

# Production Complementarity and Investment Incentives: Does Asset Ownership Matter?

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## Abstract

We extend the results of the Coase theorem to the relationships where, due to contractual incompleteness, agents are unable to bargain over all aspects of the transaction. We show that the initial allocation of ownership rights is irrelevant if a sufficiently large surplus is created by cooperation. Our result contrasts with Grossman and Hart (1986), who, using a similar model, obtain that the ownership rights should be allocated to minimize ex-ante inefficiencies in production. The critical element behind these two different results is that while Grossman and Hart (1986) model uses the Nash bargaining solution treating status quo payoffs as disagreement points, here they are treated as outside options. Our model also shows that, when relevant, asset ownership may provide disincentive to invest as in De Meza and Lockwood (1998) and Chiu (1998).

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## 1 Introduction

There has been contrasting theoretical results on the relationship between asset ownership and investment incentives. The seminal work of Grossman and Hart (1986) (GH, hereafter), followed by Hart and Moore (1990), and Hart (1995), argued that asset ownership boosts incentives to invest. However, De Meza and Lockwood (1998) and Chiu (1998), (DLC, hereafter) have shown that in certain environments, asset ownership may in fact reduce incentives to invest. This paper, in addition to obtaining DLC results in a different setting, presents a case where asset ownership does not affect investment incentives, despite the fact that investments are productive. It also contributes to understanding the relationship between asset ownership and investment incentives by disentangling the effects of production complementarity and investment productivity.

The model in this paper draws on GH where a theory of ownership rights is developed and is applied to a firm's decision to integrate vertically or horizontally. They consider a relationship between two firms whose productive activities are dependent on each other. Ex-ante both firms make a relationship-specific investment and, ex-post they make a decision regarding the production process. Due to high transaction costs, ex-ante contracts contingent on the choice variables cannot be written. However, once the ex-ante investments have been made, the ex-post production decision becomes contractible. Thus the agents can bargain over the division of surplus before the production decisions are made. In the model, they define a firm as a set of property rights over the physical assets that it owns. Ownership confers *residual control rights* over the assets in the sense that, the owner of the asset has the right to use it in whichever way he desires unless specific rights are contracted away. Since none of the variables are ex-ante contractible, the initial contract only specifies the allocation of the residual control rights. Through its effect on the use of the asset in uncontracted states, ownership rights influence agent's bargaining power and the division of ex-post surplus, which in turn affects the parties' incentives to invest in that relationship. If there is a reciprocal dependency between the production of both firms, integration improves the incentives of the new owner while it weakens the incentives of the acquired firm's ex-owner. This trade-off between the costs and benefits of ownership determines the optimal allocation of control rights, hence ownership.

The main conclusion of GH is that the ownership rights should be allocated to minimize the ex-ante inefficiencies in production and assets should be owned by the agent whose ex-ante investment is the most productive in the relationship. This result is driven by the particular equilibrium of the bargaining game that GH have considered. They considered Nash bargaining where the status quo payoffs, which is the payoff received by an agent prior to bargaining, is treated as the disagreement point to the Nash solution<sup>1</sup>. When the status quo payoffs are taken as the disagreement point of the Nash solution, the agent's equilibrium payoff, which we call as the "*split-the-difference*" payoff, is the sum of his status quo payoff and half of the difference between the total surplus and both agents' status quo payoffs. This can be an equilibrium of a bargaining game in which the agents receive their status quo payoffs at every period where an agreement has not been reached. However, if the agents do not receive an income flow in the course of the bargaining or there is no exogenous risk of breakdown, then the disagreement payoff, which is the payoff from a perpetual negotiation without an agreement, should be zero. In this case, the agents can obtain their status quo payoff only if they quit the bargaining game unilaterally to implement the status quo. When the status quo payoffs are taken as outside options, they determine the range of validity for the Nash solution. When neither agents' outside option is binding, both receive half of the total surplus, which we call "*split-the-surplus*" payoff. When only one agent's outside option is binding, he receives his outside option and the opponent claims the residual. In this paper, we replace the Nash bargaining with an explicit alternating offers bargaining game where status quo payoffs are treated as outside options.

This has been argued earlier by DLC and they have shown that asset ownership does not necessarily boost incentive to invest. In some cases, asset ownership may act as "stick" rather than "carrot" in De Meza and Lockwood (1998) terminology. In this paper, we reinforce the results of DLC on the disincentive effects of asset ownership. However, our model is based on the original model used in GH rather than the model in Hart (1995) which is a special case of the former in some sense. In Hart (1995) the ex-post production decisions are merely a decision on the choice of

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<sup>1</sup>As it has been previously argued in Binmore, Shaked, and Sutton (1989), Sutton (1986), a dynamic bargaining game differentiates between a disagreement point and an outside option.

the trading partner, i.e.: whether to trade with the existing partner or outsider. Hence the right to exercise residual control rights is rather limited.

Moreover, our model also characterizes a case where the initial allocation of ownership is irrelevant. If production complementarity and investment productivity are both important in the relationship, then we have a Coasean prediction that applies to an incomplete contracting environment. In other words, this model predicts the irrelevance of the ownership structure in a vertical structure like a mine-mouth electricity generation where production complementarity and investment productivity are equally important to both parties. In GH model the Coase theorem fails to apply because of the existence of transaction costs that are created by the agents' opportunistic behavior during bargaining. In this paper, we show that the Coase theorem partly applies despite the presence of contractual incompleteness, for the agents cannot reach to Pareto efficiency through bargaining. Regardless of the allocation of ownership rights, when the size of the surplus is endogenous, the bargaining results in an inefficient equilibrium because of the free rider problem. As parties do not receive the full benefit of their actions, their incentives to invest are distorted. In this paper, we are not interested in inefficiencies of this kind but in those that are solely driven by the allocation of ownership rights. Our model predicts that, while the ex-ante investments are inefficient, the initial allocation of ownership rights is irrelevant.<sup>2</sup>

The paper also unveils the two intertwined factors that affect the optimal distribution of ownership; investment productivity and complementarity in production. Whether an outside option is binding or not depends on both the degree of complementarity between the ex-post productive activities and ex-ante investment levels. If complementarity

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<sup>2</sup>In this model, we adopt the definition of ownership, which is the power to exercise control, used in GH. Ownership, however, can also be identified with the rights to the residual income stream. As argued by Holmstrom and Tirole (1989), the definition of ownership can be a critical element in analyzing the efficiency properties of the initial allocation of ownership rights. Several papers, such as Holmstrom and Tirole (1989), Bolton and Whinston (1993), find that the initial allocation of ownership rights over physical assets have efficiency implications. In these papers, however, ownership is defined as the rights to the both residual control and return stream. It would be interesting to examine the extent to which the irrelevance result depends on the definition of ownership.

is reciprocal and large then, regardless of the distribution of ownership, it is more likely that ex-post renegotiation produces a sufficiently large surplus at almost all investment levels. In this case, neither party's outside option is binding and both parties get “*split-the-surplus*” payoff. Hence, the distribution of ownership is irrelevant. However, if the relationship is asymmetric, in the sense that  $i$ 's action is significant for  $j$  but not vice versa, it is more likely that  $j$ 's outside option will be binding under  $j$ 's ownership. It is optimal to give the assets to agent  $j$  only if  $i$ 's investment is more productive relative to  $j$ 's investment. Therefore optimal ownership of an asset does not just depend on whether investments are productive but also whether ex-post production exhibits significant complementarity as well. These two effects cannot be separated in the De Meza and Lockwood (1998) model but rather their combined affect is represented by assets being productive or unproductive outside relationship. This is because, in their model, the ex-post production decision is reduced down to the choice of trading partner, inside or outside, whereas in many situation ex-post production may involve complex design decisions that are not contractible ex-ante.

The paper is organized as follows. In Section 2, the formal model is introduced. In section 3, the equilibrium to the induced bargaining subgame is derived in the case of non-integration. Section 4 contains the equilibrium of the investment-choice game in the case of non-integration. Section 5 considers integration, in particular we analyze the case in which firm 1 owns firm 2. In Section 6, we compare the two ownership structures and discuss the relationship between asset ownership and investment incentives. In Section 7, we look at the comparative statics as we change the level of complementarity between the two firm, in order to characterize when the equilibrium exist. Section 8 contains concluding remarks.

## 2 The Model

As in the GH model, we consider two firms, 1 and 2, that are engaged in a relationship which lasts 2 periods. Each firm is managed by an agent who receives the full return of the firm where he is employed. At the beginning of date 1, the two agents sign a contract that specifies the distribution of ownership rights over each firm's assets. After the contract is signed, the two agents make a relationship-specific investment which is denoted by  $a_i$  for  $i = 1, 2$ . We assume that the relationship-specific

investments require special skills so that the investment  $a_i$  in firm  $i$  can only be made by agent  $i$ . At date 2, the investments become observable to both agents and some further decisions regarding the production process are made, which are denoted by  $q_i$ . Although  $a_i$  is chosen by agent  $i$ , the ex-post decision,  $q_i$ , is made by the agent who owns firm  $i$ . If the firms are separately owned, that is if agent  $i$  owns firm  $i$ , each agent is an owner-manager who has residual control rights over its firm's physical assets, so agent  $i$  chooses  $a_i$  and  $q_i$  of firm  $i$ . If the firms are integrated under  $i$ 's ownership then agent  $i$  owns both firms 1 and 2 then agent  $j$  becomes his employee. For example, under 1's ownership, agent 1 chooses  $a_1$ ,  $q_1$  and  $q_2$  and agent 2 chooses  $a_2$ . The private benefit to agent  $i$  is written as  $B_i[a_i, \phi_i(q_1, q_2)]$ . The function  $\phi_i$  can be thought of as a monetary payoff from second stage production net of costs. There is a disutility associated with ex-ante investment, which is given by  $v_i(a_i)$ .<sup>3</sup> All costs and benefits are measured in date 1 dollars. The benefits and costs are the same under any ownership structure. Moreover, ownership does not provide any additional benefit.

None of the variables  $a_i$ ,  $q_i$  and  $B_i$  is contractible ex-ante. We assume that the non-contractibility of the variables arises either as a result of high transaction costs associated with writing comprehensive contracts, or because of enforcement problems. We regard  $a_i$  as the non-verifiable managerial effort which is non-contractible because of the enforcement problem. The variable  $q_i$  is ex-ante non-contractible because it stands for complex production decision and it is difficult to describe ex-ante.

Since the decision variables are ex-ante non-contractible, the date 0 contract can only allocate ownership rights between the two agent. Ownership of an asset grants the beholder the right to use it in any way he desires unless these rights are contracted away. In GH's terminology, the owner of the asset has the *residual control rights* over that asset.<sup>4</sup> Although  $q_i$  is ex-ante non-contractible, once the state of the world is observed,  $q_i$  becomes contractible and the owner of firm  $i$  may give up his residual control rights in exchange of a side-payment.

A summary of the sequence of events is as follows. At date 0, a

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<sup>3</sup>In GH model,  $B_i[a_i, \phi_i(q_1, q_2)]$  denotes the benefits net of all costs including the disutility of *ex ante* investment. In our model we preferred to separate the disutility from the benefit function to simplify the analysis. This structural change in the payoff function would not change the GH result.

<sup>4</sup>Note that in this model, financial returns are not transferable with ownership. For an example of this, see Holmstrom and Tirole (1989).

contract is signed. After that,  $a_1$  and  $a_2$  are chosen simultaneously and independently. At date 1, each agent learns the amount invested by his opponent. Before the actual choices of  $q_i$  are made they become contractible. If there is no further negotiation, the agent who owns firm  $i$  chooses  $q_i$  independently. The second stage decision,  $q$ , however, becomes contractible at date 1. Thus, a new contract may be negotiated that implements different choices of  $q_1$  and  $q_2$ , and specifies how the surplus is divided. Then  $B_1$  and  $B_2$  are realized and the necessary transfers are made between the two agents according to the new contract.

The following technical assumptions guarantee that the optimization problems have unique solutions and first-order necessary conditions are sufficient. We assume that  $B_i [a_i, \phi_i(q_1, q_2)]$  and  $v_i(a_i)$  are twice continuously differentiable and satisfy the following assumptions for all  $a_i \in A_i$  and  $q_i \in Q_i$ .

Assumption 1:  $B_i(\cdot)$  is increasing in  $\phi_i$  and  $a_i$ .  $B_1[\cdot] + B_2[\cdot]$  is strictly concave in its four arguments,  $(a_1, a_2, q_1, q_2)$ .

Assumption 2: The cost function  $v_i(a_i)$  is increasing and convex in  $a_i$ .

Assuming that monetary transfers between agents are available, the optimal contract maximizes the total ex-ante net benefits of the two agents

$$W = B_1 [a_1, \phi_1 (q_1, q_2)] + B_2 [a_2, \phi_2 (q_1, q_2)] - v_1 (a_1) - v_2 (a_2). \quad (1)$$

If we assume that  $a_1$  and  $a_2$  are verifiable, and  $q_1$  and  $q_2$  are ex-ante contractible, the first best solution which is obtained by maximizing (1) with respect to  $a_i$  and  $q_i$  for  $i = 1, 2$ , can be implemented. We denote  $a_1^F$ ,  $a_2^F$ ,  $q_1^F$ , and  $q_2^F$  as the unique maximizers of  $W$  subject to  $a_i \in A_i$ , and  $q_i \in Q_i$  for  $i = 1, 2$ .

Since we have assumed that all date 1 variables are non-contractible as of date 0, the first-best cannot be implemented. The initial contract only allocates ownership rights over firms' assets. There are three cases to consider. In the first case which we call non-integration, the firms are separately owned. In the second and third cases the firms are integrated under the ownership of a single agent, 1 and 2 respectively.<sup>5</sup>

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<sup>5</sup>We perceive ownership as a discrete variable which takes the value either 0 or 1 for each agent. Either agent 1 or agent 2 owns the firm. Two agent cannot own the same firm at the same time. Therefore we do not consider any type of joint ownership structure.

### 3 Non-integration

We solve for the subgame perfect Nash equilibrium of the full game which is characterized by a vector of  $(a, q) \in A \times Q$  and transfer payments. Each vector  $a = (a_1, a_2)$  induces a proper subgame where agent 1 and 2 bargain over the division of total surplus. We call these subgames as the induced bargaining subgames. In the next section, we characterize the equilibrium payoffs in these bargaining subgames.

#### 3.1 The induced bargaining subgames

In the case where the firms are separately owned, agent  $i$  has the right to choose  $q_i$ . At date 1, the two agents choose  $q_1$  and  $q_2$  to maximize  $B_1[a_1, \phi_1(q_1, q_2)]$  and  $B_2[a_2, \phi_2(q_1, q_2)]$ , respectively.<sup>6</sup> We assume that there exists a unique Nash equilibrium to the simultaneous  $q$ -choice subgame which is

$$\begin{aligned}\hat{q}_1 &= \arg \max_{q_1 \in Q_1} \phi_1(q_1, \hat{q}_2) \\ \hat{q}_2 &= \arg \max_{q_2 \in Q_2} \phi_2(\hat{q}_1, q_2).\end{aligned}\tag{2}$$

In general, the non-cooperative solution  $(\hat{q}_1, \hat{q}_2)$  is ex-post inefficient.<sup>7</sup> Therefore the two parties can gain from negotiating a new contract that specifies  $(q_1(a), q_2(a))$  as the actions to be taken, where

$$(q_1(a), q_2(a)) = \arg \max \{B_1[a_1, \phi_1(q_1, q_2)] + B_2[a_2, \phi_2(q_1, q_2)]\}\tag{3}$$

is the equilibrium of the cooperative  $q$ -choice subgame. The vector of equilibrium actions  $(q_1(a), q_2(a))$  is unique given that  $B(\cdot)$  function is concave. The new contract is feasible, since  $q_1$  and  $q_2$  are ex-post contractible. Let  $B[a, q(a)]$  denote the value function of this problem. The division of  $B[a, q(a)]$  among the two agents is determined by an alternating offers bargaining game. In the next section we explain the details of the game.

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<sup>6</sup>We assume that the  $q$ -choice game is a Cournot game. If agent  $i$  chooses to quit the bargaining game first and implement his outside option, he chooses  $q_i$  to maximize  $B_i[a_i, \phi_i(a_1, a_2)]$ . Agent  $j$  chooses  $q_j$  before she observes  $q_i$ . Thus, in the  $q$ -choice game  $q_1$  and  $q_2$  are chosen simultaneously. Alternatively, we could model the  $q$ -choice game as a Stackelberg game. If, in the bargaining game,  $q_i$  becomes observable before  $q_j$  is chosen, in the  $q$ -choice game  $q_1$  and  $q_2$  are chosen sequentially. This modification, however, would not change the results obtained.

<sup>7</sup>The noncooperative choices are efficient when  $\phi_i$  is a function of only  $q_i$  or when  $\phi_i = \phi_j$ , that is the both agents have the same payoff function.

### 3.1.1 Bargaining Game

In the GH model the solution to the contract negotiation is characterized by the Nash bargaining solution where the status quo payoffs are treated as disagreement payoffs. In this equilibrium each agent receives half of the increase in the total surplus which can be written as

$$\mathcal{B}_i = \frac{1}{2} \left[ B[a, q(a)] - B_i(a_i, \hat{\phi}_i) - B_j(a_j, \hat{\phi}_j) \right] \quad (4)$$

Such an equilibrium can be obtained as the solution to an alternating offers bargaining game where each agent receives the status quo payoffs at each period in which the agreement has not been reached. However if one considers a bargaining game where the agents do not receive any income flow until they reach an agreement or there is no exogenous risk of breakdown then it is more natural to treat the status quo payoffs as outside options. While a disagreement payoff directly influence the division of the surplus in the equilibrium, an outside option influences the division of the surplus only when it is a credible threat. In other words, if a player obtains a higher payoff from exercising his outside option than the equilibrium payoff he receives when he continues to bargain, his outside option constitutes a credible threat. Otherwise, quitting is not a credible threat. In the former case, he should at least receive the value of his outside option in any subgame-perfect equilibrium of the bargaining game. In the latter case, his outside option does not influence the equilibrium of the game.<sup>8</sup>

**Lemma 1** *Given the initial ownership structure, and the vector  $a = (a_1, a_2)$  of ex-ante investment levels, the induced bargaining subgame has a unique equilibrium in which the agreement is reached immediately, and firm 1 receives  $p$ , given by*

$$p = \begin{cases} B[a, q(a)] & \text{if } B_1(a_1, \hat{\phi}_1), B_2(a_2, \hat{\phi}_2) \leq \frac{B[a, q(a)]}{2} \\ B_1(a_1, \hat{\phi}_1) & \text{if } B_1(a_1, \hat{\phi}_1) > \frac{B[a, q(a)]}{2} \\ B[a, q(a)] - B_2(a_2, \hat{\phi}_2) & \text{otherwise.} \end{cases}$$

**Proof.** See Shaked and Sutton (1984). ■

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<sup>8</sup>This issue has been discussed by Binmore, Shaked and Sutton (1989). See also Sutton (1986), and Shaked and Sutton (1984).

When both outside options are small relative to the “*split-the-surplus*” solution, as in the first case, both agents prefer to continue bargaining than quitting. This would generally be the case when the surplus created by cooperation is large. In the second case agent 1 quits because he receives greater payoff in the status quo than if they split the surplus. In other words, his outside option imposes a credible threat, so that he receives his outside option in a perfect equilibrium. In the third case, agent 2 prefers quitting. She receives a share equal to her outside option, while agent 1 claims the residual. In general, when  $\delta \in (0, 1)$ , regardless of the preferences of agent 1, agent 2 has the first mover advantage in using her outside option as a credible threat. When  $\delta$  approaches 0 this advantage disappears. When agent 1’s outside option is binding, agent 2’s outside option cannot be binding. This contradicts with the assumption that cooperation generates greater surplus.

As opposed to GH’s “*split-the-difference*” solution, we find that the outside option has no effect on the bargaining outcome if it does not constitute a credible threat. Since the optimal allocation of ownership rights heavily depends on the outcome of the negotiation, the way outside option is incorporated into the model is critical.

Given the initial ownership structure and the ex-ante choice of  $(a_1, a_2)$ , we let  $\Pi_i(a_1, a_2)$  denote the overall payoff to agent  $i$  obtained from the induced bargaining subgame. In the rest of the paper, we analyze the game from agent  $i$ ’s perspective, where  $j$  denotes

$$H_i(a_1, a_2) = B[a, q(a)] - B_j(a_j, \hat{\phi}_j) \quad (5)$$

as the agent  $i$ ’s residual payoff after paying the agent  $j$  the value of her outside option, and

$$C_i(a_1, a_2) = \frac{B[a, q(a)]}{2} \quad (6)$$

as agent  $i$ ’s share in the “*split-the-surplus*” solution. Then, using Lemma 1 we obtain

$$\Pi_i(a_1, a_2) = \begin{cases} H_i(a_1, a_2) - v_i(a_i) & \text{if } j\text{'s o. o. is binding,} \\ C_i(a_1, a_2) - v_i(a_i) & \text{if neither o. o. are binding,} \\ B_i(a_i, \hat{\phi}_i) - v_i(a_i) & \text{if } i\text{'s o. o. is binding.} \end{cases} \quad (7)$$

There is a qualitative difference in the way the ex-ante investments affect the payoffs of the parties in this model compared to the GH model.

In the GH model, the two agent receives the “*split-the-difference*” payoff in the equilibrium. As the opponent’s action changes agent  $i$  responds by maximizing the “*split-the-difference*” payoff. In our model, the opponent’s level of investment first determines the payoff function that agent  $i$  is facing. Then it influences the value of this function. We first note that the opponent’s investment does not affect the value of agent  $i$ ’s outside option because the second period payoff  $\widehat{\phi}_i$  is independent of ex-ante investment choices of both agents. Then we fix an  $a_j$ , such that agent  $i$ ’s outside option gives him the highest payoff. As we increase the opponent’s investment, agent  $i$ ’s response remains constant until the “*split-the-surplus*” payoff becomes equal the value of his outside option. At this point, agent  $i$  is indifferent between maximizing the value of his outside option and the “*split-the-surplus*” payoff. As the opponent’s investment continues to increase it becomes more profitable for agent  $i$  to maximize the “*split-the-surplus*” payoff until the region where the opponent’s outside option is binding is reached. From this point on, the agent responds by maximizing the residual payoff. It is worth to note that the status quo payoffs do not affect the agents’ payoffs when neither of the firm’s outside option is binding because both receive the “*split-the-surplus*” payoff. On the other hand, in a region where the opponent’s outside option is binding, the status quo payoff both influences the level of payoff agent  $i$  receives and constrains the validity of the payoff function.

In finding the agents’ response functions, we first need to characterize the three regions of interest in  $\Pi_i(a_1, a_2)$  as a function of  $a$ . However, in doing that we assume that firms are symmetric in order to simplify the calculations.

**Lemma 2** *If  $C_i(0, 0) > B_i(0, \widehat{\phi}_i)$  and*

$$\max_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} < 2 \min_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} \quad (8)$$

*for every  $a_i \in A_i$ , then there exists a monotonically increasing function  $\alpha_i : A_j \rightarrow A_i$  such that*

- i.  $j$ ’s outside option is binding if  $a_i \leq \alpha_j^{-1}(a_j)$ ,*
- ii. neither outside option is binding if  $\alpha_j^{-1}(a_j) < a_i \leq \alpha_i(a_j)$*

iii.  $i$ 's outside option is binding if  $\alpha_i(a_j) < a_i$ .

**Proof.** See Appendix 1. ■

The first assumption of the Lemma 2 is automatically satisfied when the firms are symmetric. Otherwise, cooperation generates a smaller total surplus than non-cooperation. The second assumption requires that the marginal benefit from  $a_i$  does not change much with  $\phi_i$ . In other words, the marginal private benefit of ex-ante investment must not be very sensitive to the second period payoff.

(Insert Fig.1)

The  $\alpha$  function divides the  $(a_1, a_2)$  plane into three regions as in Figure 1. In the northwest corner, agent 1's outside option is binding, in the southeast corner, agent 2's outside option is binding and in the between region, neither agent's outside option is binding. On the  $45^\circ$  line both agents invest the same amount,  $a_1 = a_2$ . Given that they are symmetric, the non-cooperative choices of  $q$ 's will be the same, so will be the value of the status quo payoffs. If agent 1's outside option is binding then agent 2's outside option has to be binding because of symmetry. Both outside options, however, cannot be binding at the same time. Therefore, on the  $45^\circ$  line neither outside options are binding. We now consider keeping  $a_2$  at the same level as before but increasing  $a_1$ . Since  $B_1(\cdot)$  is increasing in  $a_1$ , if we increase  $a_1$  enough we reach to a point where agent 1's outside option is just binding. That's why the region where agent 1's outside option is binding should be on the northwest corner. The similar argument applies for agent 2; the region where her outside option is binding should be on the southeast corner. Thus, we have

**Claim 3**  $\alpha_i(a_j) > a_i$ . *The area in which agent  $i$ 's outside option is binding always lies above  $45^\circ$  line.*

## 4 Equilibria to the investment-choice game

Given the solution to the induced bargaining subgame, we have defined  $\Pi_i(a_1, a_2)$  as the reduced form payoff to the bargaining subgame. The ex-ante investments  $a_1$  and  $a_2$  are chosen simultaneously and independently at date 0 taking into account the outcome of the negotiation

between agents 1 and 2. Given the reduced form payoffs obtained from bargaining subgame, the Nash equilibrium to the investment choice game is a perfect subgame Nash equilibrium of the full game. We concentrate on the investment-choice game and characterize its equilibrium. A Nash equilibrium in date 0 investments is a pair  $(a_1^N, a_2^N) \in A_1 \times A_2$  such that

$$\begin{aligned} \Pi_1(a_1^N, a_2^N) &\geq \Pi_1(a_1, a_2^N) \text{ for all } a_1 \in A_1 \\ \Pi_1(a_1^N, a_2^N) &\geq \Pi_1(a_1^N, a_2) \text{ for all } a_2 \in A_2. \end{aligned} \quad (9)$$

We introduce some further assumptions into the model before we proceed.

Assumption 3:  $q_1$  and  $q_2$  are complementary activities.  $\phi_i$  is increasing in  $q_j$ .

Assumption 4: The marginal benefit of  $a_i$  is increasing in second period payoff,  $\phi_i$ .

We first derive the agents' response functions. We define  $\rho_i : A_j \rightarrow A_i$  to be agent  $i$ 's response function, where

$$\rho_i(a_j) = \arg \max_{a_i \in A_i} \Pi_i(a_1, a_2). \quad (10)$$

Since  $\Pi_i(a_1, a_2)$  depends on the region of choice space considered, it is convenient to separately analyze these regions, find the optimal action in each, and then determine the optimal action which maximizes the overall payoff.

#### 4.1 Agent $i$ 's best response when his outside option is binding

In the region where agent  $i$ 's outside option is binding his best response is defined as

$$\beta_i(a_j) = \max \{\hat{a}_i, \alpha_i(a_j)\} \quad (11)$$

where

$$\hat{a}_i = \arg \max_{a_i \in A_i} \left\{ B_i(a_i, \hat{\phi}_i) - v_i(a_i) \right\}. \quad (12)$$

that is  $\hat{a}_i$  is agent  $i$ 's optimal investment choice when the initial contract is not renegotiated. Given the non-cooperative choices of  $(\hat{q}_1, \hat{q}_2)$ , agent  $i$  chooses  $a_i$  to maximize his net benefit.

Let  $\bar{a}_j \in A_j$  be defined as,  $\alpha_i(\bar{a}_j) = \hat{a}_i$ . It is the level of ex-ante investment made by agent  $j$  so that agent  $i$ 's outside option is just

binding at its optimum. We assume that  $\widehat{a}_i > \alpha_i(0)$  so that there indeed exists an  $\bar{a}_j > 0$ . Now we can rewrite  $\beta_i(a_j)$  as

$$\beta_i(a_j) = \begin{cases} \widehat{a}_i & \text{if } a_j \leq \bar{a}_j \\ \alpha_i(a_j) & \text{if } a_j > \bar{a}_j. \end{cases} \quad (13)$$

For  $a_j \leq \bar{a}_j$ , agent  $i$ 's outside option is binding hence he chooses  $\widehat{a}_i$ . For  $a_j > \bar{a}_j$ , where his outside option is not binding at its optimum, agent  $i$  chooses  $\alpha_i(a_j)$  so that his outside option just binds.

## 4.2 Agent $i$ 's best response when neither outside option is binding

In the region where neither agent's outside option is binding, agent  $i$ 's best response is defined as

$$\eta_i(a_j) = \operatorname{argmax}_{\alpha_j^{-1}(a_j) < a_i \leq \alpha_i(a_j)} \{C_i(a_1, a_2) - v_i(a_i)\}. \quad (14)$$

In this case, the agents share the total surplus, thus agent  $i$  maximizes half of the surplus net of cost of ex-ante investment. We define

$$\delta_i(a_j) = \operatorname{argmax}_{a_i \in A_i} \{C_i(a_1, a_2) - v_i(a_i)\} \quad (15)$$

as the  $i$ 's best response to the unconstrained maximization problem. By applying the implicit function theorem we can easily show that

**Claim 4** *If  $\partial q_i(a_i) / \partial a_j > 0$  for  $i, j = 1, 2$ , then  $\delta_i(a_j)$  is increasing in  $a_j$ .*

Next we show that

**Claim 5**  *$\delta_i(a_j) < \widehat{a}_i$  for all  $a_j$ . The best response to the “split-the-surplus” payoff is always smaller than the best response to the status quo payoff.*

**Proof.** See Appendix 2. ■

Essentially, ex-ante investment,  $a$ , increases the value of the second period payoff,  $\phi$ . When the agent receives the status quo payoff he obtains the full benefit of his actions so he has greater incentive to invest.

When, however, he receives half of the total surplus, he receives only half of the benefit so his incentive to invest is distorted downwards.

We have defined  $\eta_i(a_j)$  as the best response to  $C_i(a_1, a_2) - v_i(a_i)$  when  $\alpha_j^{-1}(a_j) \leq a_i \leq \alpha_i(a_j)$ . Next we define the critical values of  $a_j$  within which  $\delta_i(a_j)$  is the relevant response function.

Let  $a'_j \in A_j$  such that  $\delta_i(a'_j) = \alpha_i(a'_j)$ , and  $a''_j \in A_j$  such that  $\delta_i(a''_j) = \alpha_j^{-1}(a''_j)$ . That is,  $a'_j$  is the level of ex-ante investment of agent  $j$  where agent  $i$ 's outside option is just binding when he maximizes the “*split-the-surplus*” payoff. We assume that  $\delta_i(0) > \alpha_i(0)$  so that there exists  $a'_j$ . The critical value  $a''_j$ , on the other hand, is the level of ex-ante investment of agent  $j$  at which his outside option is just binding when agent  $i$  maximizes the “*split-the-surplus*” payoff. In other words,  $a'_j$  and  $a''_j$  limit the range where  $\delta_i(a_j)$  is valid. Since  $\delta_i(a_j)$  is increasing in  $a_j$ , then  $a'_j < a''_j$ .

Now we can rewrite agent  $i$ 's best response when neither outside options are binding as

$$\eta_i(a_j) = \begin{cases} \alpha_i(a_j) & \text{if } a_j > a'_j \\ \delta_i(a_j) & \text{if } a'_j \leq a_j \leq a''_j \\ \alpha_j^{-1}(a_j) & \text{if } a''_j \leq a_j. \end{cases} \quad (16)$$

When agent  $j$  invests at small levels,  $a_j < a'_j$ , agent  $i$  chooses along  $\alpha_i(a_j)$  so that his outside option just binds. For  $a'_j < a_j < a''_j$ , he chooses  $\delta_i(a_j)$ , where neither of the firm's outside option is binding. When agent  $j$  invests at large levels,  $a_j > a''_j$ , agent  $i$  chooses along  $\alpha_j^{-1}(a_j)$  so that the opponent's outside option just binds.

### 4.3 Agent $i$ 's best response when opponent's outside option is binding

In the region where agent  $j$ 's outside option binds,  $i$ 's best response is defined as

$$\xi_i(a_j) = \arg \max_{\alpha_j^{-1}(a_j) \geq a_i} \{H_i(a_1, a_2) - v_i(a_i)\}. \quad (17)$$

Agent  $i$ 's outside option does not bind whenever agent  $j$ 's outside option binds. Agent  $i$  claims the residual and chooses  $a_i$  to maximize the total surplus net of the cost of ex-ante investment and the payment to the agent  $j$ . We define

$$\epsilon_i(a_j) = \arg \max_{a_i \in A_i} H_i(a_1, a_2) - v_i(a_i) \quad (18)$$

to be the maximizer of the unconstrained problem. By applying the implicit function theorem we can easily show that

**Claim 6**  $\epsilon_i(a_j)$  is increasing in  $a_j$ .

Let  $\tilde{a}_j \in A_j$  such that  $\alpha_j^{-1}(\tilde{a}_j) = \epsilon_i(\tilde{a}_j)$ . Here  $\tilde{a}_j$  is the level of ex-ante investment of agent  $j$  at which his outside option just binds when agent  $i$  maximizes the “*split-the-surplus*” payoff. Hence, we rewrite agent  $i$ ’s response function when agent  $j$ ’s outside option is binding as

$$\xi_i(a_j) = \begin{cases} \alpha_j^{-1}(a_j) & \text{if } a_j > \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } a_j < \tilde{a}_j. \end{cases} \quad (19)$$

For small  $a_j$ ’s agent  $i$  chooses along  $\alpha_j^{-1}(a_j)$  to make  $j$ ’s outside option just binding. For large  $a_j$ ’s, he chooses  $\epsilon_i(a_j)$ .

#### 4.4 The best-response function

Now we evaluate the payoff function  $\Pi_i(a_1, a_2)$  at the optimum of each region and compare them to find the best response function of agent  $i$ . We have shown that  $\hat{a}_i > \delta_i(a_j)$  for all  $a_j$ . The function  $C_i(a_i, a_j) - v_i(a_i)$  reaches its maximum at  $\delta_i(a_j)$ . Then, the function  $C_i(a_i, a_j) - v_i(a_i)$  must be decreasing for all  $a_i > \delta_i(a_j)$ , so it is for  $\hat{a}_i$ , that is  $\partial C_i(\hat{a}_i, a_j) / \partial a_i - \partial v_i(\hat{a}_i) / \partial a_i < 0$ . This implies that, when agent  $j$  invests at  $\bar{a}_j$ , the value of agent  $i$ ’s “*split-the-surplus*” payoff at its maximum,  $C_i(\delta_i(\bar{a}_j), \bar{a}_j) - v_i(\delta_i(\bar{a}_j))$ , is higher than the value of his outside option at its maximum,  $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$ . Since  $C_i(\cdot) - v_i(\cdot)$  is increasing in  $a_j$ , there exists a level of ex-ante investment, say  $\check{a}_j$ , which is lower than  $\bar{a}_j$  and at  $\check{a}_j$  agent  $i$  is indifferent between choosing  $\hat{a}_i$  or  $\delta_i(a_j)$ . In other words, at  $\check{a}_j$  agent  $i$ ’s outside option at its maximum just equals his “*split-the-surplus*” payoff evaluated at its maximum. Thus we have

**Definition 7** There exists  $\check{a}_j \in A_j$ , such that,

$$C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j)) = B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i).$$

We next need to locate  $\check{a}_j$ . Below we show that the jump in agent  $i$ ’s response function occurs at the region where his outside option is not binding.

**Claim 8**  $a'_j < \check{a}_j$ .

**Proof.** See Appendix 2. ■

We know that  $\widehat{a}_i$  is greater than  $\delta_i(a_j)$  for all  $a_j$ . Therefore,  $\delta_i(a_j)$  can only be equal to  $\alpha_i(a_j)$  when  $B_i(a_i, \widehat{\phi}_i) - v_i(a_i)$  is increasing. This means that  $C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j))$  is less than  $B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i)$ . In order to increase the value of  $C_i(\cdot) - v_i(\cdot)$  to be equal to  $B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i)$ ,  $a_j$  has to increase. Thus,  $a'_j < \check{a}_j$ . In other words, when agent  $j$ 's investment is small, agent  $i$  can obtain a higher payoff in status quo than the “*split-the-surplus*” payoff by investing at high levels. However, as agent  $j$ 's investment increases, “*split-the-surplus*” payoff increases because of the complementarity assumption and generates higher payoffs than the status quo payoff. Therefore, for small levels of  $a_j$ , agent  $i$  continues to choose  $\widehat{a}_i$  even though his outside option is not binding.

Whether  $\check{a}_j$  is greater or smaller than  $a''_j$  depends on the gains from cooperation. Here  $a''_j$  is the point where agent  $j$ 's outside option is just binding when agent  $i$  chooses  $\delta_i(a_j)$ . In other words,  $a''_j$  is the agent  $j$ 's investment level beyond which agent  $i$  chooses to maximize the residual. If  $\check{a}_j$  is smaller than  $a''_j$ , then agent  $i$  responds by choosing along  $\delta_i(a_j)$  for  $a_j \in [\check{a}_j, a''_j]$ . If  $\check{a}_j$  is greater than  $a''_j$  and agent  $i$  responds with  $\delta_i(a_j)$ , agent  $j$ 's outside option becomes binding that implies that agent  $i$  does not receive the “*split-the-surplus*” payoff but claims the residual. In fact, he maximizes his payoff if he continues to choose  $\widehat{a}_i$  for values of  $a_j < a^*_j$ . At  $a^*_j$ , the maximum value of agent  $i$ 's status quo payoff,  $B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i)$ , just equals the maximum value of his payoff when he receives the residual. For any value  $a_j \geq a^*_j$ , agent  $i$  responds by maximizing the residual,  $H_i(\cdot) - v_i(\cdot)$ .

**Definition 9** *There exists  $a^*_j \in A_j$ , such that,*

$$C_i(\alpha_j^{-1}(a^*_j), a^*_j) - v_i(\delta_i(a^*_j)) = B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i).$$

Before we present agent  $i$ 's response function it is important to note that both  $\check{a}_j$  and  $a^*_j$  are smaller than  $\bar{a}_j$ .

**Claim 10**  $\check{a}_j < \bar{a}_j$ .

**Proof.** See Appendix 2. ■

**Claim 11**  $a^*_j < \bar{a}_j$ .

**Proof.** See Appendix 2. ■

The above analysis is summarized in the following lemma that describes agent  $i$ 's response function.

**Lemma 12** *If  $\check{a}_j < a_j''$ , then agent  $i$ 's response function is*

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq \check{a}_j \\ \delta_i(a_j) & \text{if } \check{a}_j < a_j \leq a_j'' \\ \alpha_j^{-1}(a_j) & \text{if } a_j'' < a_j \leq \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } \tilde{a}_j < a_j \end{cases}.$$

*If  $\check{a}_j \geq a_j''$  then agent  $i$ 's response function is*

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq a_j^* \\ \alpha_j^{-1}(a_j) & \text{if } a_j^* < a_j \leq \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } \tilde{a}_j < a_j \end{cases}.$$

Agent  $i$  can have two types of response function depending on whether or not he switches from maximizing the status quo payoff to maximizing the “split-the-surplus” payoff in the region where the opponent’s outside option is binding. If  $\check{a}_j < a_j''$ , then the jump in the response function occurs in the region where agent  $j$ 's outside option is not binding. For small  $a_j$ , agent  $i$  chooses  $\hat{a}_i$ . At  $\check{a}_j$  there is a downward jump in the response function. From this point on, agent  $i$  chooses along  $\delta_i(a_j)$  until  $a_j''$  is reached. At  $a_j''$ , agent  $j$ 's outside option becomes binding. Agent  $i$  responds by choosing along  $\alpha_j^{-1}(a_j)$  so that agent  $j$ 's outside option just binds. After  $\tilde{a}_j$  is reached, agent  $i$  responds by choosing  $\epsilon_i(a_j)$  (see Figure 2 for the case of high degree of complementarity and Figure 3 for the case of low degree of complementarity).

(Insert Fig. 2 and Fig. 3)

If  $\check{a}_j \geq a_j''$ , that is, when the jump occurs in the region where agent  $j$ 's outside option is binding, agent  $i$  chooses  $\hat{a}_i$  for small  $a_j$ . For  $a_j \in [a_j^*, \tilde{a}_j]$  he responds along  $\alpha_j^{-1}(a_j)$  so that agent  $j$ 's outside option is just binding. For  $a_j > \tilde{a}_j$ , he responds along  $\epsilon_i(a_j)$  (see Figure 4). Whether  $\check{a}_j$  is smaller or greater than  $a_j''$  depends on the  $B$  and  $v$  functions.

(Insert Fig. 4)

In general, the game will have either a unique pure strategy Nash equilibrium in which both agents maximize the “*split-the-surplus*” payoff (this occurs if  $\check{a}_j < \delta_i(\check{a}_j)$ ) or no pure strategy equilibrium (when  $\check{a}_j > \delta_i(\check{a}_j)$ ). The following proposition describes the equilibrium of the investment-choice game.

**Proposition 13** *If there exists a Nash equilibrium to the investment-choice game in which neither agent’s outside option is binding, then it is the unique equilibrium (in pure strategies).*

**Proof.** Let  $(a_1^N, a_2^N)$  be the Nash equilibrium in which neither agent’s outside option is binding. First we show that regardless of the existence of  $(a_1^N, a_2^N)$ ,  $(\delta_i(\hat{a}_j), \hat{a}_j)$  and  $(\alpha_j^{-1}(\hat{a}_j), \hat{a}_j)$  cannot be equilibria. Since  $\bar{a}_j > \check{a}_j$ , it is also true by symmetry that  $\bar{a}_i > \check{a}_i$ . This implies that agent  $j$  switches to  $\delta_j(a_i)$  at some investment level,  $a_i$ , which is below  $\bar{a}_i$ . Therefore,  $\alpha_j^{-1}(a_j)$  never intersects the response function at  $\hat{a}_j$ . Moreover,  $\delta_j(a_i)$  is part of the response function when it is above  $\alpha_j^{-1}(a_j)$ . Since  $\hat{a}_j < \alpha_j^{-1}(\hat{a}_j)$ , then  $\hat{a}_j < \delta_i(\hat{a}_j)$ . Thus,  $(\delta_i(\hat{a}_j), \hat{a}_j)$  can never be an equilibrium, either. Given that  $(a_1^N, a_2^N)$  is the Nash equilibrium of the game, it must be true that  $\check{a}_i < \delta_i(\check{a}_j) < \delta_i(\hat{a}_j)$  since  $\delta_i(a_j)$  is monotonically increasing in  $a_j$ . It is also true that  $\delta_i(\hat{a}_j) < \epsilon_i(\hat{a}_j)$  which in turn implies that  $\hat{a}_j < \epsilon_i(\hat{a}_j)$ . Thus,  $(\epsilon_i(\hat{a}_j), \hat{a}_j)$  cannot be an equilibrium. ■

The uniqueness of the Nash equilibrium depends on the positive slope of the  $\delta_i(\cdot)$  function which arises from the complementarity assumption that we made. As  $a_j$  increases, there is a direct effect on  $C_i$ , but also an indirect effect since the second period payoff to both firms,  $\phi_i$ , increases in response to the increase in  $a_j$ . The increase in  $\phi_i$ , in return, causes  $a_i$  to increase.

Proposition 13 refers to the uniqueness, but not the existence of the pure strategy Nash equilibrium. In order to determine if the equilibrium exists we perform a comparative statics, the result of which we present its results in Section 7.

## 5 Integration (1’s Ownership)

We consider only agent 1’s ownership under integration since the case for agent 2’s ownership is symmetric. Under agent 1’s ownership, agent 1 owns both firms and agent 2 becomes his employee. Thus, agent 1

has the residual control rights over both firms' assets. In our model, this amounts to agent 1 choosing both  $q_1$  and  $q_2$  at date 1 under the provisions of the initial contract. It is, however, still necessary that both agents make the relationship-specific investment at date 0. Regardless of the ownership structure, each agent receives the full private benefit of the firm where they are employed. Being an owner does not change the structure of the payoff function or change the ex-ante distribution of surplus. Ownership only entitles the beholder the right to control assets in unspecified contingencies.

Besides having the right to choose both  $q_1$  and  $q_2$  at date 1, agent 1 is also the only agent in the bargaining game who can credibly use his outside option. The residual control rights give him the right to both choose and implement  $q_1$  and  $q_2$ . Agent 2 can bribe agent 1 to choose her favorite  $q$  but she cannot quit the bargaining game and implement the status quo choices of  $q_1$  and  $q_2$ . Essentially her outside option is not a credible threat in the bargaining game. Now the variable  $x_{ki}$  denotes the choice of agent  $i$  under  $k$ 's ownership.

At date 1, agent 1 chooses  $q_1$  and  $q_2$  to maximize  $B_1 [a_1, \phi_1 (q_1, q_2)]$ . We assume that there exists a unique equilibrium to the  $q$ -choice sub-game under 1's ownership. Let

$$(\widehat{q}_{11}, \widehat{q}_{12}) = \arg \max_{\substack{q_1 \in Q_1 \\ q_2 \in Q_2}} \phi_1 (q_1, q_2) \quad (20)$$

be the unique Nash equilibrium to this game. In general, the non-cooperative solution  $(\widehat{q}_{11}, \widehat{q}_{12})$  is ex-post inefficient. Therefore, the two parties can gain from negotiating a new contract. The rest of the analysis is similar to the case of non-integration. The payoff function for agent 1 is given by

$$\Pi_1 (a_1, a_2) = \begin{cases} C_1 (a_1, a_2) - v_1 (a_1) & \text{if neither o. o. is binding,} \\ B_1 (a_1, \widehat{\phi}_{11}) - v_1 (a_1) & \text{if 1's o. o. is binding,} \end{cases} \quad (21)$$

and for agent 2 it is

$$\Pi_2 (a_1, a_2) = \begin{cases} C_2 (a_1, a_2) - v_2 (a_2) & \text{if neither o. o. is binding,} \\ H_2 (a_1, a_2) - v_2 (a_2) & \text{if 1's o. o. is binding.} \end{cases} \quad (22)$$

The assumptions of Lemma 2 are sufficient to prove the existence of  $\alpha_{11} (a_2)$  which divides the space of  $(a_1, a_2)$  into two regions such that,

for  $a_1 > \alpha_{11}(a_2)$  agent 1's outside option is binding and  $a_1 < \alpha_{11}(a_2)$  it is not binding. The following lemma describes the agents' response functions under agent 1's ownership.

**Lemma 14** *Agent 1's response function is*

$$\rho_{11}(a_2) = \begin{cases} \hat{a}_{11} & \text{if } a_2 \leq \check{a}_{12} \\ \delta_{11}(a_2) & \text{if } \check{a}_{12} < a_2 \end{cases}.$$

*Agent 2's response function is*

$$\rho_{12}(a_1) = \begin{cases} \delta_{12}(a_1) & \text{if } a_2 \leq a''_{11} \\ \alpha_{11}^{-1}(a_1) & \text{if } a''_{11} < a_2 \leq \tilde{a}_{11} \\ \epsilon_{12}(a_1) & \text{if } \tilde{a}_{11} < a_2 \end{cases}.$$

**Proof.** See Proof of Lemma 12. ■

Under 1's ownership, both agents have a unique response function for any parameter values. This is because the agent 2's outside option is never binding. The jump in the response function always occurs at  $\check{a}_{12}$ . For small ex-ante investment levels, agent 1 responds by choosing  $\hat{a}_{11}$ . At  $\check{a}_{12}$ , there is a downward jump in the response function. For values greater than  $\check{a}_{12}$ , agent 1 chooses  $\delta_{11}(a_2)$ . Agent 2's response function is  $\delta_{12}(a_j)$  for small values of  $a_1$ . At  $a''_{11}$  there is a jump in her response function. For values greater than  $a''_{11}$ , agent 2 chooses  $\alpha_{11}^{-1}(a_1)$ , so that agent 1's outside option is just binding. For  $a_1 > \tilde{a}_{11}$ , she chooses  $\epsilon_{12}(a_1)$ .

(Insert Fig. 5)

As in the case of non-integration the game has either a unique pure strategy Nash equilibrium in which both agents maximize the “*split-the-surplus*” payoff or no equilibrium in pure strategies. The unique Nash equilibrium exists if  $\check{a}_{12} < \delta_{11}(\check{a}_{12})$ , that is, if the jump in agent 1's response function occurs to the left of 45° line (see Figure 5). An argument similar to that used in the proof of proposition 13 shows that if there exists a Nash equilibrium to the investment-choice game in which neither agent's outside option is binding, then it is a unique equilibrium in pure strategies.

## 6 Asset Ownership and Incentives to Invest

We examined two ownership types; non-integration and agent 1's ownership. In both cases the equilibrium to the investment-choice game is identical. Provided that it exists, there is a unique Nash equilibrium to the investment-choice game under both integration and non-integration in which neither agent's outside option is binding. Since the same equilibrium is obtained regardless of the initial distribution of the ownership rights we conclude that asset ownership does not affect the incentives to invest in the case considered. There is inefficiency in the model because of underinvestment. However, this inefficiency cannot be remedied by reallocating the ownership rights. In deriving the above result we have assumed that firms are symmetric. Now, instead, we consider cases where firms are asymmetric.

**Proposition 15** (A) *If  $\phi_i$  primarily depends on  $q_i$ , then optimal ownership structure is irrelevant. (B) If  $\phi_i$  hardly depends on  $q_1$  and  $q_2$  then  $j$ 's ownership is approximately first best ownership structure.*

**Proof.** First consider part (A). If there is no complementarity in ex-post production, that is  $\phi_1(\cdot)$  is only a function of  $q_1$  and  $\phi_2(\cdot)$  is only a function of  $q_2$  in the sense that  $\phi_i(\cdot) = f_i(q_i) + \epsilon_i g_i(q_j)$ ,  $i, j = 1, 2$  with  $i \neq j$  and  $\epsilon_i > 0$  is small, then under non-integration there is no room for negotiation since non-cooperative choices are almost identical to cooperative choices and ex-ante investments will be efficient. Under 1's ownership, 1's outside option is almost binding for any level of investment and 2 will be residual claimant. Under 2's ownership, 2's outside option is almost binding for any level of investment and 1 will be the residual claimant. In both cases, ex-ante investments are almost efficient because for the agent whose outside option is binding non-cooperative choice of  $q$  is almost identical to cooperative one and the other agent maximizes the residual. Therefore, the optimal ownership structure is irrelevant. In Part (B), let  $\phi_i(\cdot) = b_i + \epsilon_i h_i(q_i, q_j)$ ,  $i, j = 1, 2$  with  $i \neq j$  and  $\epsilon_i > 0$  is small. In this case, under  $j$ 's ownership there is almost no room for negotiation and ex-ante investments are almost efficient. Under  $i$ 's ownership  $j$  is willing to pay  $i$  to implement first best choices. In this case, if neither players outside option is binding players receive “*split-the-surplus*” payoffs and ex-ante investments are inefficient. If  $i$ 's outside option is binding then  $j$  is the residual claimant. In this case  $j$ 's ex-ante

investment is efficient and  $i$ 's is almost efficient. Under non-integration, there is still room for negotiation and the result is similar to the case of  $i$ 's ownership. Therefore  $j$ 's ownership is always approximately the first best ownership structure. ■

Result (A) contrasts with GH who finds that if there is no complementarity in production, asset ownership must be given to the agent who undertakes the ex-ante investment. Result (B) is in line with their conclusion.

Now consider the case where  $\phi_1(\cdot)$  is independent of  $q_2$ , while  $\phi_2(\cdot)$  depends on  $q_1$  as well as  $q_2$ . There is room for negotiation under any ownership structure because the non-cooperative choices of  $q$  differ from the cooperative choices. However, the difference in the surplus created by cooperation differs depending on the initial allocation of ownership. In particular, the difference in the surplus is bigger the more distorted the ex-post production choices are for any level of ex-ante investments. The bigger the difference the more likely that outside options are not binding. If  $q_1$  is not important for firm 2 then we have a situation similar to part (A) of Proposition 15 above. If, however,  $q_1$  is important for firm 2, then it is more likely that firm 2's outside option is binding under firm 2's ownership than under firm 1's ownership. When firm 2's outside option is binding firm 1 becomes residual claimant. If firm 1's investment is more productive and ex-ante investments are more important than ex-post production decisions, then making firm 1 residual claimant, hence, giving asset ownership to firm 2 is better. If, however, firm 2's ex-ante investment is more important than firm 1's ownership may be optimal. While this result contradicts with GH, it is line with DLC in that disowning an asset may give more incentives to invest in a relationship. For, by removing an asset, bargaining power of the significant player is reduced while the bargaining power of the insignificant one is increased, hence the significant player becomes residual claimant and his ex-ante investment is less distorted.

We stress the distinct roles played by production complementarity ( $\partial\phi_i/\partial q_j$ ) and productivity of ex-ante investment ( $\partial^2 B_i/\partial a_i \partial \phi_i$ ) in determining the optimal ownership structure. The production complementarity essentially determines whether outside option is binding or not. If production complementarity is not significant then the initial distribution of asset ownership does not change the outcome of ex-post bargaining significantly. However, if production complementarity is im-

portant, then ex-post negotiation may improve the surplus created. It is then the initial allocation of ownership that determines the magnitude of the improvement and whether outside options are binding or not. The optimal allocation of ownership is determined by the relative significance of production complementarity and productivity of the ex-ante investments. If the degree of complementarity is high then we have the Coasean result. Hence the model predicts the irrelevance of the ownership structure in a vertical structures where the production complementarity and investment productivity is equally important to both parties.

So far we have we have shown that the investment-choice game has a unique Nash equilibrium if it exists. In order to determine if the equilibrium exists we perform a comparative statics, the result of which we present in the next section.

## 7 On the Existence of the Equilibrium

It is apparent that the divergence between the cooperative and non-cooperative equilibria is completely driven by the interdependency in the second period production. The extent to which total surplus can be increased through negotiation depends on the degree of complementarity. We introduce a parameter into the second period payoff function  $\phi$  to capture the level of complementarity among the two firms.<sup>9</sup> Let  $\gamma$  be an index of complementarity where  $\gamma \in [0, 1]$ . When  $\gamma = 0$  there is no production complementarity. In that case, the second period payoff function  $\phi_i$  depends solely on  $q_i$ . As  $\gamma$  increases the degree of complementarity in the production of the two firms increases. In the case when there is no complementarity, the cooperative and non-cooperative solutions are identical. Thus there is no need for renegotiation. As a benchmark, we first examine the equilibrium when there is no complementarity, i.e.  $\gamma = 0$ . We then analyze how the equilibrium evolves as complementarity is introduced. We make following assumptions:

Assumption 5:  $\partial q_i(a) / \partial \gamma > 0$  for  $i = 1, 2$ . The cooperative choice of ex-post production increases as complementarity increases.

Assumption 6:  $\partial B_i(a_i, \hat{\phi}_i) / \partial \gamma < \partial C_i(a_1, a_2) / \partial \gamma$ , i.e., as the complementarity between the two firm's production increases the “*split-the-surplus*” payoff increases by more than the non-cooperative payoff.

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<sup>9</sup>In this section we restrict the analysis to the symmetric firms case.

## 7.1 No Complementarity (Non-integration Case)

Assume that  $\gamma = 0$ , that is, the second period payoff is independent of the opponent's production decision. Then the optimal cooperative and non-cooperative choices of  $q_i$  are the same and in both cases the value of the second period payoff,  $\widehat{\phi}_i$  and  $\phi_i^c$  are identical. As a result, regardless of whether or not he cooperates, the payoff to agent  $i$  when he claims the residual is the same as the status quo payoff. Thus,  $H_i(\cdot) = B_i(a_i, \widehat{\phi}_i)$ , and they are maximized at the same level of ex-ante investment,  $\widehat{a}_i = \epsilon_i > \delta_i$ . Even though  $\delta_i$  is independent of the opponent's ex-ante investment level, it is still lower than  $\widehat{a}_i$  since in the "split-the-surplus" solution the agent does not receive the full benefit of his actions. In this non-complementarity case,  $\alpha_i(a_j) = \alpha_j^{-1}(a_j) = a_j$ , which implies that agent  $i$ 's outside option is binding in the area above 45° line while agent  $j$ 's is binding below. In other words there is no region in which neither of the agents' outside option is binding. Below we characterize the equilibrium to this game.

**Lemma 16** *If there is no complementarity between the two firms' production then  $(\widehat{a}_1, \widehat{a}_2)$  is the unique equilibrium of the above game.*

**Proof.** See Appendix 3 ■

Given that  $\check{a}_j$  is in the region where the opponent's outside option is binding, the jump must occur at  $a_j^*$ . Here  $a_j^*$  is equal to  $\widehat{a}_j$  because of the fact that  $H_i(\cdot) = B_i(a_i, \widehat{\phi}_i)$ . The best response of agent  $i$  is to always play  $\widehat{a}_i$ . In fact the jump in the response function is fictitious. Because of symmetry  $\widehat{a}_i$  intersects 45° line at  $\widehat{a}_j$ , so we have an equilibrium (see Figure 6).

(Insert Fig.6)

As we introduce complementarity, all the relevant functions and critical points in the response function change. By using the implicit function theorem it is easy to prove that  $\widehat{a}_i$ ,  $\delta_i(a_j)$ ,  $\alpha_i(a_j)$ , and  $\epsilon_i(a_j)$  increase as  $\gamma$  increases. In other words, as we introduce complementarity into the model, we obtain an area in which neither agents' outside option is binding.

We argue that for low levels of complementarity, there is no equilibrium in pure strategies. When  $\gamma$  is small, the response function is

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq a_j^* \\ \alpha_j^{-1}(a_j) & \text{if } a_j^* \leq a_j \leq \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } \tilde{a}_j \leq a_j. \end{cases}$$

When  $\gamma$  is zero,  $a_i''$  is smaller than  $\check{a}_i$ . Thus, for a small degree of complementarity  $a_i''$  is still smaller than  $\check{a}_i$  by continuity. For that reason, the best response function is the one above where the agent switches at  $a_i^*$  rather than  $\check{a}_i$ . The only possible equilibrium is the one in which neither parties' outside option is binding. Because of the reasons discussed in the proof of proposition 13, none of them chooses  $\alpha_j^{-1}(a_j)$  in the equilibrium. For small  $\gamma$ ,  $\tilde{a}_i$  is greater than  $a_i''$  which means that  $\epsilon_i(a_j)$  is a part of the response function when it is greater than  $\alpha_j^{-1}(a_j)$ . Since  $\alpha_j^{-1}(a_j)$  cannot be an equilibrium then  $\epsilon_i(a_j)$  cannot be an equilibrium, either. With small complementarity, however,  $\check{a}_i$  will be greater than  $\delta_i(\check{a}_j)$ . That implies that the jump in the response function occurs to the right of 45° line, so there does not exist an equilibrium where neither firm's outside option is binding.

**Proposition 17** *For low levels of complementarity, there is no equilibrium to the investment-choice game in pure strategies. If the complementarity between the two firm is sufficiently large, then there is a unique Nash equilibrium.*

**Proof.** In the case where there is no complementarity,  $\gamma = 0$ , the equilibrium to this game is  $(\hat{a}_1, \hat{a}_2)$ . Now suppose that we force the agents to receive the “split-the-surplus” payoff. The unique equilibrium of this forced game is  $(\delta_1, \delta_2)$ . Let  $\mathcal{A}_i$  be the payoff to agent  $i$  in this forced equilibrium and  $\mathcal{B}_i$  be the payoff to agent  $i$  from deviating to a point which enforces outside option.  $\mathcal{B}_i$  is greater than  $\mathcal{A}_i$  since  $B_i(\cdot)$  is increasing in  $a_i$  and  $\hat{a}_i > \delta_i$ . When the complementarity is small, an interior equilibrium, if it exists, has to be close to the equilibrium of the forced division game when there is no complementarity.<sup>10</sup> Let  $\mathcal{C}_i$  denote the payoff to agent  $i$  in an equilibrium where both agents receive the “split-the-surplus” payoff when  $\gamma > 0$ . Finally let  $\mathcal{D}_i$  denote the payoff to agent  $i$  from deviating to a point which enforces outside option. We

<sup>10</sup>It follows from that the response function has a closed graph.

know that  $\mathcal{B}_i$  is greater than  $\mathcal{A}_i$ .  $\mathcal{A}_i$  is close to  $\mathcal{C}_i$  and  $\mathcal{B}_i$  is close to  $\mathcal{D}_i$  which implies that  $\mathcal{D}_i$  is greater than  $\mathcal{C}_i$ . This implies that agent  $i$  has an incentive to deviate from the  $(\delta_1, \delta_2)$  equilibrium when there is small complementarity. Therefore  $(\delta_1, \delta_2)$  cannot be an equilibrium. There also cannot be an equilibrium where both agents' outside options are binding. Therefore there is no equilibrium when the firms' production exhibits small complementarity. The second part of the lemma is proved in Proposition 13. ■

The intuition behind proposition 17 is the following. When the agent receives the “*split-the-surplus*” payoff, his incentives are distorted downwards. If we keep the opponent's action fixed, it is profitable for the agent to deviate and choose  $\hat{a}_i$  to maximize the status quo payoff. This is true for both agents because of symmetry. We cannot, however, have an equilibrium where both agents' outside options are binding. If we do, this would imply that cooperation generates a smaller surplus than non-cooperation. In fact, the only case when we obtain an equilibrium where both outside options are binding is when there is no complementarity between the two firms. In this case, the surplus under cooperation and non-cooperation is identical. As complementarity increases, the agents' outside options become non-binding, so the deviations described above do not occur. Then the game has the unique equilibrium where neither of the agents' outside options are binding.

## 8 Concluding Remarks

In this paper we analyze the role of the initial allocation of ownership rights in transactions where parties make relationship-specific investments and the contracts are incomplete. We compare two ownership structures. First, we consider the case where the firms are separately owned by agent 1 and 2 respectively. Then we analyze the case where agent 1 owns both firms and agent 2 is employed in firm 2. In both cases, when firms are symmetric and the degree of complementarity between the two firms' production is high, cooperation generates large surplus. In this case, the investment-choice game has a unique Nash equilibrium where neither agents' outside option is binding. When the agents' outside options are not binding, agent 1 and 2 split the total surplus in the equilibrium of the bargaining game. Since we obtain the same equilibrium regardless of the ownership structure, the distortions in the ex-ante investments are independent of the initial allocation of ownership rights.

Thus, we conclude that the initial allocation of ownership rights does not lead to ex-ante inefficiencies in the production. If, however, the degree of complementarity between the two firms' production is low, then the equilibrium in pure strategies does not exist.

Our conclusion that the allocation of initial ownership rights is irrelevant partially extends the result of the Coase theorem to the relationships in which agents are unable to bargain ex-ante over all aspects of the transaction, due to contractual incompleteness. This irrelevance result also contrasts the results of GH who obtain that the initial allocation ownership rights have efficiency implications. They argue that even though ex-post bargaining is costless the impossibility of ex-ante bargaining leads to the inefficiencies by distorting parties' incentives to invest in the relationship. The ownership rights should be allocated to minimize these distortions. Thus, the Coase theorem fails to apply if there is contractual incompleteness. The critical element behind these two different results is that while GH model uses the Nash bargaining solution treating status quo payoffs as disagreement points, in our model they are treated as outside options. It is worthwhile to note that, there is an ex-ante inefficiency in our model, too. This inefficiency, however, does not arise from the initial allocation of ownership rights but as a result of free-rider problem.

In case of asymmetric relationship, we find that, as in DLC, there may be cases where taking away the assets from significant partner may boosts his incentives to invest. By removing an asset, bargaining power of the significant player is reduced while the bargaining power of the insignificant one is increased, hence the significant player becomes residual claimant and his ex-ante investment is less distorted. It is important to stress the distinct roles played by production complementarity and productivity of ex-ante investment in determining the optimal ownership structure. The production complementarity essentially determines whether the outside option is binding or not. If the production complementarity is not significant then the initial distribution of asset ownership does not change the outcome of ex-post bargaining significantly. However, if production complementarity is important, then optimal allocation of ownership is determined by the relative significance of production complementarity and productivity of the ex-ante investments.

An implicit assumption in our model is regarding the definition of ownership. We define ownership as the power to exercise control. It

would be interesting to examine the extent to which the irrelevance result depends on the definition of ownership. In other words, if we broaden this definition to include the rights to the residual income stream, does the irrelevance result continue to hold? Another assumption in the model is that the relationship lasts only two periods. If, however, the relationship lasts longer and the bargaining takes place concurrently with the production, our results may differ. In this case, the status quo payoffs become the income flow accruing to the agents in the course of the bargaining. Then status quo payoffs can be interpreted as the disagreement points. This bargaining game, however, may have many equilibria, some of which are inefficient (see Fernandez and Glazer (1991)).

## **9 Figures**

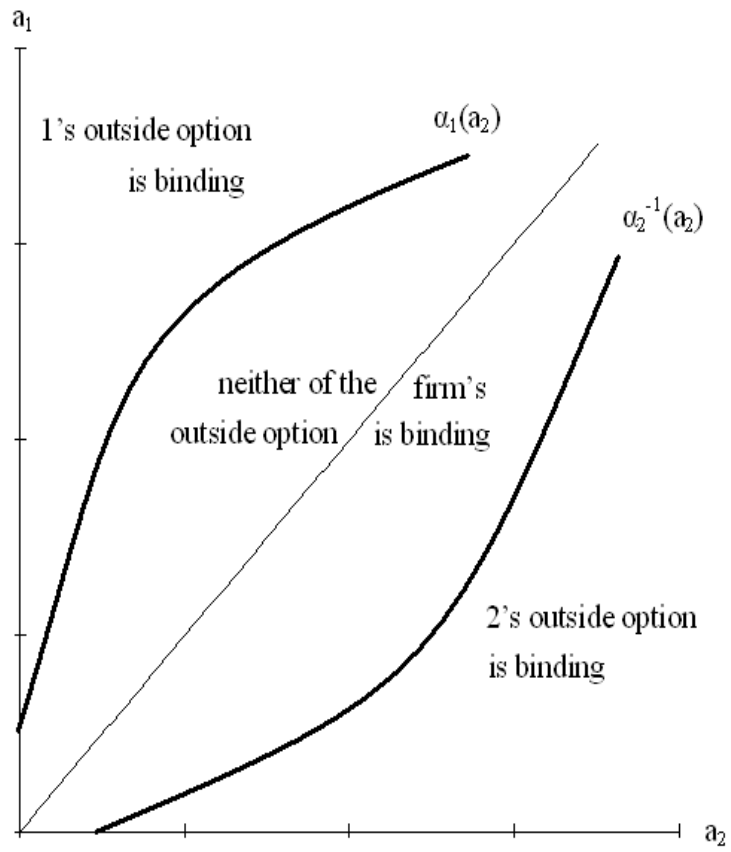


Figure 1: The functions  $\alpha_1(a_1)$  and  $\alpha_2^{-1}(a_2)$ .

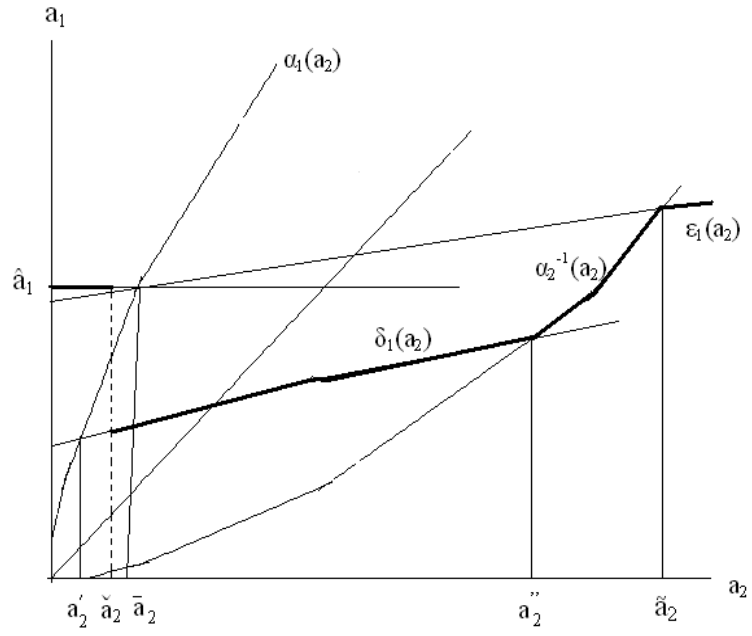


Figure 2: Agent 1's response function:  $\check{a}_2 < a_2''$  (the complementarity between  $q_1$  and  $q_2$  is large).

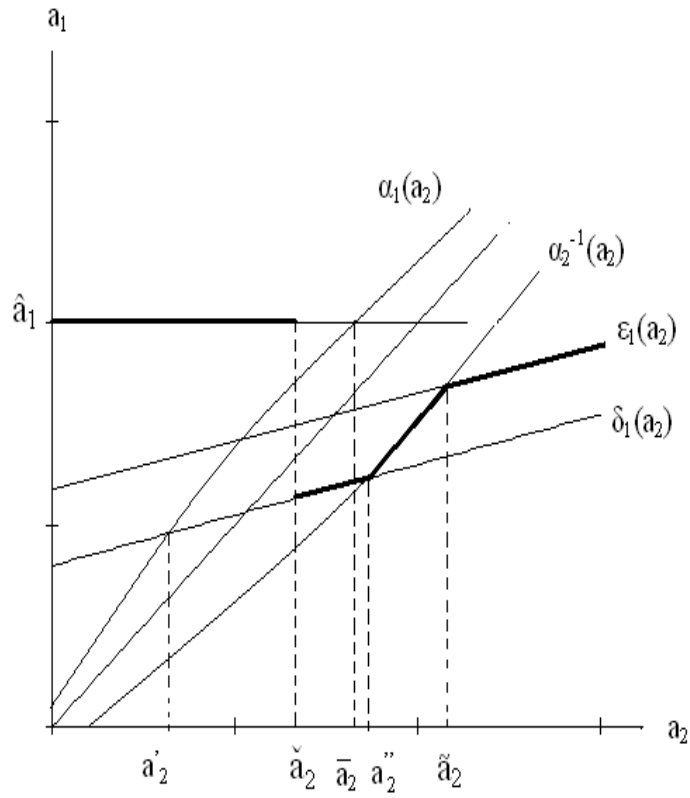


Figure 3: Agent 1's response function:  $\check{a}_2 < \check{\check{a}}_2$  (the complementarity between  $q_1$  and  $q_2$  is small).

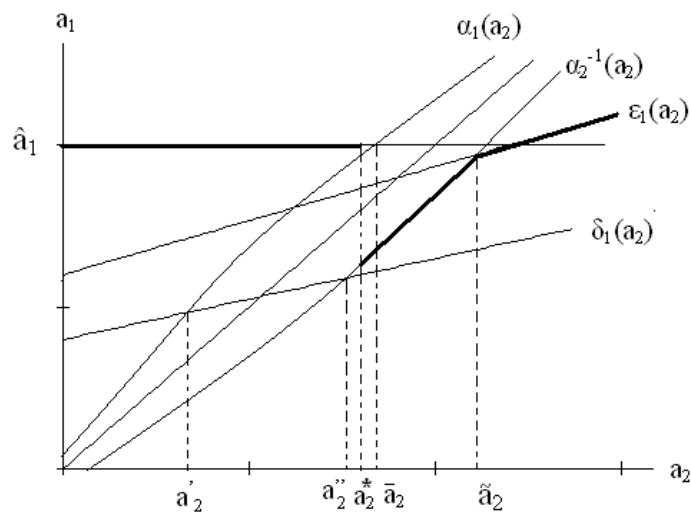


Figure 4: Agent 1's response function:  $\check{a}_2 > a_2''$  (the complementarity between  $q_1$  and  $q_2$  is small).

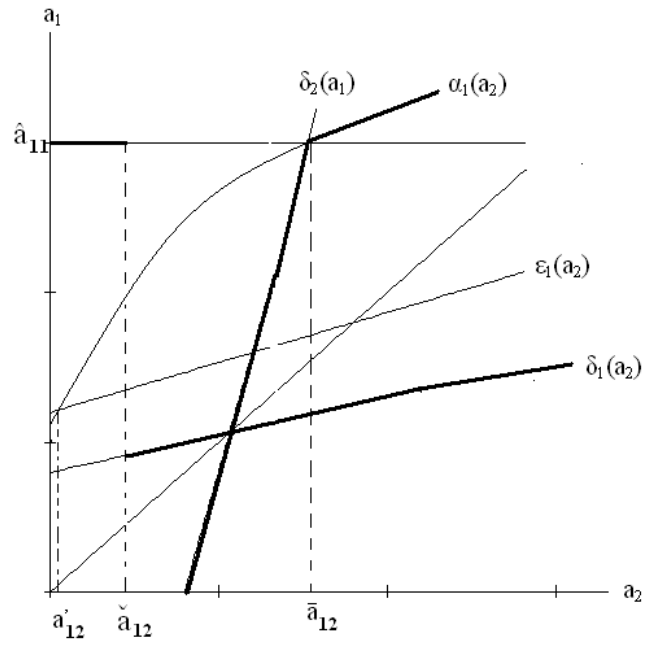


Figure 5: Player's response function under 1's ownership.

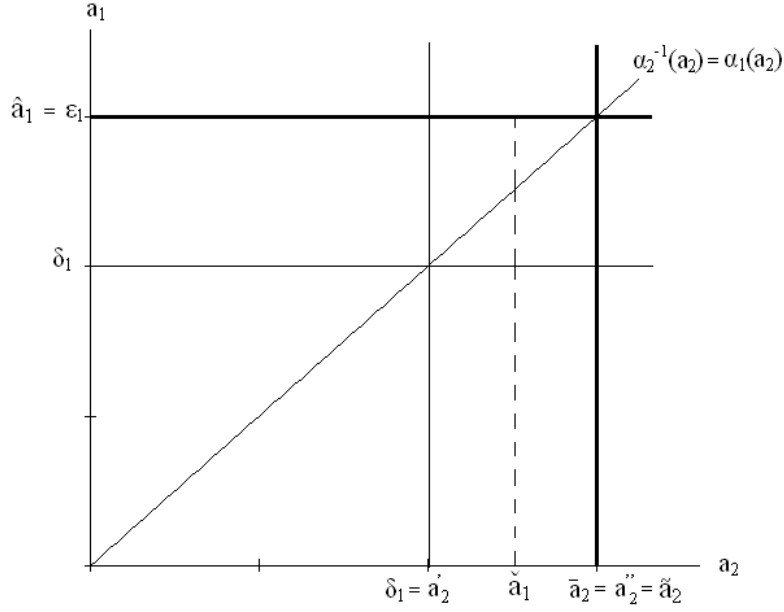


Figure 6: Players' response function when  $\gamma = 0$ .

## 10 Appendix 1

**Proof of Lemma 2:** The lemma is proven for the case of  $i = 1$ , and it is symmetric for the case  $j = 1$ . It is first shown that condition 8 implies that for every  $a_2$ , there exist a  $\delta_1(a_2)$  and a unique  $a_1$ , such that,

$$\frac{1}{2} \frac{\partial B_1(a_1, \phi_1^c)}{\partial a_1} + \delta_1(a_2) \leq \frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1} \quad (23)$$

where  $\phi_1^c$  is the value of function  $\phi$  evaluated at the cooperative choices. Note that  $\phi_1^c$  is a function of  $q_1$  and  $q_2$ .

Let  $M^* = \max_{\phi_1} \partial B_1(a_1, \phi_1) / \partial a_1$  and  $m^* = \min_{\phi_1} \partial B_1(a_1, \phi_1) / \partial a_1$ .

Then by definition  $\partial B_1(a_1, \phi_1^c) / \partial a_1 \leq M^*$  and  $\partial B_1(a_1, \hat{\phi}_1) / \partial a_1 \geq m^*$ .

We have assumed that  $\frac{1}{2}M^* < m^*$ . One can find a sufficiently small  $\delta_1(a_2)$  for each  $\phi_1^c(a_1, a_2)$  such that  $\frac{1}{2}M^* + \delta_1(a_2) < m^*$ . Then by substitution we obtain 23.

Next we show that condition 23 implies that for every  $a_2$  there exist a unique  $a_1$  such that  $B_1(a_1, \widehat{\phi}_1) = C_1(a_1, a_2)$ . We define  $D(a_1, a_2) = C_1(a_1, a_2) - B_1(a_1, \widehat{\phi}_1)$ . By 23,  $D(a_1, a_2)$  is a monotonically decreasing function of  $a_1$  and  $a_2$  and  $D(0, a_2) > 0$ . We show that there exists a sufficiently large  $a_1$  for which  $D(a_1, a_2) < 0$ . We rewrite  $B_1(a_1, \widehat{\phi}_1)$  as

$$B_1(a_1, \widehat{\phi}_1) = B_1(0, \widehat{\phi}_1) + \int_0^{a_1} \frac{\partial B_1(a_1, \widehat{\phi}_1)}{\partial a_1} da_1.$$

Substituting 23, we obtain

$$B_1(a_1, \widehat{\phi}_1) \geq B_1(0, \widehat{\phi}_1) + \int_0^{a_1} \left[ \frac{\partial C_1(a_1, a_2)}{\partial a_1} + \delta_i(a_j) \right] da_1.$$

which can be rewritten as

$$B_1(a_1, \widehat{\phi}_1) \geq B_1(0, \widehat{\phi}_1) - C_1(0, a_2) + C_1(a_1, a_2) + \delta_i(a_j) a_1.$$

If  $B_1(0, \widehat{\phi}_1) - C_1(0, a_2) + \delta_i(a_j) a_1 > 0$  then  $B_1(a_1, \widehat{\phi}_1) > C_1(a_1, a_2)$ . Thus we find an  $a_1^H > \left[ B_1(0, \widehat{\phi}_1) - C_1(0, a_2) \right] / \delta$ , such that,  $D(a_1^H, a_2) < 0$ . Using the intermediate value theorem, there exist a point  $a_i^* \in [0, a_1^H]$  such that  $D(a_i^*, a_2) = 0$ . It is unique since  $D(a_1, a_2)$  is monotonically decreasing for all  $a_i \in A_i$ .

Having shown the existence of a unique  $a_i^*$  for all  $a_2$ , we define a function  $\alpha_1 : A_2 \rightarrow A_1$  such that

$$B_1(\alpha_1(a_2), \widehat{\phi}_1) = C_1(\alpha_1(a_2), a_2) \quad (24)$$

By condition 23,  $\partial C_1(a_1, a_2) / \partial a_1 - \partial B_1(a_1, \widehat{\phi}_1) / \partial a_1 \neq 0$ , hence we can apply the implicit function theorem. Differentiating both sides of 24 with respect to  $a_2$  we obtain

$$\frac{\partial \alpha_1(a_2)}{\partial a_2} = - \frac{\frac{1}{2} \frac{\partial B_2(a_2, \phi_2(q_1(a), q_2(a)))}{\partial a_2}}{\frac{1}{2} \frac{\partial B_1(a_1, \phi_1(q_1(a), q_2(a)))}{\partial a_1} - \frac{\partial B_1(a_1, \widehat{\phi}_1)}{\partial a_1}}$$

which is strictly greater than zero, since the numerator is positive and the denominator is negative.

The existence of  $\alpha_2(a_1)$  can be shown in a similar manner. Since it is a monotonic function, its inverse,  $\alpha_2^{-1}(a_2)$ , is a well defined function. By definition,  $D(\alpha_1(a_2), a_2) > 0$  if  $a_1 > \alpha_1(a_2)$ , hence agent 1 maximizes  $B_1(a_1, \widehat{\phi}_1) - v_1(a_1)$ . For  $a_1 < \alpha_1(a_2)$ , agent 1's outside option is not binding and for  $a_1 > \alpha_2^{-1}(a_2)$ , agent 2's outside option is also not binding. Thus, agent 1 receives  $C_1(a_1, a_2) - v_1(a_1)$ . For  $a_1 \leq \alpha_2^{-1}(a_2)$ , agent 2's outside option binds, therefore agent 1 claims the residual and receives  $H_1(a_1, a_2) - v_1(a_1)$ .

## 11 Appendix 2

**Proof of Claim 5:**  $\delta_i(a_j)$  is defined by the following first order condition,

$$\frac{1}{2} \frac{\partial B_i(\delta_i(a_j), \phi_i^c)}{\partial a_i} = \frac{\partial v_i(\delta_i(a_j))}{\partial a_i}.$$

By assumption 8,  $\frac{1}{2} [\partial B_i(\delta_i(a_j), \phi_i^c) / \partial a_i] < \partial B_i(\delta_i(a_j), \widehat{\phi}_i) / \partial a_i$  for any  $a_i \in A_i$ . Therefore,  $\partial v_i(\delta_i(a_j)) / \partial a_i < \partial B_i(\delta_i(a_j), \widehat{\phi}_i) / \partial a_i$ . The right hand side is decreasing and the left hand side is increasing in  $a_i$ . To reach to an equilibrium  $a_i$  has to increase, hence  $\delta_i(a_j) < \widehat{a}_i$  for all  $a_j$ .

**Proof of Claim 8:** At  $a'_j$ ,  $\delta_i(a'_j) = \alpha_i(a'_j)$ . Therefore, the following condition

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) = B_i(\delta_i(a'_j), \widehat{\phi}_i) - v_i(\delta_i(a'_j))$$

is satisfied. Then,

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) < B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i)$$

since  $\widehat{a}_i$  is the unique maximum. By using the definition of  $\check{a}_j$ , we replace the right hand side of the inequality by  $C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j))$  and obtain

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) < C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j)).$$

Since  $\partial C_i / \partial a_j > 0$ , then it must be true that  $a'_j < \check{a}_j$ .

**Proof of Claim 10:** Using the definitions of  $\check{a}_j$  and  $\bar{a}_j$ ,

$$B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i) = C_i(\widehat{a}_i, \bar{a}_j) - v_i(\widehat{a}_i) = C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j))$$

which is less than  $C_i(\delta_i(\bar{a}_j), \bar{a}_j) - v_i(\delta_i(\bar{a}_j))$ . This implies that  $\check{a}_j < \bar{a}_j$ .

**Proof of Claim 11:** At  $\bar{a}_j$ ,  $C_i(\widehat{a}_i, \bar{a}_j) = B_i(\widehat{a}_i, \widehat{\phi}_i)$  by definition.  $a_j^*$  cannot be equal to  $\bar{a}_j$  because  $H_i(a_i, a_j) > B_i(a_i, \widehat{\phi}_i)$  for all  $(a_i, a_j)$ . Since  $H_i(a_i, \bar{a}_j)$  cannot intersect  $C_i(a_i, \bar{a}_j)$  at  $\widehat{a}_i$ , it must intersect it at some  $a_i$  which is less than  $\widehat{a}_i$ . Hence  $\alpha_j^{-1}(\bar{a}_j) < \widehat{a}_i$ . Then at  $\alpha_j^{-1}(\bar{a}_j)$  the following must hold

$$C_i(\alpha_j^{-1}(\bar{a}_j), \bar{a}_j) - v_i(\alpha_j^{-1}(\bar{a}_j)) > B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i).$$

Since  $\alpha_j^{-1}(a_j)$  is increasing in  $a_j$ ,  $a_j^* < \bar{a}_j$ .

## 12 Appendix 3

**Proof of Lemma 16:** Note that  $\bar{a}_i = \widehat{a}_i$  by the fact that  $\alpha_i(a_j)$  is the 45° line and by the definition of  $\bar{a}_i$ . In claim 10 we have shown that  $\check{a}_j < \bar{a}_j$ . Thus it follows that,  $\check{a}_j < \widehat{a}_j$ .

We next show that  $\check{a}_i > \delta_i$ . We consider the opposite, that is  $\check{a}_i \leq \delta_i$ . Then since  $\alpha_i(a_i) = a_i$  in the case of no complementarity,  $B_i(\check{a}_i, \widehat{\phi}_i) - v_i(\check{a}_i) = C_i(\check{a}_i, \check{a}_j) - v_1(\check{a}_i)$ . In other words,  $C_i$  intersects  $B_i$  at  $\check{a}_i$ . By the single crossing property in lemma 2,  $C_i(a_i, \check{a}_j) - v_1(a_i) \leq B_i(a_i, \widehat{\phi}_i) - v_1(a_i)$  for all  $a_i \geq \check{a}_i$ . Hence  $C_i(\delta_i, \check{a}_j) - v_1(\delta_i) \leq B_i(\delta_i, \widehat{\phi}_i) - v_1(\delta_i) < B_i(\widehat{a}_i, \widehat{\phi}_i) - v_i(\widehat{a}_i)$ , which contradicts with the definition of  $\check{a}_i$ . Therefore it must be true that  $\check{a}_i > \delta_i$ .

## 13 References

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