y_{t-2} = \phi y_{t-3} + u_{t-2} \]

- Substituting into (3) yields:

\[ y_t = \phi (\phi y_{t-2} + u_{t-1}) + u_t \]
\[ = \phi^2 y_{t-2} + \phi u_{t-1} + u_t \]

- Substituting again for \( y_{t-2} \):

\[ y_t = \phi^2 (\phi y_{t-3} + u_{t-2}) + \phi u_{t-1} + u_t \]
\[ = \phi^3 y_{t-3} + \phi^2 u_{t-2} + \phi u_{t-1} + u_t \]

- Successive substitutions of this type lead to:

\[ y_t = \phi^T y_0 + \phi u_{t-1} + \phi^2 u_{t-2} + \phi^3 u_{t-3} + \ldots + \phi^T u_0 + u_t \]
IMPACTS OF SHOCKS TO STATIONARY AND NON-STATIONARY SERIES

We have 3 cases:
1. $\phi < 1 \Rightarrow \phi^T \to 0$ as $T \to \infty$
   So the shocks to the system gradually die away.
2. $\phi = 1 \Rightarrow \phi^T = 1 \forall T$
   So shocks persist in the system and never die away. We obtain:
   as $T \to \infty$
   So just an infinite sum of past shocks plus some starting value of $y_0$.
3. $\phi > 1$. Now given shocks become more influential as time goes on, since if $\phi > 1$, $\phi^3 > \phi^2 > \phi$ etc.
Going back to our 2 characterizations of non-stationarity, the r.w. with drift:

\[ y_t = \mu + y_{t-1} + u_t \]  

(1)

and the trend-stationary process

\[ y_t = \alpha + \beta t + u_t \]  

(2)

The two will require different treatments to induce stationarity. The second case is known as deterministic non-stationarity and what is required is detrending.

The first case is known as stochastic non-stationarity. If we let

\[ \Delta y_t = y_t - y_{t-1} \]

and

If we take (1) and subtract \( y_{t-1} \) from both sides:

\[ y_t - y_{t-1} = \mu + u_t \]

\[ \Delta y_t = \mu + u_t \]

We say that we have induced stationarity by “differencing once”.
DETRENDING A SERIES WITH DETERMINISTIC TREND

Recall the deterministic trend process:

\[ y_t = \alpha + \beta t + u_t \]  

(2)

More generally, a time series may have the polynomial trend:

\[ y_t = \alpha + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \ldots + \beta_n t^n + u_t \]

Detrending is accomplished by running such a regression and obtaining the series of residuals. The residuals will give you the detrended series.

*How is the appropriate degree of the polynomial determined?*

- By standard t-tests, F-tests, and/or using statistics such as the AIC or the SBIC.
DETRENDING A SERIES: USING THE RIGHT METHOD

- Trend-stationary vs difference-stationary
- Although trend-stationary and difference-stationary series are both “trending” over time, the correct approach needs to be used in each case.
- If we first difference the trend-stationary series, it would “remove” the non-stationarity, but at the expense on introducing an MA(1) structure into the errors, and such a model will not be estimable.
- Conversely if we try to detrend a series which has stochastic trend, then we will not remove the non-stationarity.
- We will now concentrate on the stochastic non-stationarity model since deterministic non-stationarity does not adequately describe most series in economics or finance.
DEFINITION OF NON-STATIONARITY

- Consider again the simplest stochastic trend model:

\[ y_t = y_{t-1} + u_t \]

or

\[ \Delta y_t = u_t \]

- We can generalize this concept to consider the case where the series contains more than one “unit root”. That is, we would need to apply the first difference operator, \( \Delta \), more than once to induce stationarity.

Definition

If a non-stationary series, \( y_t \) must be differenced \( d \) times before it becomes stationary, then it is said to be integrated of order \( d \). We write \( y_t \sim I(d) \).

So if \( y_t \sim I(d) \) then \( \Delta^d y_t \sim I(0) \).

An \( I(0) \) series is a stationary series

An \( I(1) \) series contains one unit root,

\[ \text{e.g. } y_t = y_{t-1} + u_t \]
CHARACTERISTICS OF I(0), I(1), AND I(2) SERIES

- An I(2) series contains two unit roots and so would require differencing twice to induce stationarity.

- I(1) and I(2) series can wander a long way from their mean value and cross this mean value rarely.

- I(0) series should cross the mean frequently.

- The majority of economic and financial series contain a single unit root, although some are stationary and consumer prices have been argued to have 2 unit roots.
The early and pioneering work on testing for a unit root in time series was done by Dickey and Fuller (Dickey and Fuller 1979, Fuller 1976). The basic objective of the test is to test the null hypothesis that \( \phi = 1 \) in:

\[
y_t = \phi y_{t-1} + u_t
\]

against the one-sided alternative \( \phi < 1 \). So we have

\( H_0: \) series contains a unit root

vs. \( H_1: \) series is stationary.

We usually use the regression:

\[
\Delta y_t = \psi y_{t-1} + u_t
\]

so that a test of \( \phi = 1 \) is equivalent to a test of \( \psi = 0 \) (since \( \phi - 1 = \psi \)).
THE Dickey-Fuller TEST

Dickey Fuller tests are also known as \( \tau \) tests: \( \tau, \tau_\mu, \tau_\tau \).

The null (\( H_0 \)) and alternative (\( H_1 \)) models in each case are

i) \( H_0: y_t = y_{t-1} + u_t \)
   \( H_1: y_t = \phi y_{t-1} + \mu + u_t, \phi < 1 \)
   This is a test for a random walk against a stationary autoregressive process of order one (AR(1))

ii) \( H_0: y_t = y_{t-1} + u_t \)
    \( H_1: y_t = \phi y_{t-1} + \mu + u_t, \phi < 1 \)
    This is a test for a random walk against a stationary AR(1) with drift.

iii) \( H_0: y_t = y_{t-1} + u_t \)
    \( H_1: y_t = \phi y_{t-1} + \mu + \lambda t + u_t, \phi < 1 \)
    This is a test for a random walk against a stationary AR(1) with drift and a time trend.
Computing the DF test statistic:

- We can write
  \[ \Delta y_t = u_t \]
  where \( \Delta y_t = y_t - y_{t-1} \), and the alternatives may be expressed as
  \[ \Delta y_t = \psi y_{t-1} + \mu + \lambda t + u_t \]
  with \( \mu = \lambda = 0 \) in case i), and \( \lambda = 0 \) in case ii) and \( \psi = \phi - 1 \). In each case, the tests are based on the t-ratio on the \( y_{t-1} \) term in the estimated regression of \( \Delta y_t \) on \( y_{t-1} \), plus a constant in case ii) and a constant and trend in case iii). The test statistics are defined as
  \[ \text{test statistic} = \frac{\hat{\psi}}{\hat{SE}(\psi)} \]

- The test statistic does not follow the usual t-distribution under the null, since the null is one of non-stationarity, but rather follows a non-standard distribution. Critical values are derived from Monte Carlo experiments in, for example, Fuller (1976). Relevant examples of the distribution are shown in table 4.1 below.
THE DICKEY-FULLER TEST

Critical Values:
The null hypothesis of a unit root is rejected in favour of the stationary alternative in each case if the test statistic is more negative than the critical value.

<table>
<thead>
<tr>
<th>Significance level</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.V. for constant but no trend</td>
<td>-2.57</td>
<td>-2.86</td>
<td>-3.43</td>
</tr>
<tr>
<td>C.V. for constant and trend</td>
<td>-3.12</td>
<td>-3.41</td>
<td>-3.96</td>
</tr>
</tbody>
</table>

Table 4.1: Critical Values for DF and ADF Tests (Fuller, 1976, p373).
The tests above are only valid if $u_t$ is white noise. In particular, $u_t$ will be autocorrelated if there was autocorrelation in the dependent variable of the regression ($\Delta y_t$) which we have not modelled. The solution is to “augment” the test using $p$ lags of the dependent variable. The alternative model in case (i) is now written:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t$$

The same critical values from the DF tables are used as before. A problem now arises in determining the optimal number of lags of the dependent variable. There are 2 ways

- use the frequency of the data to decide
- use information criteria
Consider the simple regression:
\[ \Delta y_t = \psi y_{t-1} + u_t \]
We test \( H_0: \psi = 0 \) vs. \( H_1: \psi < 0 \).

If \( H_0 \) is rejected we simply conclude that \( y_t \) does not contain a unit root.

But what do we conclude if \( H_0 \) is not rejected? The series contains a unit root, but is that it? No! What if \( y_t \sim I(2) \)? We would still not have rejected. So we now need to test

\[ H_0: y_t \sim I(2) \text{ vs. } H_1: y_t \sim I(1) \]

We would continue to test for a further unit root until we rejected \( H_0 \).

We now regress \( \Delta^2 y_t \) on \( \Delta y_{t-1} \) (plus lags of \( \Delta^2 y_t \) if necessary).

Now we test \( H_0: \Delta y_t \sim I(1) \) which is equivalent to \( H_0: y_t \sim I(2) \).

So in this case, if we do not reject (unlikely), we conclude that \( y_t \) is at least \( I(2) \).
Phillips and Perron have developed a more comprehensive theory of unit root nonstationarity. The tests are similar to ADF tests, but they incorporate an automatic correction to the DF procedure to allow for autocorrelated residuals.

The tests usually give the same conclusions as the ADF tests, and the calculation of the test statistics is complex.
Main criticism is that the power of the tests is low if the process is stationary but with a root close to the non-stationary boundary.

e.g. the tests are poor at deciding if \( \phi = 1 \) or \( \phi = 0.95 \), especially with small sample sizes.

If the true data generating process (dgp) is

\[
y_t = 0.95y_{t-1} + u_t
\]

then the null hypothesis of a unit root should be rejected.

One way to get around this is to use a stationarity test as well as the unit root tests we have looked at.
Stationarity tests have

\[ H_0: y_t \text{ is stationary} \]

versus

\[ H_1: y_t \text{ is non-stationary} \]

So that by default under the null the data will appear stationary.

One such stationarity test is the KPSS test (Kwaitowski, Phillips, Schmidt and Shin, 1992).

Thus we can compare the results of these tests with the ADF/PP procedure to see if we obtain the same conclusion.
A researcher estimates $\Delta y_t = \mu + \psi y_t + u_t$ where to test for the unit root of $y$. She obtains $\psi = 0.02$ with standard error 0.31.

i) What are the null and alternative hypotheses for this test?

ii) Given the data and a critical value -2.88 perform the test.

iii) What is the conclusion from this test and what should be the next step?

İv) redo ii-iii with $\psi = 0.52$ and standard error 0.16
Answer

(i) The null hypothesis is of a unit root against a one sided stationary alternative, i.e. we have

- \( H_0 : y_t \sim I(1) \)
- \( H_1 : y_t \sim I(0) \)

which is also equivalent to

- \( H_0 : \psi = 0 \)
- \( H_1 : \psi < 0 \)

(ii) The test statistic is given by which equals \(-0.02 / 0.31 = -0.06\)

Since this is not more negative than the appropriate critical value, we do not reject the null hypothesis.

(iii) We therefore conclude that there is at least one unit root in the series (there could be 1, 2, 3 or more). What we would do now is to regress \( \Delta^2 y_t \) on \( \Delta y_{t-1} \) and test if there is a further unit root. The null and alternative hypotheses would now be

- \( H_0 : \Delta y_t \sim I(1) \) i.e. \( y_t \sim I(2) \)
- \( H_1 : \Delta y_t \sim I(0) \) i.e. \( y_t \sim I(1) \)

If we rejected the null hypothesis, we would therefore conclude that the first differences are stationary, and hence the original series was \( I(1) \). If we did not reject at this stage, we would conclude that \( y_t \) must be at least \( I(2) \), and we would have to test again until we rejected. We cannot compare the test statistic with that from a \( t \)-distribution since we have non-stationarity under the null hypothesis and hence the test statistic will no longer follow a \( t \)-distribution.

(iv) The value of the test statistic = \(-0.52 / 0.16 = -3.25\). We therefore reject the null hypothesis since the test statistic is smaller (more negative) than the critical value. We conclude that the series is stationary since we reject the unit root null hypothesis. We need do no further tests since we have already rejected.
Question

1.) What kinds of variables are likely to be non-stationary?

2.) Why is it important to test for non-stationarity before estimation?
Answer

1. Many series in finance and economics in their levels (or log-levels) forms are non-stationary and exhibit stochastic trends. They have a tendency not to revert to a mean level, but they “wander” for prolonged periods in one direction or the other. Examples would be most kinds of asset or goods prices, GDP, unemployment, money supply, etc. Such variables can usually be made stationary by transforming them into their differences or by constructing percentage changes of them.

2. Non-stationarity can be an important determinant of the properties of a series. Also, if two series are non-stationary, we may experience the problem of “spurious” regression. This occurs when we regress one non-stationary variable on a completely unrelated non-stationary variable, but yield a reasonably high value of $R^2$, apparently indicating that the model fits well. Most importantly therefore, we are not able to perform any hypothesis tests in models which inappropriately use non-stationary data since the test statistics will no longer follow the distributions which we assumed they would (e.g. a $t$ or $F$), so any inferences we make are likely to be invalid.
Stationarity and Unit Root in Eviews

- i) Click the series twice
- ii) Click View and Unit Root test
- iii) Choose Levels,
- iv) Choose Augmented Dickey Fueller and “with intercept”
- iv) If the test statistic < critical value (i.e. less than the negative value) reject Ho. No unit root
- Otherwise choose first difference and continue with iv) until you reject Ho. The amount of differencing required to reject Ho=order of integration=number of unit roots
Augmented Dickey-Fuller Unit Root Test on CPI

<table>
<thead>
<tr>
<th>Test Statistic</th>
<th>1% Critical Value*</th>
<th>5% Critical Value</th>
<th>10% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>17.62461</td>
<td>-2.5742</td>
<td>-1.9410</td>
<td>-1.6164</td>
</tr>
</tbody>
</table>

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CPI)
Method: Least Squares
Date: 09/25/00  Time: 22:00
Sample(adjusted): 1980:02 1999:12
Included observations: 239 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI(-1)</td>
<td>0.002765</td>
<td>0.000157</td>
<td>17.62461</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared    -0.176150
Adjusted R-squared -0.176150
S.E. of regression 0.206321
S.D. dependent var 0.190244
Sum squared resid 10.13125
Schwarz criterion -0.314593
Log likelihood 38.59389
Durbin-Watson stat 0.970671
Stationary Testing Procedure

1) Start with no intercept and no trend. Check both the autocorrelation (i.e. DW statistic and ADF statistic). If DW is close to 2 then the test is reliable (no autocorrelation among residuals) If ADF is less (even more negative) than the negative critical value, then the series is stationary.

2) If DW is not close to 2, try with an intercept redo 1)

3) If DW is still not close to 2, try with an intercept and trend redo 1)

4) If ADF statistic is greater than the negative critical value then the series is not stationary. Difference the series once and redo 1 (and 2 and 3 if necessary). If you find the new series to be stationary then the original series has one unit root

5) If not, difference again and redo 1-4 until you achieve stationarity.
Introduction:

- In most cases, if we combine two variables which are I(1), then the combination will also be I(1).

- More generally, if we combine variables with differing orders of integration, the combination will have an order of integration equal to the largest. i.e.,

  \[ X_{i,t} \sim I(d_i) \text{ for } i = 1, 2, 3, ..., k \]

so we have \( k \) variables each integrated of order \( d_i \).

Let

\[ z_t = \sum_{i=1}^{k} \alpha_i X_{i,t} \tag{1} \]

Then \( z_t \sim I(\text{max } d_i) \)
Linear Combinations of Non-stationary Variables:

- Rearranging (1), we can write
  \[ X_{1,t} = \sum_{i=2}^{k} \beta_i X_{i,t} + z'_t \]

  where \[ \beta_i = -\frac{\alpha_i}{\alpha_1} \], \[ z'_t = \frac{z_t}{\alpha_1} \], \[ i = 2, \ldots, k \]

- This is just a regression equation.

- But the disturbances would have some very undesirable properties: \( z'_t \) is not stationary and is autocorrelated if all of the \( X_i \) are I(1).

- We want to ensure that the disturbances are I(0). Under what circumstances will this be the case?
Definition of Cointegration (Engle & Granger, 1987)

- Let $z_t$ be a $k \times 1$ vector of variables, then the components of $z_t$ are cointegrated of order $(d,b)$ if
  - i) All components of $z_t$ are I(d)
  - ii) There is at least one vector of coefficients $\alpha$ such that $\alpha' z_t \sim I(d-b)$

- Many time series are non-stationary but “move together” over time.
- If variables are cointegrated, it means that a linear combination of them will be stationary.
- There may be up to $r$ linearly independent cointegrating relationships (where $r \leq k-1$), also known as cointegrating vectors. $r$ is also known as the cointegrating rank of $z_t$.
- A cointegrating relationship may also be seen as a long term relationship.
Cointegration and Equilibrium:

- Examples of possible Cointegrating Relationships in finance and economics:
  - spot and futures prices
  - ratio of relative prices and an exchange rate
  - equity prices and dividends
  - real money balances, real GDP and the determinants of velocity

- Market forces arising from no arbitrage conditions should ensure an equilibrium relationship. (UIP, etc.)

- No cointegration implies that series could wander apart without bound in the long run.
EQUILIBRIUM CORRECTION OR ERROR CORRECTION MODELS

When the concept of non-stationarity was first considered, a usual response was to independently take the first differences of a series of I(1) variables.

The problem with this approach is that pure first difference models have no long run solution.

e.g. Consider \( y_t \) and \( x_t \) both I(1).
The model we may want to estimate is
\[
\Delta y_t = \beta \Delta x_t + u_t
\]
But this collapses to nothing in the long run.

The definition of the long run that we use is where
\( y_t = y_{t-1} = y; \) \( x_t = x_{t-1} = x. \)

Hence all the difference terms will be zero, i.e. \( \Delta y_t = 0; \Delta x_t = 0. \)
EQUILIBRIUM CORRECTION OR ERROR CORRECTION MODELS

Specifying an ECM:

- One way to get around this problem is to use both first difference and levels terms, e.g.

\[ \Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \gamma x_{t-1}) + u_t \]  

(2)

- \( y_{t-1} - \gamma x_{t-1} \) is known as the error correction term.

- Providing that \( y_t \) and \( x_t \) are cointegrated with cointegrating coefficient \( \gamma \), then \( (y_{t-1} - \gamma x_{t-1}) \) will be I(0) even though the constituents are I(1).

- We can thus validly use OLS on (2).

- The Granger representation theorem shows that any cointegrating relationship can be expressed as an equilibrium correction model.
TESTING FOR COINTEGRATION

- The model for the equilibrium correction term can be generalized to include more than two variables:
  \[
y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \ldots + \beta_k x_{kt} + u_t \tag{3}
  \]

- \( u_t \) should be I(0) if the variables \( y, x_{2t}, \ldots, x_{kt} \) are cointegrated.

- So what if we want to test is the residuals of equation (3) to see if they are non-stationary or stationary. We can use the DF / ADF test on \( u_t \).
  
  So we have the regression
  \[
  \Delta \hat{u}_t = \psi \hat{u}_{t-1} + v_t \quad \text{with} \quad v_t \sim iid.
  \]

- However, since this is a test on the residuals of an actual model, \( \hat{u}_t \), then the critical values are changed.
En­gle and Granger (1987) have tabulated a new set of critical values and hence the test is known as the Engle Granger (E.G.) test.

We can also use the Durbin Watson test statistic or the Phillips Perron approach to test for non-stationarity of $\hat{u}_t$.

What are the null and alternative hypotheses for a test on the residuals of a potentially cointegrating regression?

$H_0 :$ unit root in cointegrating regression’s residuals

$H_1 :$ residuals from cointegrating regression are stationary
There are (at least) 3 methods we could use: Engle Granger, Engle and Yoo, and Johansen.

**The Engle Granger 2 Step Method**

This is a single equation technique which is conducted as follows:

**Step 1:**
- Make sure that all the individual variables are I(1).
- Then estimate the cointegrating regression using OLS.
- Save the residuals of the cointegrating regression, $\hat{u}_t$.
- Test these residuals to ensure that they are I(0).

**Step 2:**
- Use the step 1 residuals as one variable in the error correction model e.g.

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (\hat{u}_{t-1}) + u_t$$

where $\hat{u}_{t-1} z = y_{t-1} x_{t-1}$
Procedure for Testing for Cointegration in Eviews

1) Run ADF tests on each series (x and y). If both are stationary then OLS is fine. If only dependent variable, y, is stationary then it is OK to run simple OLS, provided that you check the residuals for being white noise. If not, you might prefer to make x stationary first and run \( Y(t) = a + Dx(t) + e \) where \( Dx = x(t) - x(t-1) \) if \( X \) is I(1).

2) If \( Y \) is non-stationary only you can make \( Y \) stationary and run \( DY(t) = a + x(t) + e \) if \( Y \) is I(1).

3) If both are non-stationary then you need to test for cointegration and use a cointegration model to make forecasts.

4) First run an OLS of \( y \) on \( x \) (here Lft500 on LDlv) with intercept. (Step 1 of Engle Granger)
Procedure for Testing for Cointegration in Eviews

5) Save first stage residuals

6) Check if the residuals are stationary. If they are I(0) the the variables are cointegrated of order (1,1)

7) Regress Dy on Dx and Residuals from step 1. Dy=a.Dx +b(lagged residuals from step1) +e

Ls dLFT500 dLDIV RES(-1)
Procedure for Testing for Cointegration in Eviews

8) You can also run the following

\[ \text{DY = c + a.dY(-1) + b.dX + f.dX(-1) + g.RES(-1) + e} \]
A Model for Non-stationary Variables: Lead-Lag Relationships between Spot and Futures Prices

Background

- We expect changes in the spot price of a financial asset and its corresponding futures price to be perfectly contemporaneously correlated and not to be cross-autocorrelated.
  
  i.e. expect \( \text{Corr}(\Delta \ln(F_t), \Delta \ln(S_t)) \approx 1 \)

  \( \text{Corr}(\Delta \ln(F_t), \Delta \ln(S_{t-k})) \approx 0 \quad \forall \ k \)

  \( \text{Corr}(\Delta \ln(F_{t-j}), \Delta \ln(S_t)) \approx 0 \quad \forall \ j \)

- We can test this idea by modelling the lead-lag relationship between the two.

EXAMPLE (CONTINUED)

Methodology:

- The fair futures price is given by

\[ F_t^* = S_t e^{(r-d)(T-t)} \]

where \( F_t^* \) is the fair futures price, \( S_t \) is the spot price, \( r \) is a continuously compounded risk-free rate of interest, \( d \) is the continuously compounded yield in terms of dividends derived from the stock index until the futures contract matures, and \((T-t)\) is the time to maturity of the futures contract. Taking logarithms of both sides of equation above gives

\[ f_t^* = s_t + (r - d)(T - t) \]

- First, test \( f_t \) and \( s_t \) for nonstationarity.
Dickey-Fuller Tests on Log-Prices and Returns for High Frequency FTSE Data

<table>
<thead>
<tr>
<th></th>
<th>Futures</th>
<th>Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dickey-Fuller Statistics for Log-Price Data</td>
<td>-0.1329</td>
<td>-0.7335</td>
</tr>
<tr>
<td>Dickey Fuller Statistics for Returns Data</td>
<td>-84.9968</td>
<td>-114.1803</td>
</tr>
</tbody>
</table>
Cointegration Test Regression and Test on Residuals:

- Conclusion: \( \log F_t \) and \( \log S_t \) are not stationary, but \( \Delta \log F_t \) and \( \Delta \log S_t \) are stationary.
- But a model containing only first differences has no long run relationship.
- Solution is to see if there exists a cointegrating relationship between \( F_t \) and \( S_t \) which would mean that we can validly include levels terms in this framework.

**Potential cointegrating regression:**

\[
S_t = \gamma_0 + \gamma_1 F_t + z_t
\]

where \( z_t \) is a disturbance term.

- Estimate the regression, collect the residuals, \( \hat{z}_t \), and test whether they are stationary.
Estimated Equation and Test for Cointegration for High Frequency FTSE Data

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.1345</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.9834</td>
</tr>
</tbody>
</table>

DF Test on residuals

| Test Statistic | -14.7303 |
Conclusions:

- Conclusion: $\hat{z}_t$ are stationary and therefore we have a cointegrating relationship between log $F_t$ and log $S_t$.

- Final stage in Engle-Granger 2-step method is to use the first stage residuals, $\hat{z}_t$ as the equilibrium correction term in the general equation.

- The overall model is

$$\Delta \ln S_t = \beta_0 + \delta \hat{z}_{t-1} + \beta_1 \Delta \ln S_{t-1} + \alpha_1 \Delta \ln F_{t-1} + \nu_t$$
# EXAMPLE (CONTINUED)

The estimated ECM model:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}_0$</td>
<td>9.6713E-06</td>
<td>1.6083</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>-8.3388E-01</td>
<td>-5.1298</td>
</tr>
<tr>
<td>$\hat{\beta}_1$</td>
<td>0.1799</td>
<td>19.2886</td>
</tr>
<tr>
<td>$\hat{\alpha}_1$</td>
<td>0.1312</td>
<td>20.4946</td>
</tr>
</tbody>
</table>

Look at the signs and significances of the coefficients:

- $\hat{\alpha}_1$ is positive and highly significant
- $\hat{\beta}_1$ is positive and highly significant
- $\hat{\delta}$ is negative and highly significant
THE ENGLE-GRANGER APPROACH: SOME DRAWBACKS

This method suffers from a number of problems:

1. Unit root and cointegration tests have low power in finite samples.
2. We are forced to treat the variables asymmetrically and to specify one as the dependent and the other as independent variables.
3. Cannot perform any hypothesis tests about the actual cointegrating relationship estimated at stage 1.

- Problem 1 is a small sample problem that should disappear asymptotically.
- Problem 2 is addressed by the Johansen approach.
- Problem 3 is addressed by the Engle and Yoo approach or the Johansen approach.
THE ENGLE-YOO 3-STEP METHOD

- One of the problems with the EG 2-step method is that we cannot make any inferences about the actual cointegrating regression.

- The Engle & Yoo (EY) 3-step procedure takes its first two steps from EG.

- EY add a third step giving updated estimates of the cointegrating vector and its standard errors.

- The most important problem with both these techniques is that in the general case above, where we have more than two variables which may be cointegrated, there could be more than one cointegrating relationship.

- In fact there can be up to \( r \) linearly independent cointegrating vectors (where \( r \leq g-1 \)), where \( g \) is the number of variables in total.
THE ENGLE-YOO 3-STEP METHOD

- So, in the case where we just had $y$ and $x$, then $r$ can only be one or zero.

- But in the general case there could be more cointegrating relationships.

- And if there are others, how do we know how many there are or whether we have found the “best”?

- The answer to this is to use a systems approach to cointegration which will allow determination of all $r$ cointegrating relationships - Johansen’s method.