Measures of Volatility and Aggregate Investment†

C. Emre Alper††
Department of Economics and
Center for Economics and Econometrics
Bogaziçi University

Gwen Eudey
Department of Economics
Georgetown University

N. Tarhan Feyzioglu
International Monetary Fund

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Recent studies on irreversible investment literature suggest that levels of interest rates and tax rates are relatively less important than various uncertainty measures as determinants of aggregate investment. We attempt to find empirical evidence that aggregate uncertainty and, in particular, variability of inflation as an index of instability matters in aggregate investment decisions. We first follow the methodology of Solimano and Pindyck and, using cross-sectional data for nine OECD countries, observe very low cross-section correlation of inflation volatility and marginal profitability of capital volatility. When we allow for time-varying uncertainty and model it as a GARCH process, we uncover that volatility of marginal profitability of capital indeed changes over time and that GARCH processes of marginal profitability and inflation are the same for eight out of nine OECD countries in the sample. This suggests that the threshold required rate of return and the uncertainty proxied by the volatility of inflation are correlated and that they are moving over time. We conclude by noting the inadequacy of cross-sectional analysis for the testing the implications of the model when uncertainty follows a stochastic process.

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†† Corresponding author. Department of Economics, Bogazici University, P. K. 2, Bebek 80815 Istanbul, Turkey. Fax: +90 (212) 287-2453. E-mail: alper@boun.edu.tr
I. Introduction

Aggregate investment is the most important component of GDP for achieving sustainable growth. Traditional investment theory has investment as being costlessly reversible and that the standard "net-present-value rule" dictates optimal investment behavior: invest when the marginal revenue product of capital is greater than the Jorgensonian user cost of capital, and disinvest when the marginal revenue product is less than the cost. Traditional investment theory implies that each firm sets its marginal revenue product of capital equal to its user cost of capital. Thus policies affecting either marginal revenue product or user cost of capital should be effective in determining level of investment. Empirical studies of investment behavior, however, show little or no response of investment to changes in real interest rates and tax incentives, which are primarily aimed at affecting the user cost of capital.

Recent literature has considered the possibility that most investments at the firm level, are irreversible or reversible at a cost. The irreversible nature of investment creates a wedge between the Jorgensonian user costs -including delivery, depreciation, adjustment costs, etc.- and the value of the marginal revenue product of capital. 1 Optimal investment policy under this type of framework can be characterized as a trigger-strategy investment behavior taken by firms. Specifically, firms should invest when the marginal revenue product equals the threshold required rate of return for undertaking an investment project. This threshold required rate is higher than the user cost of capital and uncertainty about future values of marginal revenue product raises this

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1 Abel and Eberly (1995) show that introducing a tiny wedge between the purchase price of capital and the sale price of capital leads to a substantial effect on the threshold required rate of return and introduces a substantial amount to option value or opportunity cost of undertaking an investment. Therefore, for the purposes of this paper, we shall use 'irreversible' investment as encompassing both cases.
threshold. Thus, a policy implication toward achieving higher level of investment would be to have macroeconomic stability that would reduce uncertainty, rather than to lower the real interest rates or to give tax incentives.

Solimano and Pindyck (1993) make use of a model developed in Caballero and Pindyck (1992) to analyze the aggregate investment behavior of thirty countries which are distributed such that half of the countries are LDCs and the other half are OECD countries. Solimano and Pindyck specifically investigate the relation between the uncertainty, threshold, and level of aggregate investment. Despite utilizing a model based on an economy with perfect competition, their results are surprisingly inconclusive for OECD countries. The analysis using LDC data however yields results that show support for the model.

In this paper we analyze the same problem and first follow the methodology of Solimano and Pindyck and use cross-section analysis. The results show only negligible support for the irreversible model for a competitive economy by Caballero and Pindyck. These results nonetheless may be misleading for cases where the uncertainty measure evolves across time stochastically. We therefore take a look at time series properties of uncertainty and indeed uncover a GARCH process. There are some conclusive evidence that is obtained through time series analysis.

This paper is divided into five sections. In section II, we describe the basic model of Caballero and Pindyck as well as furnish some of the basic intuition behind it. In Section III we discuss the properties of the data. We present the results of cross-section analysis and then compare them with time-series analysis in section IV. Finally, in section V, we present the main results of the paper.
II. The Model and Methodology

A. The Model

The basic model of McDonald and Siegel [1986] has irreversible investment which arises due to the sunk cost, $I$. This sunk cost is the amount needed to undertake an investment project. The value of the investment project, $V$, follows a geometric Brownian motion:

$$dV = \eta V \, dt + \sigma V \, dz$$
$$dz = \varepsilon \, dt$$

where $dz$ is the increment of a Wiener process. Given the properties of Brownian motion, percentage changes in $V$ are normally distributed, which implies that level changes are lognormally distributed. The change in $\ln V$ is normally distributed with mean $(\eta - \frac{1}{2} \sigma^2)T$ and variance $\sigma^2 T$. This result is a direct application of Ito's lemma. Intuitively, the geometric Brownian motion for the value of the project implies that the value of the project is known today and unknown for all future values and uncertainty about the future values increases with the length of the forecast.

The solution to the maximization of the net present value (net of sunk cost $I$) of the project is given in Pindyck [1991]. The key result is that if the investment project is irreversible, then the threshold return required to undertake an investment project is directly proportional to $\sigma$. Even though net present value of an investment project may be positive, the irreversible nature of investment adds the possibility of delaying the project rather than undertaking it today, as suggested by the standard neoclassical theory of investment. As a result, investment in the short run may be reduced as uncertainty increases. The effect of uncertainty on long-run investment
level is ambiguous.

We make use of a simplified version of the model first given in Caballero and Pindyck and later utilized in Solimano and Pindyck for aggregate data. Caballero and Pindyck analyze the affects of both industry and firm-specific uncertainty in a competitive setting, with a sunk cost of entry and constant returns to scale technology. Firm-specific shocks have symmetric effects on the expected marginal revenue product of capital, and do not matter for a risk-neutral firm owner, who only cares about average profit. Aggregate shocks, on the other hand, are asymmetric - as explained in the next section - and do affect the investment decision. They test this implication in their model by using U.S. Manufacturing investment data. Their results show that, in such a setting, only industry-level uncertainty affects competitive firm’s irreversible investment decision\(^2\). They conduct the test by analyzing the effects of industry level, as well as firm level uncertainty, on the threshold return required to undertake an investment project. Solimano and Pindyck conduct the same test using aggregate level data.

In the next section, we present the model which we use to analyze the implications of irreversible investment in a competitive setting.

### B. Methodology

We consider a competitive economy with a Cobb-Douglas aggregate production function that exhibits constant returns to scale. We express the output of a price taking firm in such a setting as:

\(^2\) Abel (1983) demonstrates that an increase in output price uncertainty leads the competitive firm to increase investment under a continuous-time model where future prices evolves stochastically. Pindyck and Caballero do not consider the possibility of marginal revenue product of capital being convex in the output price.
\[ Y_t = A_t K_t^{\alpha} L_t^{(1-\alpha)} \]

where \( Y_t \) is the real GDP, \( K_t \) is the gross capital stock at constant prices, \( L_t \) is the total civilian employment, \( \alpha \) is the share of capital, and \( A_t \) is the Solow residual.

The marginal profitability of capital is then given by:

\[ \Pi_K = \alpha (1-\alpha) \left( \frac{1-\alpha}{\alpha} \right) A_t^{\alpha} W_t^{(1-\alpha)} \]

where \( W_t \) is the real wage rate. This equation says that marginal productivity of capital is the marginal profitability of capital. It is interesting to note that marginal cost of capital is equal to zero, since entry or investment requires only a sunk cost. Next, we solve for \( A_t \) from the Cobb-Douglas production function and substitute into the marginal profitability of capital equation:

\[ \Pi_K = \alpha (1-\alpha) \left( \frac{1-\alpha}{\alpha} \right) \left( \frac{Y_t}{K_t^{\alpha} L_t^{(1-\alpha)}} \right)^{\frac{1}{\alpha}} W_t^{(1-\alpha)} \]

The marginal profitability \( A_t \) is denoted as \( B_t \) from now on following Caballero and Pindyck's notation. We then take natural logarithm of the previous equation for simplicity and with lowercase letters depict natural logarithm, the marginal profitability, \( b_t = \ln (B_t) \) is

\[ b_t = \ln \left[ \frac{1-\alpha}{\alpha} \right] + \frac{a_t}{\alpha} - \frac{(1-\alpha)}{\alpha} w_t \]

\[ a_t = Y_t - \alpha k_t - (1-\alpha) l_t \]

As shown in Caballero and Pindyck, marginal profitability has a boundary (threshold) which varies positively with the volatility parameter of the marginal profitability. We use the above equation to calculate marginal profitability of capital \( b_t \) for nine OECD countries by using
quarterly data from 1978:1 through 1994:2\(^3\) and perform cross-sectional analysis as in Solimano and Pindyck.

III. Data

We used deseasonalized quarterly data for the period 1978:1 through 1994:2. In order to calculate the Solow residuals \(a_t\), we used the real gross domestic product as aggregate output, \(Y_t\), total civilian employment, \(L_t\), and the derived gross fixed private capital stock, \(K_t\). Gross fixed private capital stock data is available annually. We used quarterly gross private capital formation and assumed an annually varying compounded depreciation rate to convert gross fixed private capital stock into quarterly frequency. The depreciation rate is increasing for all the countries throughout the period.\(^4\) The reason for this may be caused by a shift in the composition of capital stock toward short-lived capital goods such as computers.

We obtain the share of capital \(\alpha\) in the production function from dividing operating surplus by the sum of compensation of employees paid by resident producers and operating surplus. We then calculate the share of capital each year and average over the period to get \(\alpha\). Next we calculate the real wages \(W_t\), first calculating nominal wages from dividing compensation of employees by total civilian employment and then deflating by the GDP deflator\(^5\).

We report the marginal profitability of capital data as well as investment capital stock ratio

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\(^3\) The nine OECD countries are Canada, USA, Japan, Australia, Finland, France, Germany, Sweden and the UK. Details about the sources of data are available from the authors upon request.

\(^4\) A table of depreciation rates is available from authors upon request.

\(^5\) Since quarterly compensation data of employees for Sweden was not available, we used a quarterly index of hourly earnings in mining and manufacturing.
for the nine OECD countries in Table 1. We break down the data into three sub-periods, in order to look for possible structural breaks. Except for Japan, the marginal profitability of capital appears to be more volatile for years 1978-82. There are no other major variations for the three periods reported.

IV. Empirical Analysis

A. Cross-Section Analysis

The model's foremost result is that irreversible investment combined with aggregate uncertainty implies a higher threshold required rate of return to undertake an investment project. We first examine the relation between the volatility of marginal profitability of capital and the threshold. Table 2 depicts the outcome of cross-section regressions for the nine OECD countries with 66 observations for each country. The dependent variables are $DBMAX$, the maximum value of $b_i$ minus the average of $b_i$; $DBDEC$, the average of the top ten percent values of $b_i$ minus the average of $b_i$; $DBQUINT$, the average of the top twenty percent values of $b_i$ minus the average of $b_i$. $DBKMAX$, $DBKDEC$ and $DBKQUINT$ are calculated in the same manner as above except that the values of $b_i$ now correspond to the maximum, top ten and top twenty percent values of change in the real capital stock.

The dependent variable in the regressions is used as a proxy for the threshold on which when marginal profitability of capital hits, triggers investment. We use the standard deviation of the quarterly changes of $b_i$, $STD(\Delta b_i)$ as an explanatory variable to see whether the threshold varies positively with $STD(\Delta b_i)$ as predicted by the model. A problem with this analysis is that
higher volatility of $\Delta b_t$ will, itself, also imply a higher threshold. In order to prevent a spurious conclusion of a positive significant coefficient in front of the volatility term, as support for the model, another set of proxies are obtained for the threshold. We collect the values of $b_t$ that correspond to the maximum, top ten, and top twenty percentile of the change in the real total gross capital stock respectively.

The results given in Table 2 do not strongly support the irreversible investment model of Caballero and Pindyck. The coefficients of the volatility measure $\text{STD}(\Delta b_t)$ does carry the expected sign and are significant for the first two regression equations. The sign of the volatility term, however, is negative and insignificant in the latter three regressions. As mentioned above, the results concerning the relation between the volatility and the threshold from the first set of three regressions may be spurious.

Table 3 depicts the correlation between the volatility of marginal profitability and economic indicators. Indicators include the average level of inflation of consumer prices, the standard deviation of quarterly differenced inflation, and the standard deviation of changes in the real interest rate\(^6\). The results in table 3 show that the volatility of marginal profitability is negatively correlated with real interest rates and positively correlated -- though with an insignificant 6.2\% -- with volatility of inflation. The average level of inflation is negatively correlated with the volatility measure.

\(^6\) Real interest rates are computed by using the following equation:

\[
\frac{\frac{\left(p_t - p_{t-1}\right)}{\left(1 + \frac{\left(p_t - p_{t-1}\right)}{p_{t-1}}\right)} - i_t}{\left(\frac{\left(p_t - p_{t-1}\right)}{p_{t-1}}\right)}
\]

where $i_t$ is the nominal interest rate. Lending rate is used for nominal interest rate for all countries in the data set except for France which was not available and money-market rate is used instead.
Overall, cross-section evidence from our data set does not seem to support the implication of the recent literature on irreversible investment that the volatility of inflation and the marginal productivity of capital are related positively. A possible explanation for this could be that averaging over years and employing cross section analysis may yield misleading results when one or more of the variables evolve stochastically over time.

We would like to analyze the time series properties of the data and more importantly allow for a time varying variance of the value of the firm. The cross-section analysis used above and by Solimano and Pindyck assumes a time independent volatility of the marginal profitability. It follows to ask whether this assumption is supported by the data. If not, how might it affect the findings of the cross-section evidence?

B. Time Series Analysis

We allow for the following diffusion process for the value of a project:

\[ dV = \eta V \, dt + \sigma_i V \, dz \]

\[ dz = \varepsilon_i \sqrt{dt} \]

\[ \phi(L)\Pi_{K,t} = \theta(L)\varepsilon_i , \quad \varepsilon_i \sim N(0,\sigma_i^2) \]

\[ \sigma_i^2 = \alpha(L)e_i^2 + \beta(L)\sigma_i^2 \]

where \( dz \) is the increment of a Wiener process, \( L \) is a lag operator, and \( \Phi(L), \theta(L), \alpha(L), \beta(L) \) are polynomials of order \( p, q, gq \) and \( arp \) respectively, such that
As before, the value of the project is known today and future values are lognormally distributed. However, unlike the previous case, variance does not grow linearly with time horizon. We assume that the volatility of marginal profitability varies across time and uncertainty is modeled as the variance of the residuals of the ARMA process for marginal productivity of capital.\footnote{The conditional mean of the residuals of the ARMA process, however, are still equal to zero.} We next explore the time series properties of this unexpected variance of volatility.

1. Order of Integration at Seasonal Frequencies: The Generalized form of AutoRegressive Conditional Heteroskedasticity (GARCH) procedure requires the use of stationary variables or cointegrated variables so that the residuals are stationary. There are a number of unit-roots test available for testing the order of integration. One of the eminent unit-root tests, based on testing residuals, is the Dickey and Fuller [1979] approach. The Augmented-Dickey-Fuller (ADF) test comprises the long-run or the zero frequency aspects of economic time series.

Many economic time series with quarterly frequency exhibit seasonality, that is, they tend to follow a regularity over the course of every year. The unit-root tests have recently been extended to consider the possibility of unit roots at different frequencies, such as seasonals. In
this paper, the unit-root test by Hylleberg, Engle, Granger, and Yoo [1990] for quarterly time series is utilized. Intuitively, the test is based upon analyzing the underlying seasonal structure of the data.

The results of the seasonal unit roots test for the marginal profitability indicate that the marginal profitability for Finland has no unit roots; for Canada, USA, Australia, France, Germany, and U.K. are ordinary $I(1)$ variables with no unit roots at the seasonals, and for Japan and Sweden are $I(1)$ with additional unit root at the frequency $\theta = \frac{1}{2}$.

For countries that have unit roots at the frequencies $\theta = 0$ and $\frac{1}{2}$ it is necessary to filter out the unit roots since GARCH cannot be estimated with a unit root process in the error terms. To filter out the underlying unit-root components at 0 frequency involves transforming $x_t$ to $x_s = (1-L)x_t$, where $x_s$ is the filtered series. In order to filter out the unit root at the $\frac{1}{2}$ frequency as well, we transform from $x_s$ to $x^{2s} = (1+L)x_s$. Next, we check and see if there are anymore unit roots left in the processes of marginal profitability. Finally, we filter the nonstochastic seasonality in the data by estimating the processes with three centered seasonal dummy variables that sum to zero over a year and subtracting any significant seasonal dummy from the process.

2. ARMA GARCH Modelling: We made use of the Kalman filter to estimate $b_t$ as an ARMA(p,q) process with a GARCH(gq, arp) error process\(^8\). We estimated the following model:

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\(^8\) We estimated the process by writing the Kalman filter and executing the program using GAUSS. The maximum likelihood estimation is exact, if there are no GARCH effects or if there exist only nonstochastic GARCH effects. Otherwise, the procedure will give approximated likelihood estimation procedure.
\begin{align*}
(1 - \sum_{i=1}^{p} \phi_i L^i)(y_t - \gamma_j) &= (1 + \sum_{i=1}^{q} \theta_i L^i) \varepsilon_t \\
\varepsilon_t &\sim N(0, h_t) \\
h_t &= \alpha_0 + \sum_{i=1}^{gq} \alpha_i L^i \varepsilon^2_t + \sum_{i=1}^{arp} \beta_i L^i h_t
\end{align*}

the obvious problem in estimating the series is deciding on the values of \( p, q, gq, \) and \( arp \). We use a two step method: we first estimate the model as an ARMA process for various levels of \( p \) and \( q \). Examining the significance of the lags and the Schwarz-Bayesian Information Criterion, we chose the values for \( p \) and \( q \) while making sure that no autocorrelation remains in the residuals. The second step is to run GARCH on the ARMA\((p,q)\) process for \( gq, arp = 0, 1, 2 \). Again, by inspecting at the Schwarz-Bayesian Information Criterion and significance of the coefficients, we decided on the level of \( gq \) and \( arp \). We present the results of the final regressions in Table 4.

We see that for countries Canada, USA, and Sweden, our volatility measure of the marginal profitability follows a GARCH model and Japan, Finland, and UK have a nonstochastic conditional variance that converges to some number say \( \sigma \), monotonically. For Australia, France, and Germany there exists no GARCH at the error term, thus a non-time varying volatility of the residual terms is suitable. These initial results imply that for only the latter three countries out of nine in our sample, averaging across time and using cross-sectional analysis is appropriate.
3. Volatilities of Marginal Profitability and Inflation: One of the disappointing outcomes of the cross-section analysis was the low correlation (approximately 6%) between the volatilities of marginal profitability and inflation. We now analyze the time series properties of the variance of inflation which in general is a good proxy for aggregate uncertainty and hence must be important for influencing investment behavior.

We used monthly consumer price indices for the eight OECD countries\(^9\) to derive inflation. We checked for the stationarity of the data and detected unit roots at the levels of inflation and found out that differencing once sufficed to eliminate the unit root.

Next, we identified the differenced inflation process that is an AR process \(p\) and an MA process \(q\). After deciding on the ARMA\((p, q)\) processes by examining the SBIC, the significance of coefficients and having detected no autocorrelation in the residuals, we run the GARCH\((gq, arp)\) regressions. We report the final results in Table 5.

Analyzing Table 5, we found out that Canada, USA, Sweden, and UK have stochastic component in the variance of their residuals. Japan, Finland, and France have monotonically declining nonstochastic variance, and Australia and Germany have non-time varying constant variance. When we compared the GARCH process of marginal profitability with that of inflation, we noticed that except for U.K. they are identical. Those countries that have i) constant; ii) nonstochastic, monotonically decreasing and converging; and iii) stochastic residual variance of marginal profitability processes, respectively, have also i) constant; ii) nonstochastic, monotonically decreasing and converging; and iii) stochastic residual variance of inflation processes.

\(^9\) We could not get monthly CPI data for Australia, so we used quarterly CPI instead.
We next run simple correlations and ordinary least squares regressions for countries that
do not have constant marginal profitability variance; the countries are *Canada, USA, Japan, Finland, Sweden* and the *UK*. We presented the results in tables 6 and 7. In contrast to the
previous correlation of 6.2% using cross-section analysis, time series analysis yielded significantly
higher correlations between inflation and marginal profitability volatilities. When we regressed
inflation volatility on marginal profitability volatility with some lags of inflation volatility to
eliminate any autocorrelation, we see that the marginal profitability volatility has a positive and
significant coefficient in all cases except for *Canada* which has a 15% p-value.

Assuming the underlying theory holds true, if we model a time varying uncertainty then it
must be true that the threshold required rate of return also changes over time. Unless one has
data on the threshold, one cannot test whether it varies positively and significantly with
uncertainty. One can, however, test the direct effect of aggregate uncertainty on threshold by
examining the behavior of aggregate investment after making certain assumptions about the cross
sectional distribution of firms' marginal profitability. Under fairly reasonable assumptions, most
of the firms tend to lie very close to the threshold. Therefore, an increase in the threshold would
imply a decrease in aggregate investment in the short run. This points is taken up by Alper,

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\(^{10}\) See Caballero [1993].
V. Conclusion

Traditional investment theory prescribes a reduction of real interest rates and/or tax incentives to promote investment, however, empirical investigation shows little and in some cases no response of investment to these policies. Recent literature emphasizes the role irreversibility plays in investment decisions. This suggests that macroeconomic policies aimed at reducing aggregate uncertainty may stimulate aggregate investment.

Empirical analysis using averages across time and cross-section data for nine OECD countries present negligible support for Caballero and Pindyck model. The threshold required rate of return to undertake an investment project does not seem to be positively related to aggregate uncertainty as the theory suggests.

We use time-varying uncertainty by employing GARCH methodology and show that volatility of marginal profitability of capital and volatility of inflation are positively related in the short run, thus an increase in uncertainty or instability implies a higher threshold required rate of return for investment. This in turn leads to lower aggregate investment in the short run under reasonable assumptions about the cross-sectional density of firms' marginal profitability.

We believe that the cross-section analysis of taking simple averages may yield misleading results when time-varying variables are involved. One of the priorities of the empirical agenda of aggregate investment should be to model a time varying threshold required rate of return. This would be a more direct test of the model that says threshold varies positively with uncertainty in a more direct manner.
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