

Moving Holidays and Seasonality: An Application in the Time and the Frequency Domains For Turkey

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Abstract

When holiday variation is present so that the dates of certain holidays change from year to year, the relatively automatic seasonal adjustment procedures may fail to extract the seasonal component from a series since the holiday effects are not confined to that component. Turkey, a predominantly Muslim country, constitutes a good example of moving holidays since the official calendar is Gregorian, based on the cycles of the earth around the sun, even though significant Islamic holidays are tied to the Hegirian calendar, based on the lunar cycles. The existence of residual deterministic seasonal effects on monthly series that have already been conventionally seasonally adjusted as well as the consequences of ignoring this type of seasonality is analyzed. Based on analyses in the time and the frequency domains, the main intuitive conclusion is that one should first check to see if there exists “residual” deterministic seasonality left in the “conventionally” deseasonalized series and remove it if it does so. Estimation results point to the importance of paying special attention to such residual deterministic seasonality.

Keywords : Moving Holidays, Calendar Variation, Seasonality, Time Domain, Frequency Domain, Hegirian Calendar

JEL Classification : C22, E30

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1. Introduction

Certain kinds of economic activity and their associated time series exhibit regular seasonal fluctuations. Existing research on seasonality in economic time series have focused on a wide range of issues from viewing the existence of seasonal effects as noise in variables (Sims, 1974) to treating them as worthy of study in their own right (Barsky and Miron, 1989). Consistent with the first view, the choice of the “optimal” method that should be used to eliminate seasonal fluctuations and the consequences of using different seasonal adjustment methods has also been studied extensively. (See for example, Lovell (1963), Jorgenson (1964), Grether and Nerlove (1970), Gersovitz and MacKinnon (1978), Barsky and Miron (1989), Jaeger and Kunst (1990), and a more recent survey by Hylleberg (1992a).)

The isolation or the extraction of the seasonal component of an economic time series is a difficult issue.¹ This issue is further complicated when the date of a holiday changes from year to year and therefore the effect of the holiday is not confined to the “conventionally” extracted seasonal component of the series. The moving holiday phenomenon occurs when the dates for certain holidays, for example those that are tied to the lunar calendar, change from year to year and thus may affect different months across years. Bell and Hillmer (1983), and Findley and Soukup (2000) analyze this subject by focusing on the Easter effects in the U.S. as seasonal effects with calendar variation.

The purpose of this study² is to address the issue of identifying and eliminating deterministic seasonality due to holidays tied to the lunar calendar as well as analyzing the consequences of ignoring them by utilizing methods in the time and the frequency domains using data from a predominantly Muslim country, Turkey.³ Conventional methods of deseasonalization that are suitable for the Gregorian calendar will not detect seasonality of the religious events that have fixed dates according to the Hegirian calendar, which is essentially a lunar calendar⁴, since these events move approximately 11 days earlier every Gregorian year. Three significant Islamic events

¹ For the importance of seasonal adjustment, see Granger (1978), Sims (1974), Grether and Nerlove (1972) and Bell and Hillmer (1983).

² This paper extends the analysis of Alper and Aruoba (2001).

³ Turkey has been following the Gregorian calendar since the passing of the law #698 in December 26, 1925.

⁴ The Islamic lunar calendar is called the Hegirian, dating from the migration of the prophet Mohammed to Medina in 622. It is based on cycles of the moon around the earth while the Gregorian calendar is based on the cycles of the earth around the sun.

take place according to the Hegirian calendar system: The holy month of Ramadan⁵, the Feast of Ramadan, and the Feast of Sacrifice. The feast of Ramadan, also referred to as Eid ul-Fitr, lasts for 3.5 days following the end of the month of Ramadan and the feast of Sacrifice, also referred to as Eid ul-Adha, lasts for 4.5 days.⁶ Frequently, when the feast of Ramadan or Sacrifice happens in the middle of the week, the Turkish government decrees the remaining days of the week as a holiday and all civil servants enjoy a 9-day holiday. These holidays clearly affect retail trade, production, and financial markets. For example, when people refuse to use credit cards for transactions for the Feast of Sacrifice due to religious reasons, liquidity demand increases.

There are basically two questions that will be addressed in the paper. First, whether these aforementioned holidays cause regular, seasonal, deterministic fluctuations in the main macroeconomic indicators in Turkey is probed for in the time and the frequency domains.⁷

Analysis in the time domain consists of linearizing, detrending and extracting the deterministic seasonal components of each series using the conventional methods as explained in the methodology. Dummy variables for each of the holidays are used to detect the existence of any regular seasonal, deterministic patterns.

For analysis in the frequency domain, we follow the criterion due to Nerlove (1964) and Granger (1978), and regard a seasonal adjustment procedure as “good” if it removes the peaks around the seasonal frequencies in the spectrum of the series without affecting the rest of the spectrum. Nerlove (1964) cites two more properties that a “good” seasonal adjustment procedure should satisfy, namely that the coherence of the original and the seasonally adjusted series should be as close to unity as possible, except around seasonal frequencies and that following the seasonal adjustment, phase shift should be minimized especially at low frequencies. Nerlove (1965) applies spectral methods to the Bureau of Labor Statistics deseasonalization method (currently called Census X-11 and the modified “Hannan” method and concludes that both methods remove more than just the seasonal component. Instead of comparing the results of the two different

⁵ Ramadan is a month of ritual fasting during which believers do not partake of food, drink or pleasures of the senses between daybreak and sunset. Ramadan occurs during the ninth month of the Hegirian calendar.

⁶ The official durations of the holidays are decreed by law number 2429, article 1B. With respect to the duration, the feasts of Ramadan and Sacrifice are referred to as the lesser and the greater feasts, respectively.

⁷ What is evident in the weekly data may average out in the monthly frequency. Indeed, with quarterly data this effect is not evident.

deseasonalization procedures directly, he compares them with the unadjusted series and makes informal inferences using the three criteria cited above.⁸ Granger (1978) who summarizes the properties of “good” seasonal adjustment procedures mentioned in the literature, includes “the spectrum of the adjusted series should not have peaks around the seasonal frequencies” among the properties.⁹ The criterion we use in this paper encompasses all three of Nerlove’s criteria and Granger’s criterion.

The second question that the paper addresses regards the consequences, if any, of ignoring the residual deterministic seasonality due to religious holidays tied to the lunar calendar. Analysis on the conventionally seasonally adjusted series and further adjusted series is done to check for changes in persistence of the series as well as cyclical properties by analyzing autocorrelations and cross-correlations with output.

Analyses in the time and the frequency domains reveal the existence of residual seasonality due to religious holidays in the “conventionally” seasonally adjusted series for a number of variables. These variables include the industrial production index, its subgroups, imports, and nominal variables such as reserve money and government revenues. We also find that “further” deseasonalizing the series with significant residual seasonality improve the estimated power spectra according to Nerlove and Granger’s criterion, whereas for variables without significant residual deterministic seasonality, no improvement is detected. When we analyze the consequences of ignoring this type of residual deterministic seasonality, we find that persistence is lower for variables with significant residual deterministic seasonality and volatility is higher for all variables. Furthermore, we also find spurious correlation among variables containing residual seasonality based on the cross correlations.

The rest of the paper is organized as follows. Section 2 gives a brief description of the methodology. Section 3 presents the data and the estimation results. Section 4 concludes. The details of the methods used for the frequency domain analysis are presented in the Appendix.

⁸ Grether and Nerlove (1970) show that the phenomenon observed in Nerlove (1965), namely dips created near the seasonal frequencies after deseasonalization, is obtained as a result of “optimal” deseasonalization procedure as well.

⁹ Other properties he cites are no change-of-scale effect, coherence between the original series and the seasonal estimate must be unity and the phase must be zero (the series must be almost unchanged), the cross-spectrum of the original and the adjusted series must be equal to the spectrum of the adjusted series, sum-preserving and the sum of the adjusted series must be equal to the sum of the original series. Some other criteria proposed in the literature are being product-preserving, orthogonality, idempotency and symmetry (Lovell, 1963).

2. Methodology

Traditional univariate methods of analyzing economic time series are mainly concerned with decomposing the variation in a particular series into trend, seasonal, cyclical and irregular components. The decomposition method for a series is not unique and certain systematic assumptions about the nature of and the interaction among the trend, seasonal, cyclical and irregular components are needed to identify the series. For example, the seasonal component may be deterministic/stochastic, multiplicative/additive, or separable/inseparable in nature. We follow the standard practice of the real business cycle literature and assume separable trend and deterministic seasonality once a series is linearized. Thus, we start out by taking the natural logarithm of the series and then detrend and “conventionally” deseasonalize the series in succession for further analysis. Throughout the paper, “conventional” deseasonalization refers to applying methods such as regression on seasonal dummy variables or X-11, and “proper” or “further” deseasonalization refers to removing the “residual” seasonality that remains in the series due to religious events by the method explained below. Initially, we analyze the series that is obtained after “conventional” deseasonalization and detrending, which is intended to contain only the cyclical and irregular components, for the existence of any residual deterministic seasonality. We claim that standard methods of deseasonalization are unable to extract certain seasonal components when calendar variation is present. Our ultimate aim is to show the effects of ignoring this “residual” seasonality, due to moving holidays, in the time series by comparing the cyclical properties of “properly” deseasonalized and “conventionally” deseasonalized time series.¹⁰

Let Y_t be a series of interest. We assume that it is possible to decompose the series into three parts: a trend component, a deterministic seasonal component, and an irregular component. We also assume that the specification is a multiplicative one so that after taking the natural logarithm of the series, it is possible to detrend and deseasonalize the series by isolating the trend and seasonal components and then subtracting them in succession.

¹⁰ Ignoring this type of seasonality may also have other side effects that is beyond the focus of this paper: it will reduce the forecasting ability of a model fit to such time series. Especially in the context of a regression problem, if such seasonal fluctuations affect the dependent and the independent variables differently, the precision of coefficient estimates will decrease. Moreover, the coefficient estimates may be biased because ignoring the “residual” seasonality can be regarded as omission of a relevant variable in the regression.

We first wish to remove the trend and to that end we employ the spline function proposed by Hodrick and Prescott (1997) that extracts the long-run component of the $\ln Y_t$ series, \mathbf{t}_t , leaving $\ln Y_t$ stationary up to the fourth order. The trend component is chosen to minimize the following quadratic expression over \mathbf{t}_t :

$$\sum_{t=1}^T (\ln Y_t - \mathbf{t}_t)^2 + 14,400 \sum_{t=2}^T [(\mathbf{t}_{t+1} - \mathbf{t}_t) - (\mathbf{t}_t - \mathbf{t}_{t-1})]^2$$

and the detrended variable is equal to the difference between $\ln Y_t$ and \mathbf{t}_t . The filter proposed by Hodrick and Prescott allows the trend component to change slowly across time.¹¹

Next, we carry out the seasonal adjustment of the trend-free series by estimating its seasonal deterministic component and then removing this component from the trend-free series. To remove the deterministic seasonal component, we use the regression method due to Lovell (1963) and Jorgenson (1964)¹² and estimate the following model:

$$(\ln Y_t - \mathbf{t}_t) = \sum_{i=1}^{12} \mathbf{a}_i D_{it} + \sum_{j=1}^s \mathbf{b}_j P_{jt} + u_t$$

where u_t is a stochastic component that may or may not be white noise, D_{it} , $i=1, \dots, 12$, are monthly dummies and P_{jt} , $j=1, \dots, s$ are polynomial terms in time up to order $s \geq 1$. The latter variables are included to account for the non-seasonal deterministic component. We get the “conventionally” seasonally adjusted variable, c_t , as

$$c_t = (\ln Y_t - \mathbf{t}_t) - \sum_{i=1}^{12} \hat{\mathbf{a}}_i D_{it}$$

We suspect that c_t still contains some deterministic seasonality, that is, regular seasonal peaks and troughs, which still exists due to the moving holidays tied to the lunar calendar. Detection of this “residual” deterministic seasonality will be based on analyses in the time and the frequency domains.

¹¹ The Hodrick-Prescott filter has been subject to criticisms; see for example, King and Rebelo (1993), and Cogley and Nason (1995). However, previous research on the Turkish data by Alper (1998) reveals insignificant differences in results when an alternative filter is considered.

¹² The X-11 method of the U.S. Bureau of Census, which is a variant of the moving average method, is also used as an alternative method to deseasonalize the series when possible stochastic seasonality is present. See Hylleberg (1992b) for the details of this method. The results turned out to be quite similar. We chose the regression method over X-11 due to the loss of reliability at the end series as well as the ‘excess persistence’ findings by Jaeger and Kunst (1990) of the X-11 adjusted data compared to data adjusted by regression on dummies.

For detection in the time domain we estimate the equation

$$c_t = \sum_{i=1}^4 \mathbf{d}_i d_{it} + \sum_{k=1}^r \mathbf{f}_k c_{t-k} + \mathbf{e}_t$$

where \mathbf{e}_t is a stochastic component that is serially uncorrelated, d_{it} is a monthly seasonal dummy variable that takes the value 1 if a religious event tied to the lunar calendar takes place that particular month, zero otherwise. Initially, ignoring the religious dummy variables, we identify r , the order of the autoregressive process at the right hand side of the equation, by choosing the minimum value that makes \mathbf{e}_t serially uncorrelated and improves the Schwarz criterion.¹³ We then estimate the autoregressive process including the religious intercept dummy variables and check for the significance of the dummy variables. Significant coefficient(s) of the dummy variables is an indication of “leftover” or “residual” deterministic seasonality, since with the removal of trend and seasonality and a reasonably well-fit autoregressive process, what remains should be a pure random component, not explained by any variable.

Next, we further deseasonalize the series by estimating the following equation

$$c_t = \sum_{i=1}^4 \mathbf{g}_i d_{it} + \sum_{j=1}^s \mathbf{b}_j P_{jt} + u_t$$

and then subtracting the effects of the religious dummy variables from c_t .

$$f_t = c_t - \sum_{i=1}^4 \hat{\mathbf{g}}_i d_{it}$$

Seasonality can be defined as the systematic or unchanging intra-year movements that are caused by climatic changes, timing of religious holidays, business practices and expectations that give rise to spectral peaks around the seasonal frequency and its harmonics. So, detection in the frequency domain involves the estimation of the spectra of the original series after both the trend and the seasonal components have been removed, c_t , and the “further” deseasonalized series, f_t . We use the Blackman-Tukey Periodogram Smoothing Method (1959) with Blackman Lag window for estimating the spectrum. The details of this estimator and important concepts of spectral analysis used in this paper are given in the Appendix. After comparing the two spectra, if

¹³ No serial correlation is necessary since we want to obtain the random component of the series and show that the random component of a series contains “residual” seasonality.

the peaks in the estimated spectrum of f_t at the seasonal frequencies are reduced without creating other peaks or troughs at other frequencies, we call the seasonal adjustment a “successful” one.

Finally, we analyze the consequences of ignoring this “residual” deterministic seasonality. As mentioned previously, improperly identifying and eliminating regular seasonal fluctuations from variables used in time series analyses reduce the precision of the coefficient estimates since seasonal regularities impose additional variation on variables used in the estimations. For a number of macroeconomic monthly time series, we calculate the autocorrelation functions and check whether or not persistence increases since there exists less noise in the data once the deterministic “Hegirian Seasonality” is eliminated. We also check to see whether the volatility of each series decreases once the leftover seasonality is removed. Finally, we calculate monthly cross-correlations with the industrial production index and look for any emerging patterns after the residual deterministic seasonality is eliminated.

3. Data and Empirical Results

A broad range of monthly variables for the period 1985-2000 is obtained from the database of the Central Bank of the Republic of Turkey. These variables are: the Industrial Production Index, its sub-categories and imports as a proxy for aggregate economic activity; fiscal variables; monetary variables; price indices and inflation; financial variables including the stock exchange indices and the TL/USD exchange rate.¹⁴

Four dummy variables are constructed to proxy the religious events.¹⁵ The first and the second dummy variables represent the feast of Ramadan and the feast of Sacrifice, respectively, and take on the value 1 if a month contains at least half of the respective feasts and zero otherwise. The third dummy variable is for the 9-day holiday, and it takes on the value 1 if the government has decreed a 9-day holiday for the feast and zero otherwise. The fourth dummy variable is for the Holy month of Ramadan and takes on the value 1 if a month contains at least 5 business days of it and zero otherwise. While the first three dummy variables cannot take on the value 1 for two consecutive “Gregorian” months, this is not necessarily true for the Ramadan dummy.

¹⁴ Details of the dataset are given in Table 1.

¹⁵ The exact dates of these events for the post-1985 period are obtained from the Directorate of Religious Affairs of Turkey. The dummy variables are available from the authors upon request.

3.1 Detection of “Residual Seasonality”

As explained in the methodology section, we first take the natural logarithm of the variables, and obtain the trend-free series using the Hodrick-Prescott filter, and then deseasonalize the series using the regression method and label the resultant series c_t . This series is regarded in the time series literature as the irregular component with no trend and no deterministic seasonal fluctuations. Since we assume that there is some “residual” deterministic seasonality in c_t , due to aforementioned reasons, further deseasonalization is necessary. As explained in the methodology the constructed religious dummy variables tied to the lunar calendar are used to further deseasonalize c_t .¹⁶ The resultant series is labeled as f_t and is claimed to better represent the seasonality-free irregular component.

Next, we identify the order of the autoregressive process for each of the 23 trend-free, “conventionally” deseasonalized and linearized series, c_t . The order of the series is chosen to be the minimum value making the residuals from the estimation serially uncorrelated. After the autoregressive order of each series is identified, the Islamic dummy variables are appended to the model; and following the estimation, the significance of these dummy variables are tested based on the Wald test of coefficient restrictions and the Schwarz criterion (1978).

Figure 1 shows the Industrial production Index before and after the removal of “residual” seasonality to illustrate the consequences of this “further” deseasonalization procedure. It may be observed that some of the spikes (troughs and peaks) in the data disappear (e.g. for years 1987, 1990, 1991, 1997) once these deterministic religious events tied to the lunar calendar are accounted for.

Table 2 summarizes the results in the time domain. The first four columns give the name of the variables, order of the autoregressive process, the Q-statistic¹⁷ for testing the existence of serial correlation in the residuals up to 24 lags, the adjusted R-squared and the Schwarz criterion for the regression, respectively. All of the Q-statistics lead to failure of rejection of the null hypothesis, implying that the residuals from the autoregressive models are serially uncorrelated. The fifth

¹⁶ Instead of this procedure, one could have applied the deseasonalization method to the detrended variable using both the monthly and religious dummy variables. The results are identical.

¹⁷ The Q-statistic is due to Ljung and Box (1979). It asymptotically follows a chi-square distribution with degrees of freedom equal to the number of lags.

column gives information about the significance of the religious dummy variables, when they are included in the “identified” autoregressive regression. A minus sign indicates insignificance of the coefficients for the religious dummy variables. After all the dummy variables are added to the regression, the variables with insignificant coefficients are taken out and the test statistics for the remaining variables are reported. The values of the coefficients as well as the corresponding p-values for significance are given in the fifth column. As mentioned earlier, if the dummy variables are significant, it would imply the existence of unaccounted-for seasonality in the data. Of the 23 variables examined, 8 variables contain significant effects of at least one of the religious events.¹⁸ The next column reports the Wald test statistics¹⁹ for testing the joint significance of the coefficients of the dummy variables of the religious events and the corresponding p-values. The last two columns report the adjusted R-squared and the Schwarz criterion associated with the regression including the dummy variables.

Next, we turn to analysis in the frequency domain and estimate the power spectra of c_t and f_t as well as the $\{100(1-\alpha)\}$ % confidence intervals for all 23 variables. We utilize these to see if the spectrum of f_t improves over the spectrum of c_t in terms of Nerlove (1964) and Granger (1978)’s criterion described earlier, i.e. the adjusted series should not have peaks or dips in the spectrum at the seasonal frequency and its harmonics.

The estimations are carried out with the Blackman-Tukey Smoothed Periodogram estimator (1959), discussed in the Appendix. While estimating the power spectrum, we divide the interval $w \in [0, p]$ into 600 frequencies, denoted by j . We compare the spectrum estimates of c_t and f_t in the entire range, i.e. $j = 1, \dots, 600$ and pay special attention to the “religious frequency”, the frequency of a cycle that takes one lunar (Hegirian) year to complete, and its harmonics.²⁰

¹⁸ For variables like the industrial production index and its subgroups, the coefficients of the corresponding dummy variables are significantly negative due to the loss of business days. We also observe that the reserve money increases significantly for the months having the two feasts, implying that the open market operations by the central bank provide liquidity to the market during the holidays. These operations are carried out in response to an increase in the liquidity demand prior to the holidays. It is worthwhile to note that Ramadan is significant only one occasion but the feast of Ramadan and feast of Sacrifice have significant effects for almost all 8 variables.

¹⁹ The Wald statistic asymptotically follows an F distribution with $q, (n-k)$ degrees of freedom where q is the number of restrictions, n is the sample of variables and k is the number of independent variables in the regression. Equivalently, a chi-square distribution could have also been used. All our results based on the F distribution-based Wald test are also obtained by the chi-square distribution-based Wald test.

²⁰ When a spectrum has a large peak at some frequency w , then related peaks may occur in the harmonics, i.e. multiples of that frequency. For example if there is a weekly cycle, then there will be bi-weekly, tri-weekly etc. cycles.

Therefore, we denote $j^* = 102$ or $w^* = 0.17p$ ²¹ and the harmonics of it, i.e. $j = 204, 306, 408, 510$, the “religious frequencies” in this paper.²²

For each variable, we carry out the following analysis. We first estimate the spectra of c_t and f_t along with the confidence bands for those spectra.²³ Next, we identify the intervals within the whole range where the lower bound of a confidence band for the spectrum of c_t or f_t is strictly above the upper bound of the confidence band of the spectrum of the other.²⁴ If the lower bound of the confidence band for the spectrum estimate of c_t is strictly above the upper bound of the confidence band of the spectrum of f_t for some frequency band, then we call this “deterioration”. The opposite case is an “improvement”. If the confidence bands overlap then the two spectra estimates are not statistically different from each other. This case is noted as “no change”. Finally, we check if the “religious frequencies” fall into any of the improvement or deterioration bands for each variable. We only consider bands longer than 12 consecutive frequencies (1% of the range $w \in [0, 2p]$).²⁵

The last two columns of Table 3 summarize these findings. The first of these columns report if an improvement is observed in the “religious frequencies” and the other column reports the results for all other frequencies. A variable marked with a plus sign has at least one band marked as improvement and no bands marked as deterioration. Similarly a variable marked with a minus sign would have some band marked as deterioration and no bands marked as improvement. A variable is marked with a plus/minus if for certain frequencies it has improvement and some other frequencies it has deterioration bands. If the bands for the two spectra intersect for the whole range, then this is marked with a zero, signifying no change. According to this table we can classify the variables into 4 categories:

- 1) Improvement in the whole range and the “religious frequencies”: 2 variables.

²¹ This corresponds to a cycle of 11.76 months, which is the Hegirian calendar cycle, as it is approximately 353 days.

²² These correspond to cycles of length 5.88, 3.92, 2.94 and 2.35 months or 174, 118, 88, and 70 days.

²³ The confidence intervals are 90% confidence intervals. The results are more or less similar when the length of the confidence intervals is set to 95%.

²⁴ We follow this procedure which is “approximate” instead of an “exact” procedure which would involve deriving the distribution of the difference between the spectra of c_t and f_t and testing the significance of the difference. In Appendix 2 we show that whenever the procedure we follow rejects the hypothesis that the spectra are equal at a given frequency, the “exact” test would reject it as well.

²⁵ As noted in the appendix, the total variance of a series is the sum of the spectra in the range $w \in [0, 2p]$. So the above threshold eliminates bands, which correspond to less than 1% of the total variance.

- 2) No change in any frequency: 13 variables.
- 3) No change in the religious frequencies and deterioration in some other frequencies: 5 variables.
- 4) Improvement in the religious frequencies and ambiguity, deterioration or no change in the whole range: 3 variables.

We report the graph for one variable from each category.²⁶ Figures 1 through 4 report the graphs for Industrial Production Index (Category 1), Mining and Quarrying (Category 2), Istanbul Stock Exchange National 100 Index (Category 3) and Reserve Money (Category 4). These graphs include the spectrum estimates for c_t and f_t with shaded regions. A shaded region with a plus sign corresponds to a frequency band not including any of the religious frequencies where there is an improvement. A shaded region with a “+R” mark, is a frequency band including one of the religious frequencies which shows an improvement, whereas a minus sign signifies a band not including any of the religious frequencies which show a deterioration.²⁷ All figures except for the Istanbul Stock Exchange Index in TL depict the spectra for the whole range, for ISETL we start graphing from $j=100$ as a peak before that point is very significant and overshadows the following frequencies.

Table 3 summarizes the main findings. The “significant” / “not significant” entries in the first column refer to the significance of the Wald test, as given in Table 2.²⁸ The second and third column summarize the findings for the frequency domain as described above. The important result is that if any of the Islamic dummies were significant in the time domain, further deseasonalization never hurts in the frequency domain, i.e. it gives at least as good results in terms of Nerlove and Granger’s criterion.²⁹ However, for the variables that the time series analysis produced an “insignificant” result, i.e. there was no residual seasonality, further deseasonalization results in either no change or deterioration. This is due to the spurious effects of the religious dummy variables on the spectrum of the “further” deseasonalized variable.

²⁶ The rest of the graphs are available from the authors.

²⁷ Deterioration at the religious frequencies is not observed for any of the variables.

²⁸ Even though the coefficients for two religious events for CREDIT are individually significant, the joint test of significance gives a p-value of 0.06. Therefore CREDIT will be marked as “not significant”.

²⁹ Only in the case of reserve money, the result is ambiguous.

In light of this analysis, we suggest the following for researchers in Turkey, and in every country where holidays are tied to different calendars. Once the decision to deseasonalize has been made, one must apply the time domain procedure described above, i.e. model the “conventionally” deseasonalized variable and append the religious dummy variables to this regression and test for their significance.³⁰ If these dummy variables turn out to be insignificant, then proceed to further analysis using the “conventionally” deseasonalized variable. Otherwise, one must “further” deseasonalize the series, to completely remove any seasonality in the data.³¹

3.2 Consequences of Ignoring Residual Seasonality

After verifying the existence of residual deterministic seasonality due to the existence of the implicit Hegirian calendar, we next turn to consequences of ignoring these effects. For this, we obtain cross-correlation and auto-correlation tables of the detrended and deseasonalized variables before and after the removal of residual “Hegirian seasonality”. Table 4 reports the autocorrelations up to six lags for the variables that we found significant effects in the time domain analysis, namely, measures of aggregate economic activity such as the Industrial Production Index, its 3 sub-categories, imports, government revenues and reserve money.³² The upper half report results pertaining to the linearized, detrended, and deseasonalized data containing residual “Hegirian seasonality” and the lower half report autocorrelations after the residual seasonality is removed. Conforming to *a priori* expectations, the auto-correlation coefficients, giving information about the persistence³³ of the data, rise in absolute value³⁴ once the residual seasonality is removed except for five coefficients.³⁵ The average increase is 0.04 over the initial values and the largest increase is for the first lag of IPEGW (Electricity Gas and Water), which is 0.27. If we consider only the coefficients, which are significant in the upper part of the table, then following the removal of the residual seasonality they increase on the average

³⁰ Alternatively, one could directly insert the religious dummy variables in to the seasonality estimation regression and test their significance in that regression. This conforms to Jorgenson (1964)’s suggestion for making a decision on which seasonal adjustment procedure to implement.

³¹ Hence, this procedure would require one to further deseasonalize Industrial Production Index and its sub-categories, Imports, and Reserve Money among the variables that we use.

³² Rest of the variables are excluded since further deseasonalization procedure is not required from them.

³³ Informally, persistence may be defined as the long-run level effect of a 1 per cent shock on a macroeconomic time series.

³⁴ We look at changes in absolute value instead of levels, since some variables have negative correlation coefficients and an increase in correlation will be a decrease in level.

³⁵ These are the fourth lag of imports and second and fourth lags of Industrial Production Index mining and quarrying subcategory and the first lag of government revenues.

by 0.06. This shows that the deseasonalization procedure removes some noise from the data and makes the remaining series relatively more predictable.

Next, the cross correlations of the series with the Industrial Production Index and their volatilities are analyzed. The results are reported in Table 5.³⁶ Again, conforming to *a priori* expectations, the volatility of all series except the Istanbul Stock Exchange Industrial Index, is reduced once the noise from the residual seasonality is removed. When the cross correlation coefficients prior to the removal of the deterministic residual seasonality are compared to those obtained after the removal two important trends emerge. First, for the variables, which had significant results in the time domain, many of the coefficients in the lower part of the table are less than their counterparts in the upper part of the table in absolute value. On the other hand, for the variables, which didn't show any significance in the time domain, only five coefficients³⁷ out of 55 significant coefficients are less than their counterparts in the upper part of the table in absolute value. These results imply that for the variables having significant results in time domain, the cross-correlation coefficients with industrial production index are overstated since they capture the co-movement of the "residual" seasonality in the series and industrial production index. For all other variables, the cross-correlations increase in absolute value, once the "residual" seasonality is removed. Even though these variables don't have any residual seasonality, we still get an increase due to the removal of seasonality in industrial production index.

To recapitulate, while the persistence of the series with "residual" seasonality increase after its removal, volatility of all series is reduced. These two results imply that after the removal of residual seasonality, the series become more predictable and estimation results based on these variables will be more reliable. Moreover, the correlations with Industrial Production Index decrease in absolute value for the variables with "residual" seasonality, signifying the existence of a spurious relationship due to "residual" seasonality whereas for all others they increase in absolute value, strengthening the conclusion above.

³⁶ All variables except government revenues and expenditures, net domestic borrowing and foreign currency are reported in the table. All the coefficients for the mentioned variables are insignificant.

³⁷ These are the third lag of credits, stock exchange index in TL and in USD, second lag of M1 and first lag of CPI inflation.

4. CONCLUSION

Proper decomposition of a macroeconomic time series into a trend, cyclical, seasonal, and irregular components is the main concern of traditional time series analysis. The existing approaches to extracting the seasonal component of economic time series have concentrated on developing relatively automatic procedures. When holiday variation is present so that the dates of certain holidays change from year to year, these relatively automatic procedures may fail to extract the seasonal component since the holiday effects are not confined to that component. Since 1926, Turkey has been following the Gregorian calendar, based on the cycles of the earth around the sun, even though significant Islamic holidays are tied to the Hegirian calendar, based on the lunar cycles. The aim of this study was to show for Turkey, the existence of residual deterministic seasonal effects on series that have already been conventionally seasonally adjusted as well as consequences of ignoring this type of seasonality.

The analysis was based on 23 monthly macroeconomic time series for Turkey, and of these 8 are shown to contain significant moving holiday effects even after the “conventional” seasonal adjustment based on the regression method was made. These variables included measures of aggregate economic activity such as the industrial and the manufacturing production indices, imports, as well as nominal variables such as government revenues and reserve money. Prices and financial indices did not contain significant deterministic residual seasonality.

Next, further seasonal adjustment of the variables by the regression method using the religious dummy variables tied to the lunar calendar was made. Then, a comparison of the spectra estimates of these variables with that of the “conventionally” deseasonalized variables was carried out. Following Nerlove (1964) and Granger (1978)’s criterion for “good” seasonal adjustment, namely the requirement that the seasonal adjustment procedure should remove the peaks around the seasonal frequencies in the spectrum of the series without affecting the rest of the spectrum, we checked to see if the spectra of the “further” deseasonalized series lie under the spectra of the “conventional” seasonally adjusted series through statistical methods. We found that for the variables with significant moving holiday effects in the time domain analysis, the “further” deseasonalized series improved over the conventional ones in the frequency domain based on the criterion above. Specifically, the spectrum of the further deseasonalized series was statistically below the other around the “religious” frequencies and the rest of the spectra

remained statistically unchanged. On the other hand, for variables with insignificant moving holiday effects in the time domain analysis, further seasonal adjustment did not improve or even worsened based on the above criterion. The main intuitive conclusion is that one should first check to see if there exists any “residual” seasonality due to religious effects left in the “conventionally” deseasonalized series and remove it if it does so. If residual seasonality is not present, then one should proceed with the conventional adjustment since further adjustment may not improve the properties of the adjusted series.

Consequences of ignoring the “residual” seasonality were also considered. We calculated the autocorrelations, volatilities and cross-correlations with output for the variables before and after the removal of the “residual” seasonality due to moving holidays. We found that the persistence of the variables containing “residual” seasonality tend to increase with the removal of the deterministic seasonality tied to the lunar calendar. Moreover, the volatility for almost all variables decreases after the removal of residual seasonality. These two facts imply that the variables become relatively more predictable and estimation results based on these variables will be more reliable when the residual seasonality is removed. When the cross-correlations with output (industrial production index) were analyzed, we found that the relationship between the variables that contained residual seasonality is weakened implying the existence of a spurious relationship caused by common residual seasonality. For all the other variables, on the other hand, removal of residual seasonality increased the cross-correlations, strengthening our conclusions for the importance of paying special attention to such residual deterministic seasonality.

APPENDIX 1 : Spectral Density Estimation and Some Important Results

In this section we briefly review some of the important concepts in spectral analysis used in the paper.³⁸

A covariance stationary time series can be written as an infinite sum of random numbers:

$$Y_t = \mathbf{m} + \sum_{j=0}^{\infty} \mathbf{f}_j \mathbf{e}_{t-j}$$

where \mathbf{m} is a constant term and $\{\mathbf{e}_t\}$ is a purely random process with mean zero and a constant finite variance. This is called the time domain representation of the series. A typical analysis in the time domain involves inference based on the autocorrelation function or autocovariances, as in Box-Jenkins (1976) methodology.

An analytically equivalent way of analyzing time series is based on their cyclical properties. This requires the representation of the time series in the frequency domain. All covariance stationary series can be represented in the frequency domain as a combination of uncorrelated cosine and sine waves of different amplitudes:

$$Y_t = \mathbf{m} + \sum_{j=1}^{\infty} a_j \cos \mathbf{w}_j t + \sum_{j=1}^{\infty} b_j \sin \mathbf{w}_j t + \mathbf{n}_t$$

where a_j and b_j depend on the amplitude and phase of the cycle j and \mathbf{w}_j is the frequency of the cycle j and \mathbf{n}_t is some stationary random series. These two forms of data analysis are complementary in nature and provide different insights. Analysis in the time domain is useful for model selection and identification purposes while analysis in the frequency domain is helpful in analyzing issues pertaining to seasonality.

In the frequency domain, the series are decomposed into orthogonal components, each of which is associated with a frequency. Spectral density (or power spectrum) of a series records the contribution of these components belonging to a frequency, to the total variance of the process³⁹ and is defined as:

³⁸ For detailed discussions of spectral analysis see Priestley (1981), Chatfield (1984), Hayes (1996) and Hamilton (1994) Chapter 6.

³⁹ Granger (1966)

$$h(\mathbf{w}) = \frac{1}{2\mathbf{p}} \sum_{s=-\infty}^{\infty} \mathbf{g}_s e^{-i\mathbf{w}s} = \frac{1}{2\mathbf{p}} \sum_{s=-\infty}^{\infty} \mathbf{g}_s \cos(s\mathbf{w}) \text{ for } -\mathbf{p} \leq \mathbf{w} \leq \mathbf{p} \quad 40$$

where $\mathbf{g}_s = E[(Y_t - \mathbf{m})(Y_{t-s} - \mathbf{m})]$ is the autocovariance at lag s . As the above definition suggests, the variance of a series may be obtained by summing the spectra over all frequencies.

The frequency band of interest, by convention, is $\mathbf{w} \in [0, \mathbf{p}]$.⁴¹ When the plot of the spectrum is analyzed, a low frequency spike (i.e. higher contribution to total variance of a low frequency) of the density (small \mathbf{w} , large \mathbf{p}) implies a cycle which repeats only a few times in the sample, whereas a high frequency spike would be a regularity that repeats itself very often.⁴²

The problem at hand is to estimate $h(\mathbf{w})$ for $0 \leq \mathbf{w} \leq \mathbf{p}$, given observations (Y_1, Y_2, \dots, Y_T) from a trend-free time series. The obvious choice for this is the periodogram, which is obtained by replacing the population autocorrelation coefficients by their sample counterparts:

$$\hat{h}_p(\mathbf{w}) = \frac{1}{2\mathbf{p}} \sum_{s=-(T-1)}^{T-1} \hat{\mathbf{g}}_s e^{-i\mathbf{w}s}$$

It is documented that the periodogram, although asymptotically unbiased, is an inconsistent estimator for the population spectrum, since for large s very few observations are used to compute the autocovariances.⁴³

For achieving consistency, the Blackman-Tukey (1959) Periodogram Smoothing Method (the kernel estimate of the spectrum) is used. This method introduces a lag window, $\mathbf{I}(s)$, that takes the form

$$\mathbf{I}(s) = \begin{cases} \mathbf{I}^*(s) & \text{if } |s| \leq M \\ 0 & \text{if } |s| > M \end{cases}$$

This lag window is a weighting function that puts the largest weights around $s=0$ and the weights associated with lags greater than the truncation point, M , are set equal to zero. The reduction of weights in the tails significantly reduces the variance of the estimator since most of the variance

⁴⁰ Spectral density is the Fourier transform of the sequence of autocovariances.

⁴¹ No information is lost from using this restricted interval. Negative frequencies are not used, because there is no way to decide if the data are generated by a cycle with a frequency \mathbf{w} or $-\mathbf{w}$. Similarly, any cycle with a frequency greater than \mathbf{p} , say $k\mathbf{p}$ for $1 < k < 2$, is indistinguishable from a cycle with a frequency $(k-1)\mathbf{p}$. These results follow from trigonometric properties. See Hamilton (1994) p. 160, for details.

⁴² Granger (1966) argues that a typical economic time series is dominated by low frequency (trend-like) cycles even after detrending.

comes from the tail estimates that rely on less number of observations. The Blackman-Tukey estimate of the spectrum is:

$$\hat{h}_{BT}(\mathbf{w}) = \frac{1}{2\mathbf{p}} \sum_{s=-(T-1)}^{T-1} \mathbf{I}(s) \hat{\mathbf{g}}_s \cos(s\mathbf{w})$$

Equivalently, $\hat{h}_{BT}(\mathbf{w})$ can be represented as the weighted average of periodogram values of adjacent frequencies:

$$\hat{h}_{BT}(\mathbf{w}) = \int_{-\mathbf{p}}^{\mathbf{p}} \hat{h}_p(\mathbf{w}) W(\mathbf{w}-\mathbf{q}) d\mathbf{q}$$

where $W(\mathbf{q})$ is called the spectral window or kernel.⁴⁴

Under some very mild assumptions about the spectral window, the Blackman-Tukey estimator is a consistent estimator of the population spectrum at any point. Moreover, as noted by Hayes (1996), it gives, albeit slightly, better estimates in terms of a criterion based on the normalized variance.⁴⁵

It is customary to divide the interval $[0, \mathbf{p}]$ into $(T-1)/2$ frequencies. We will deviate from this convention and divide the interval into 600 frequencies, given by $\mathbf{w}_j = \frac{2\mathbf{p}j}{1200}$, $j = 1, \dots, 600$. This doesn't change the shape, the magnitude of the spectra or the locations of the peaks and troughs but smoothes out the graph. The relationship between the frequency of a cycle, \mathbf{w}_j , and the period of it is $p_j = \frac{2\mathbf{p}}{\mathbf{w}_j} = \frac{1200}{j}$. This also convenient since, for example, the $j=100$ frequency corresponds to one cycle per year, $j=200$ corresponds to two cycles per year and so on.

⁴³ $\text{var}[\hat{h}_p(\mathbf{w})] \rightarrow [h(\mathbf{w})]^2$ in the limit which is not equal to zero.

⁴⁴ The lag window is the Fourier transform of the spectral window. $\mathbf{I}(s) = \int_{-\mathbf{p}}^{\mathbf{p}} e^{is\mathbf{q}} W(\mathbf{q}) d\mathbf{q}$ for $s = -(T-1), \dots, (T-1)$.

The equivalency of the two representations is obtained by using Fourier transform and convolution theorem.

⁴⁵ Another consistent estimation method for the spectral density discussed in the literature is the Welch's Method. (see Welch, 1967 and Hayes, 1996). In this method, the data is divided into K (possibly overlapping) sections and within each section the sample periodograms are averaged using a spectral window. These K numbers are averaged to get an estimate of the spectral density at any point. This method requires less computational power as

For carrying out the estimation, one has to choose a lag window and a truncation point. Some popular lag windows used in the literature are: Bartlett, Tukey-Hamming, Tukey-Hanning, Blackman.⁴⁶ For the analysis, Blackman window is used since it shows slightly less variation compared to other lag windows, which is a desirable property.⁴⁷ The formula for the Blackman window is:

$$I_B(s) = \begin{cases} 0.42 - 0.5 \cos\left(2\pi \frac{s-1}{M-1}\right) + 0.08 \cos\left(4\pi \frac{s-1}{M-1}\right) & \text{if } |s| \leq M \\ 0 & \text{if } |s| > M \end{cases}$$

Figure A1 depicts the weight function for Blackman window with $M=50$. As argued above, the weight function attaches the highest weight (1) to lag $s=0$ and the weights decrease for higher values of s in absolute value. The lags greater than 50 in absolute value are truncated, i.e. given zero weights.

As for the selection of the truncation point, we follow Jenkins and Watts (1968) and analyze the spectra using the Blackman window with three different M 's. The smaller the truncation point (therefore the smaller the bandwidth) the smoother the estimate is. While this reduces variance of the estimate, the bias increases. Therefore, choosing an intermediate value of M will reduce the mean square error. We will use $M=40$ as the truncation point.⁴⁸ Figure A2 depicts the spectrum estimates of IPI (detrended and conventionally deseasonalized as explained in the methodology) with $M=10$, $M=40$ and $M=75$.

autocorrelations are computed for a limited number of observations but its merits are not better than the Blackman-Tukey estimator.

⁴⁶ See Priestley(1981) for detailed derivations for these windows.

⁴⁷ Estimates of spectrum using other lag windows produced only slight differences. Moreover, Blackman windows have slightly wider central lobes and less sideband leakage than equivalent length Hamming and Hanning windows. See Oppenheim and Schafer (1989) pp. 447-448

⁴⁸ The selection of the lag window and truncation point is arbitrary in the literature. However when the same analysis is carried out using different lag windows and truncation points, similar results were obtained. As noted by Priestley (1981), any $M = T^a$ with $0 < a < 1$ is necessary for consistency of the estimator of the spectral density function. $M=40$ corresponds to roughly to $a = 0.7$.

Next, some asymptotic distribution theory that will be used inference will be introduced. If

(Y_1, Y_2, \dots, Y_T) can be written as $Y_t = \mathbf{m} + \sum_{j=0}^{\infty} \mathbf{f}_j \mathbf{e}_{t-j}$ and \mathbf{e}_t comes from a Gaussian distribution, then

it can be shown that ⁴⁹

$$\left\{ \frac{\hat{h}(\mathbf{w})}{h(\mathbf{w})} \right\}^{appr.} \sim a \mathbf{c}_n^2 \text{ for } \mathbf{w} \neq 0, \pm \mathbf{p}$$

where $a = \frac{1}{\mathbf{n}}$ and $\mathbf{n} = 2T / \sum_s \mathbf{I}_T^2$ is the degrees of freedom. For calculations, using Blackman lag

window and $M=40$, \mathbf{n} is between 9 and 15 depending on the length of the series. Using this result, approximate $\{100(1-\mathbf{a})\}\%$ confidence intervals can be written as

$$\left[\frac{\mathbf{n}\hat{h}(\mathbf{w})}{b_{\mathbf{n}}(\mathbf{a})}, \frac{\mathbf{n}\hat{h}(\mathbf{w})}{a_{\mathbf{n}}(\mathbf{a})} \right]$$

where $P[\mathbf{c}_n^2 \leq a_{\mathbf{n}}(\mathbf{a})] = P[\mathbf{c}_n^2 \geq b_{\mathbf{n}}(\mathbf{a})] = \frac{\mathbf{a}}{2}$. The confidence intervals given in this section are

asymptotic results. Neave (1972) has shown that these results are also quite accurate for short series.

⁴⁹ See Chatfield (1984) p.149 and Priestley (1981) p.466 for details.

APPENDIX 2 – Justification of Using Confidence Intervals for Testing The Equality of Two Spectra at a Given Frequency

In this appendix, we show that whenever the “approximate” procedure we follow in the paper rejects the hypothesis that two spectra are equal at a given frequency, the “exact” procedure, as will be made explicit below, rejects it as well.

Consider two estimates $\hat{g}(\mathbf{w})$ and $\hat{h}(\mathbf{w})$, estimating true spectra $g(\mathbf{w})$ and $h(\mathbf{w})$ at the frequency \mathbf{w} .

We want to test

$$H_0 : g(\mathbf{w}) - h(\mathbf{w}) = 0$$

$$H_A : g(\mathbf{w}) - h(\mathbf{w}) > 0$$

by using the estimates above, which have the following distributions:

$$\hat{g}(\mathbf{w}) \overset{a}{\sim} N[g(\mathbf{w}), \mathbf{s}_{\hat{g}}^2]$$

$$\hat{h}(\mathbf{w}) \overset{a}{\sim} N[h(\mathbf{w}), \mathbf{s}_{\hat{h}}^2]$$

where $\mathbf{s}_{\hat{g}}^2$ and $\mathbf{s}_{\hat{h}}^2$ are the asymptotic variances of the respective estimates.

The “exact” test, which takes the covariance of $\hat{g}(\mathbf{w})$ and $\hat{h}(\mathbf{w})$ into account⁵⁰, can be cast in terms of the confidence interval for the variable $d \equiv g(\mathbf{w}) - h(\mathbf{w})$, which is estimated by $\hat{d} \equiv \hat{g}(\mathbf{w}) - \hat{h}(\mathbf{w})$. \hat{d} has the following distribution:

$$\hat{d} \overset{a}{\sim} N\left(0, \mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}\right)$$

The $\{100(1-\alpha)\}$ % confidence interval for d is

$$\hat{d} \pm z_{\alpha/2} \sqrt{\mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}}$$

We would reject H_0 in favor of H_A if and only if the lower limit of this confidence interval is greater than 0, i.e.

$$\hat{d} - z_{\alpha/2} \sqrt{\mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}} > 0$$

In the paper, we use an “approximate” approach in which we conduct our test using the confidence intervals of $g(\mathbf{w})$ and $h(\mathbf{w})$ which are

⁵⁰ In our case the covariance is very high between the two estimates since one of the series is derived from the other.

100(1- α)% Confidence Interval for $g(\mathbf{w})$: $\hat{g} \pm z_{\alpha/2} \mathbf{s}_{\hat{g}}$

100(1- α)% Confidence Interval for $h(\mathbf{w})$: $\hat{h} \pm z_{\alpha/2} \mathbf{s}_{\hat{h}}$

The ‘‘approximate’’ test is conducted as follows: If the lower limit of the confidence interval of $g(\mathbf{w})$ is greater than the upper limit of the confidence interval of $h(\mathbf{w})$, then the null hypothesis is rejected in favor of the alternative hypothesis. Thus if

$$\hat{g} - z_{\alpha/2} \mathbf{s}_{\hat{g}} > \hat{h} + z_{\alpha/2} \mathbf{s}_{\hat{h}} \Leftrightarrow \hat{d} - z_{\alpha/2} (\mathbf{s}_{\hat{g}} + \mathbf{s}_{\hat{h}}) > 0$$

we conclude that $g(\mathbf{w})$ is significantly greater than $h(\mathbf{w})$ at the frequency \mathbf{w} .

Claim: If the lower limit of the confidence interval of $g(\mathbf{w})$ is greater than the upper limit of the confidence interval of $h(\mathbf{w})$, then the lower limit of the confidence interval of d is greater than 0, i.e. whenever the ‘‘approximate’’ test rejects H_0 in favor of H_A , the ‘‘exact’’ test would reject it as well.

Proof: To prove this claim, it suffices to show that

$$\hat{d} - z_{\alpha/2} \sqrt{\mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}} > \hat{d} - z_{\alpha/2} (\mathbf{s}_{\hat{g}} + \mathbf{s}_{\hat{h}})$$

or equivalently

$$\mathbf{s}_{\hat{g}} + \mathbf{s}_{\hat{h}} > \sqrt{\mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}}$$

which would imply that whenever the right hand side is greater than zero, the left hand side also is. Taking squares of both sides we get

$$\mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 + 2\mathbf{s}_{\hat{g}}\mathbf{s}_{\hat{h}} > \mathbf{s}_{\hat{g}}^2 + \mathbf{s}_{\hat{h}}^2 - 2\mathbf{s}_{\hat{g}\hat{h}}$$

$$2\mathbf{s}_{\hat{g}}\mathbf{s}_{\hat{h}} > -2\mathbf{s}_{\hat{g}\hat{h}}$$

Using the definition of correlation coefficient $\mathbf{r}_{\hat{g}\hat{h}} = \frac{\mathbf{s}_{\hat{g}\hat{h}}}{\mathbf{s}_{\hat{g}}\mathbf{s}_{\hat{h}}}$, we get

$$\mathbf{r}_{\hat{g}\hat{h}} > -1$$

which is always true by the definition of the correlation coefficient.

Hence, we show that with underlying normal distributions, using the approximate procedure gives the same result in terms of rejection of the null hypothesis as the exact test. Since we have asymptotic results for the spectrum estimates, the above result will be true only asymptotically.

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TABLE 1 - Descriptions and Ranges of Variables

Acronym	Description	Starting Date	Ending Date	Number of Observations
IPROD	SIS Industrial Production Index (1992=100)	Jan 1986	April 2000	172
IPIMQ	Mining and Quarrying (subgroup of IPROD)	Jan 1986	April 2000	172
MPROD	Manufacturing Industries (subgroup of IPROD)	Jan 1986	April 2000	172
IPEGW	Electricity Gas and Water (subgroup of IPROD)	Jan 1986	April 2000	172
IMPOR	Imports (in million USD)	Jan 1985	May 2000	185
GOVRE	Government Revenues (in million TRL)	Jan 1985	June 2000	186
GOVEX	Government Expenditures (in million TRL)	Jan 1985	June 2000	186
NDOMB	Net Domestic Borrowing (in million TRL)	Jan 1985	June 2000	186
RM	Reserve Money (in million TRL)	Sep 1989	Aug 2000	132
M1	M1 (in billion TRL)	Jan 1986	April 2000	172
FXDEP	Foreign Exchange Denominated Deposit Accounts (in billion TRL)	Jan 1986	April 2000	172
WPI	SIS Whole Sale Price Index (1987=100)	Jan 1985	June 2000	186
CPI	SIS Consumer Price Index (1987=100)	Jan 1987	June 2000	162
WPIINF	WPI Inflation (year-on-year)	Jan 1986	June 2000	174
CPIINF	CPI Inflation (year-on-year)	Jan 1988	June 2000	150
CREDIT	Credits given by Deposit Banks (in billion TRL)	Jan 1986	April 2000	172
ISETL	Istanbul Stock Exchange National 100 Index (Monthly Average, TRL Based)	Jan 1986	Aug 2000	176
ISEVOL	Istanbul Stock Exchange Trade Volume (Monthly Average)	Nov 1986	Aug 2000	166
ISEUSD	Istanbul Stock Exchange National 100 Index (Monthly Average, USD Based)	Jan 1986	Aug 2000	176
ISEFIN	Istanbul Stock Exchange Financial Index (Monthly Average, USD Based)	Jan 1991	Aug 2000	116
ISEIND	Istanbul Stock Exchange Industrial Index (Monthly Average, USD Based)	Jan 1991	Aug 2000	116
ONINTR	Weighted Average of Overnight Simple Interest Rate in Interbank Market	Jan 1990	Aug 2000	128
USDTL	Exchange Rate of USD (Central Bank Buying Rate)	Jan 1985	Aug 2000	188

Table 2 – Time Domain Results

Variable	Fitted Model	Q-Statistic for 24 lags	Adjusted R-squared and SC for the Original Model		Religious Events Event / Coefficient / (P-Value)		Wald Test Statistic	Adjusted R-squared and SC for the Model with Dummies	
IPROD	AR(13)	17.47 (0.83)	0.26	-3.21	Feast of Ramadan -0.04 (0.00) Feast of Sacrifice -0.05 (0.00)	11.48 (0.00)	0.35	-3.30	
IPIMQ	AR(13)	17.87 (0.81)	0.18	-2.43	Feast of Sacrifice -0.04 (0.03)	4.68 (0.03)	0.20	-2.43	
MPROD	AR(13)	16.71 (0.86)	0.26	-2.95	Feast of Ramadan -0.04 (0.00) Feast of Sacrifice -0.05 (0.00)	10.62 (0.00)	0.35	-3.02	
IPEGW	AR(13)	17.10 (0.84)	0.09	-4.16	Feast of Ramadan -0.03 (0.00) Feast of Sacrifice -0.03 (0.00)	16.81 (0.00)	0.25	-4.30	
IMPOR	AR(11)	20.57 (0.66)	0.51	-1.69	Feast of Ramadan -0.10 (0.00) Feast of Sacrifice -0.07 (0.00)	12.60 (0.00)	0.57	-1.78	
GOVRE	AR(14)	21.37 (0.62)	0.67	-3.35	Feast of Ramadan -0.02 (0.03)	4.84 (0.03)	0.67	-3.54	
GOVEX	AR(13)	24.24 (0.45)	0.30	-2.43	-	-	-	-	
NDO MB	AR(4)	26.88 (0.31)	0.43	0.13	-	-	-	-	
RM	AR(13)	20.68 (0.66)	0.53	-4.06	Feast of Ramadan 0.02 (0.04) Feast of Sacrifice 0.02 (0.02)	4.22 (0.02)	0.55	-4.06	
M1	AR(10)	21.96 (0.58)	0.45	-3.06	-	-	-	-	
FXDEP	AR(8)	16.30 (0.88)	0.74	-3.33	-	-	-	-	
WPI	AR(2)	22.63 (0.54)	0.86	-4.87	-	-	-	-	
CPI	AR(6)	17.59 (0.82)	0.81	-7.98	-	-	-	-	
WPIINF	AR(13)	11.25 (0.99)	0.88	-3.15	-	-	-	-	
CPIINF	AR(10)	24.59 (0.43)	0.83	-4.00	-	-	-	-	
CREDIT	AR(4)	31.09 (0.15)	0.89	-4.66	Feast of Ramadan 0.02 (0.03) Ramadan -0.05 (0.03)	2.81 (0.06)	0.89	-4.64	
ISETL	AR(11)	12.96 (0.97)	0.88	-1.01	-	-	-	-	
ISEVOL	AR(12)	19.10 (0.75)	0.70	1.03	-	-	-	-	
ISEUSD	AR(11)	9.68 (0.99)	0.89	-0.93	-	-	-	-	
ISEFIN	AR(11)	9.22 (0.99)	0.80	-0.59	-	-	-	-	
ISEIND	AR(11)	14.77 (0.93)	0.80	-1.06	-	-	-	-	
ONINTR	AR(1)	17.52 (0.83)	0.46	-0.42	-	-	-	-	
USD TL	AR(13)	6.86 (0.99)	0.86	-3.69	-	-	-	-	

Notes: Numbers in parentheses below a test statistic is the p-value for that test statistic. For coefficients, the numbers in parentheses are the p-values for the simple t-statistics. In the Religious Events columns, a dash represents no significant effect of religious dummy variables. All t-statistics and the Wald test statistics are significant with 5% significance (except for CREDIT, the Wald test statistic has a p-value of 0.06). All Q-statistics are insignificant, the smallest p-value being 0.15.

TABLE 3 – Summary of Results

Series	Time Domain	Frequency Domain	
		<u>Religious Frequencies</u>	<u>All Other Frequencies</u>
IPROD	SIGNIFICANT	+	+
IPIMQ	SIGNIFICANT	0	0
MPROD	SIGNIFICANT	+	0
IPEGW	SIGNIFICANT	+	+
IMPOR	SIGNIFICANT	0	0
GOVRE	SIGNIFICANT	0	0
GOVEX	NOT SIGNIFICANT	0	0
NDOMB	NOT SIGNIFICANT	0	0
RM	SIGNIFICANT	+	+/-
M1	NOT SIGNIFICANT	0	0
FXDEP	NOT SIGNIFICANT	0	0
WPI	NOT SIGNIFICANT	0	0
CPI	NOT SIGNIFICANT	0	0
WPIINF	NOT SIGNIFICANT	0	-
CPIINF	NOT SIGNIFICANT	0	-
CREDIT	NOT SIGNIFICANT	0	0
ISETL	NOT SIGNIFICANT	0	-
ISEVOL	NOT SIGNIFICANT	0	0
ISEUSD	NOT SIGNIFICANT	0	-
ISEFIN	NOT SIGNIFICANT	0	-
ISEIND	NOT SIGNIFICANT	0	0
ONINTR	NOT SIGNIFICANT	0	0
USDTL	NOT SIGNIFICANT	+	-

Notes: For the time domain column, “significant” / “not significant” refers to the variable having significant or insignificant coefficients for at least one of the religious events, as shown in Table 2. (CREDIT is marked NOT SIGNIFICANT since the Wald test statistic has a p-value of 0.06) The frequency domain column refers to the comparison of the spectrum of yf_t over the spectrum of yc_t in terms of the criterion described in the text. “+” means that there is at least one band where an improvement is noted and no other bands noted as deterioration. Similarly “-” means that at least one band is marked as deterioration with no other bands being marked as improvement. “0” refers to the non-existence of any band with either improvement of deterioration and “+/-” refers to the case where we find at least one band with improvement and with deterioration. For this comparison only bands longer than 12 consecutive frequencies (1% of the range $w \in [0, 2p]$) is used in this table.

**Table 4 – Autocorrelations of Chosen Macroeconomic Series
Before and After the Removal of the Religious Effects**

Before Removal	t-6	t-3	t-2	t-1	t	t+1	t+2	t+3	t+6
IPROD	0.01	0.04	0.26	0.28	1.00	0.28	0.26	0.04	0.01
IPIMQ	-0.24	0.05	0.23	0.48	1.00	0.48	0.23	0.05	-0.24
MPROD	0.02	0.03	0.26	0.31	1.00	0.31	0.26	0.03	0.02
IPEGW	-0.10	-0.09	0.11	0.24	1.00	0.24	0.11	-0.09	-0.10
IMPOR	0.06	0.42	0.61	0.56	1.00	0.56	0.61	0.42	0.06
GOVRE	0.38	0.62	0.72	0.82	1.00	0.82	0.72	0.62	0.38
RM	-0.13	0.26	0.53	0.68	1.00	0.68	0.53	0.26	-0.13
Religious Effects									
Removed	t-6	t-3	t-2	t-1	t	t+1	t+2	t+3	t+6
IPROD	0.05	0.08	0.27	0.49	1.00	0.49	0.27	0.08	0.05
IPIMQ	-0.17 (*)	0.08	0.20 (*)	0.50	1.00	0.50	0.20 (*)	0.08	-0.17 (*)
MPROD	0.06	0.06	0.27	0.49	1.00	0.49	0.27	0.06	0.06
IPEGW	-0.27	0.01	0.16	0.52	1.00	0.52	0.16	0.01	-0.27
IMPOR	0.07	0.48	0.63	0.66	1.00	0.66	0.63	0.48	0.07
GOVRE	0.38	0.64	0.72	0.82 (*)	1.00	0.82 (*)	0.72	0.64	0.38
RM	-0.17	0.30	0.57	0.77	1.00	0.77	0.57	0.30	-0.17

Notes: The variables in this table are those which have significant results in the time domain. Numbers in boldface reflect significant autocorrelation coefficients at 95%. The significance is determined by threshold values (0.1524 for the first four variables, 0.147 for imports and government revenues and 0.174 for reserve money) which are calculated using the asymptotic standard error $(1/\sqrt{T})$.

(*) sign reflects a decrease in the absolute value compared to the value before the religious seasonality is removed.

Table 5 – Cross Correlations of Some Series with the Industrial Production Index Before and After the Removal of the Religious Effects

Before Removal	Volatility (Std. Error in Percentage)	t-6	t-3	t-2	t-1	t	t+1	t+2	t+3	t+6
IPROD	5.28%	0.01	0.04	0.26	0.28	1.00	0.28	0.26	0.04	0.01
IPIMQ	8.12%	-0.14	-0.02	0.06	0.05	0.31	0.02	0.03	-0.03	-0.07
MPROD	6.11%	0.02	0.03	0.26	0.29	0.99	0.29	0.27	0.04	0.01
IPEGW	3.21%	0.04	0.03	0.16	-0.02	0.49	0.02	0.06	0.07	0.05
IMPOR	13.06%	0.02	0.22	0.39	0.33	0.61	0.28	0.31	0.14	-0.09
RM	3.96%	0.02	0.22	0.17	0.19	-0.26	-0.12	-0.25	-0.18	-0.11
WPI	5.45%	-0.01	-0.15	-0.18	-0.23	-0.25	-0.25	-0.24	-0.18	-0.14
CPI	4.07%	-0.03	-0.18	-0.21	-0.23	-0.20	-0.16	-0.13	-0.03	-0.05
CREDIT	6.72%	0.07	0.25	0.28	0.35	0.38	0.42	0.43	0.43	0.29
ISETL	35.95%	0.03	0.24	0.28	0.30	0.26	0.19	0.11	0.06	0.01
ISEVOL	65.31%	0.01	0.27	0.31	0.32	0.29	0.17	0.10	0.06	-0.01
M1	6.43%	0.11	0.31	0.39	0.22	-0.01	0.09	-0.09	-0.03	-0.12
WPIINF	12.19%	-0.05	-0.20	-0.19	-0.17	-0.15	-0.14	-0.11	-0.09	-0.05
CPIINF	7.01%	0.03	-0.09	-0.16	-0.23	-0.21	-0.16	-0.12	-0.06	-0.01
ISEUSD	39.26%	0.05	0.27	0.33	0.37	0.32	0.25	0.17	0.11	0.02
ONINTR	26.87%	-0.10	-0.36	-0.44	-0.34	-0.19	0.03	0.13	0.17	0.14
ISEFIN	33.25%	-0.15	0.20	0.33	0.38	0.32	0.21	0.14	0.08	-0.02
ISEIND	26.07%	-0.07	0.23	0.37	0.43	0.36	0.24	0.15	0.07	-0.07
USDTL	8.54%	-0.10	-0.22	-0.31	-0.39	-0.39	-0.34	-0.31	-0.24	-0.06
Religious Effects Removed	Volatility (Std. Error in Percentage)	t-6	t-3	t-2	t-1	t	t+1	t+2	t+3	t+6
IPROD	4.65%	0.05	0.08	0.27	0.50	1.00	0.5	0.27	0.08	0.05
IPIMQ	7.67%	-0.07 (*)	-0.02	-0.00(*)	0.05 (*)	0.20 (*)	0.09	-0.02 (*)	-0.05	0.00 (*)
MPROD	5.44%	0.06	0.07	0.27	0.49	0.99 (*)	0.49	0.27	0.08	0.05
IPEGW	2.94%	0.02 (*)	0.11	0.16	0.24	0.43 (*)	0.20	0.10	0.11	0.02 (*)
IMPOR	12.48%	0.06	0.28	0.40	0.46	0.56 (*)	0.43	0.36	0.20	-0.08 (*)
RM	3.75%	0.06	0.19 (*)	0.17	0.05 (*)	-0.19 (*)	-0.24	-0.25	-0.21	-0.10 (*)
WPI	5.41%	-0.01	-0.15 (*)	-0.22	-0.24	-0.29	-0.27	-0.27	-0.23	-0.14
CPI	4.05%	-0.03 (*)	-0.16 (*)	-0.25	-0.26	-0.25	-0.19	-0.16	-0.07	-0.03 (*)
CREDIT	6.68%	0.07	0.25 (*)	0.33	0.36	0.41	0.48	0.47	0.47	0.33
ISETL	35.52%	0.05	0.23 (*)	0.31	0.33	0.31	0.22	0.14	0.08	-0.01
ISEVOL	64.18%	0.02	0.28	0.36	0.35	0.34	0.20	0.10	0.08	-0.05
M1	6.38%	0.14	0.33	0.33 (*)	0.23	0.01 (*)	0.04 (*)	-0.01 (*)	-0.04	-0.11 (*)
WPIINF	11.95%	-0.06	-0.18 (*)	-0.21	-0.16 (*)	-0.17	-0.13 (*)	-0.13	-0.12	-0.04 (*)
CPIINF	6.92%	-0.01 (*)	-0.07 (*)	-0.17	-0.21 (*)	-0.22	-0.15 (*)	-0.12	-0.08	-0.02
ISEUSD	38.66%	0.07	0.27 (*)	0.37	0.39	0.38	0.28	0.20	0.14	0.00 (*)
ONINTR	26.67%	-0.15	-0.40	-0.46	-0.37	-0.18 (*)	0.00 (*)	0.15	0.17 (*)	0.14 (*)
ISEFIN	33.24%	-0.15	0.24	0.39	0.42	0.36	0.24	0.15	0.09	-0.02
ISEIND	26.15%	-0.08	0.29	0.44	0.48	0.41	0.28	0.15	0.08	-0.09
USDTL	8.41%	-0.12	-0.25	-0.37	-0.42	-0.44	-0.36	-0.33	-0.27	-0.06 (*)

Notes: The first 5 variables are those with significant residual seasonality effects in the time domain analysis. Government revenues (GOVRE) and expenditures (GOVEX), net domestic borrowing (NDOMB) and foreign exchange deposits (FXDEP) are not included in this table since none of the coefficients were significant. Boldfaced coefficients are statistically significant using 0.20 as the threshold value. Coefficients with (*) next to them in the lower part of table reflect a decrease in the coefficient in absolute value compared to the corresponding coefficient in the upper part of the table.

**Figure 1 - Industrial Production Index
Before and After the Removal of Residual Seasonality**

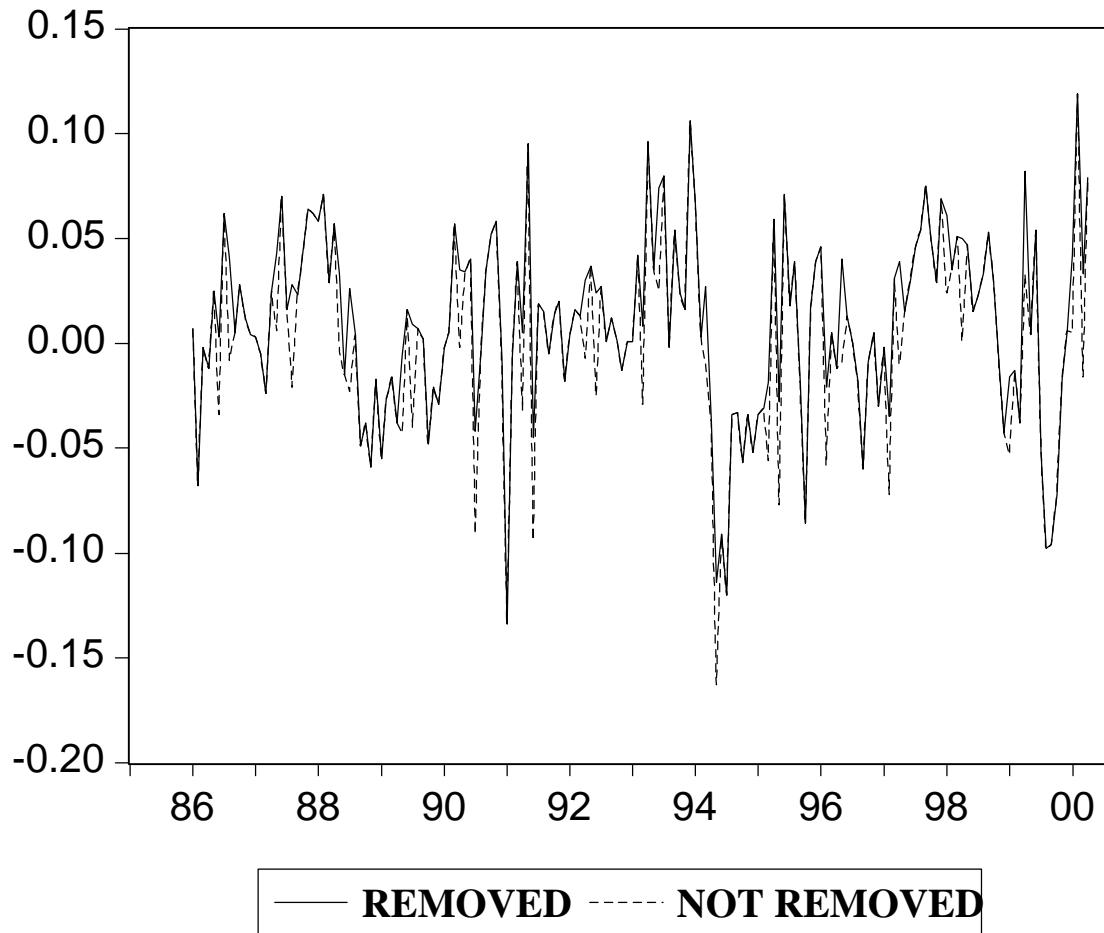


Figure 2 - The Spectrum Estimates of Industrial Production Index "Conventionally" and "Further" Deseasonalized

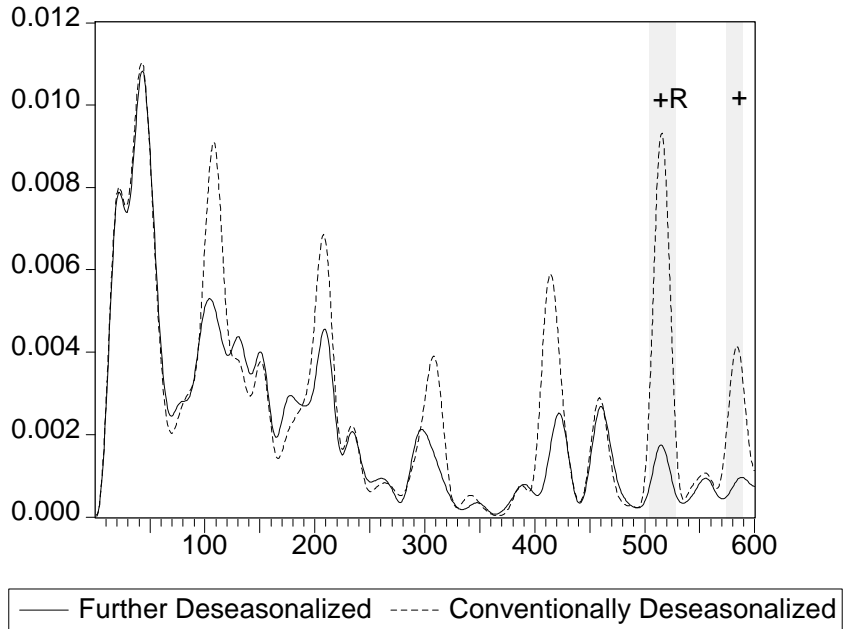
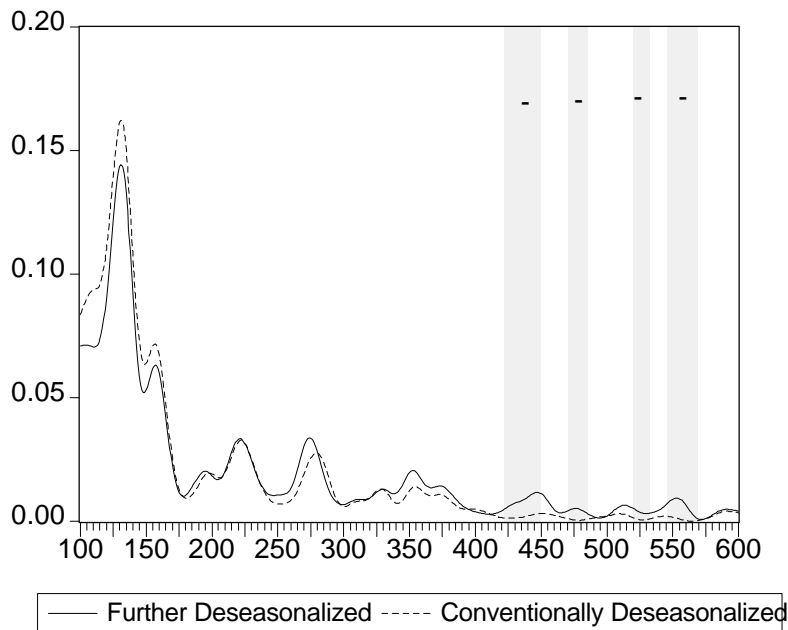


Figure 3 - The Spectrum Estimates of ISE National 100 Index "Conventionally" and "Further" Deseasonalized



Notes: The figures depict the spectrum estimates of the "conventionally" deseasonalized and "further" deseasonalized series. A shaded region with a "+" mark corresponds to a frequency band not including any of the religious frequencies where there is an improvement. A shaded region with a "+R" mark, is a frequency band including one of the religious frequencies which shows an improvement, whereas "-" mark signifies a band not including any of the religious frequencies which show a deterioration.

Figure 4 - The Spectrum Estimates of Reserve Money
"Conventionally" and "Further" Deseasonalized

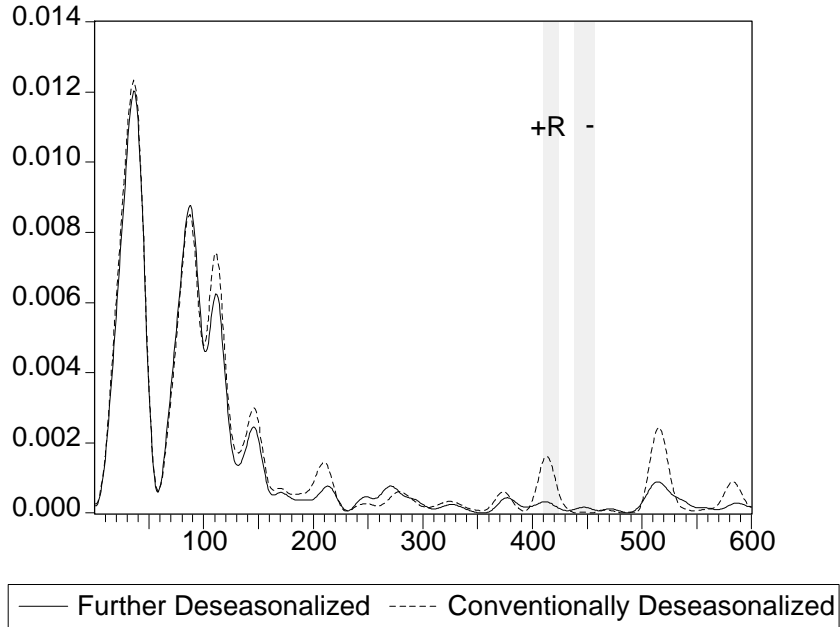
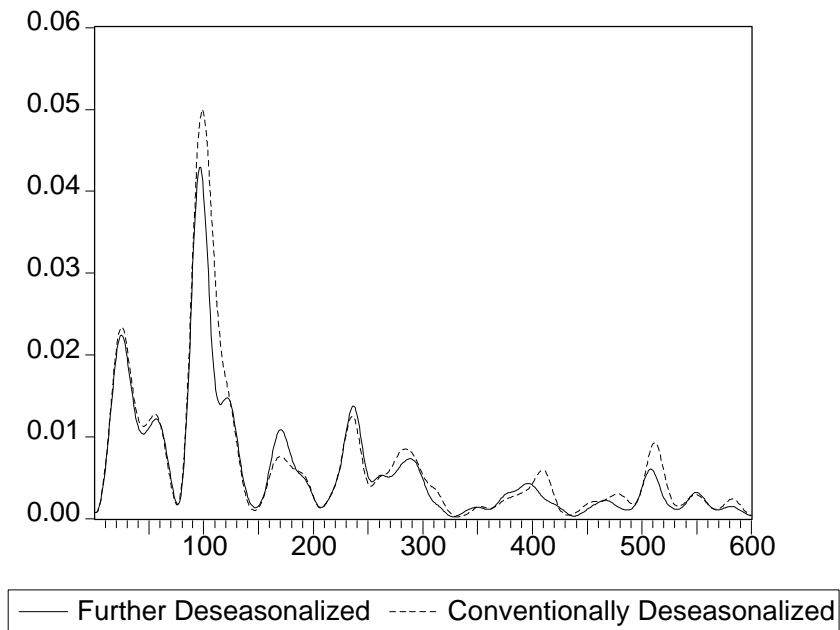


Figure 5 - The Spectrum Estimates of Mining and Quarrying
"Conventionally" and "Further" Deseasonalized



Notes: The figures depict the spectrum estimates of the "conventionally" deseasonalized and "further" deseasonalized series. A shaded region with a "+" mark corresponds to a frequency band not including any of the religious frequencies where there is an improvement. A shaded region with a "+R" mark, is a frequency band including one of the religious frequencies which shows an improvement, whereas "-" mark signifies a band not including any of the religious frequencies which show a deterioration.

Figure A1 - Weight Function for Blackman-Tukey Window with M=50

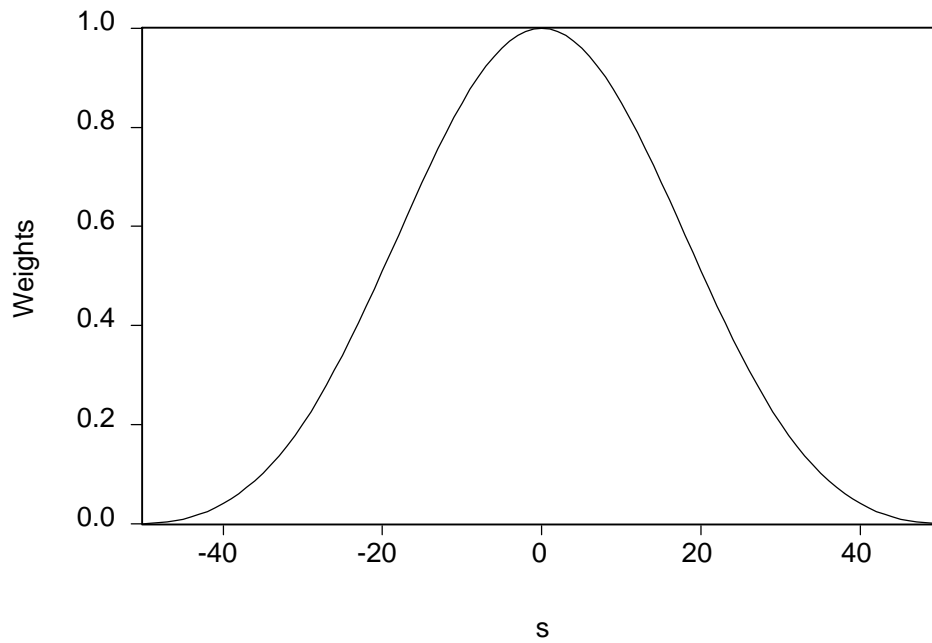


Figure A2 - Spectrum Estimates of IPI using Blackman-Tukey Method, Blackman Lag Window with M=10,40 and 75

