Harrod-Domar Model of Growth

- Developed during the early days of the post-World War II Keynesian Revolution in the 1940s: Harrod (1939) and Domar (1946)
- Extension of Keynesian theory to growth
- Emphasized potential dysfunctional aspects of growth: e.g., how growth could go hand-in-hand with increasing or constant but high unemployment
- Simple model, built around a priori Keynesian macro equations, where the critical components are all constants
- Popular model till Solow came up with the neoclassical model in response (See Solows Nobel Prize Speech for the story: http://nobelprize.org/nobel_prizes/economics/laureates/1987/solow-lecture.html)
Model - Technology

Fixed Coefficient (Leontieff) Production Function
The level of scarce input determines the output levels

\[ Y(t) = \min\left(\frac{K(t)}{v}, \frac{L(t)}{\alpha}\right) \]  \hspace{1cm} (1)

where

- \( v \): utilized capital/output ratio
- \( \alpha \): employed labor/output rate
Leontieff technology isoquants

- Note: *Isoquants* are the curves that define the set of points at which the same level of output is produced while changing input levels.
- In Leontieff, inputs are perfect complements (See the GRAPH)
- To ensure full employment of both inputs:

\[ Y(t) = \min \left( \frac{K(t)}{v}, \frac{L(t)}{\alpha} \right) \Rightarrow \frac{K}{L} = \frac{v}{\alpha} \quad (1') \]

- No substitution between capital and labor ⇒ Critical property of Leontieff
Model

- Keynesian model: $S(t) = Y(t) - C(t)$ (assuming $C$ is a constant fraction of total income)

  $$\Rightarrow S(t) = sY(t)$$  \hspace{1cm} (2)

  where $s$: saving rate (a constant fraction) where $0 < s < 1$

- Goods Market Clearing Condition:

  $$Y(t) = C(t) + I(t)$$  \hspace{1cm} (3)

- Gross Investment Definition:

  $$I(t) = \delta K(t) + \dot{K}(t)$$  \hspace{1cm} (4)

  where $\delta$: depreciation rate (a constant fraction) where $0 < \delta < 1$
Model Continued

- **Population**

\[ \frac{\dot{L}(t)}{L(t)} = n_L : \text{Constant population growth rate} \quad (5) \]

- NOTE: This is how we define the growth rate of any variable: \( \frac{\dot{X}(t)}{X(t)} \)

where dot above \( X \) represents the instantaneous change in \( X \), i.e. the time derivative of \( X \): \( \dot{X}(t) = \frac{dX(t)}{dt} \).

- In discrete time \( \Delta \) is equivalent to dot: \( X_{t+1} = (1 + n_X)X_t \).

- \( \Delta X = X_{t+1} - X_t \rightarrow \Delta X/X_t = n_X \) is the growth rate of \( X \).
Macroeconomic equilibrium (Drop $t$’s from now on to shorten exposition):

$$S = I \Rightarrow \quad (6)$$

$$sY = \delta K + \dot{K} \quad (7)$$

Assuming full employment of $K$ (and excess labor supply/unemployment) implies:

$$Y = \frac{K}{v} \quad (1')$$

Then we can write (7) as:

$$s\frac{K}{v} - \delta K = \dot{K} \Rightarrow \frac{\dot{K}}{K} = \frac{s}{v} - \delta \quad (7')$$
Macroeconomic equilibrium

- Let $g_K = \frac{\dot{Y}}{Y} =$ Actual growth rate of the economy
- By (1'): $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = g_K$ (Note: Derive!)
- At the equilibrium:
  $$\Rightarrow \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{s}{v} - \delta$$

This is called the "Warranted rate of growth": The growth rate at which all saving is absorbed into investment. $\Rightarrow$ If the goods market clear through $S = I$ such that the economy is in equilibrium, actual growth rate of the economy, $g_K$, would coincide with the warranted growth rate, $s/v - \delta$. 

Results of Harrod-Domar model

First main result of the Harrod-Domar model:

- Harrod emphasized the unstable nature of this equilibrium: if $S=I$ does not hold, the economy moves away from this equilibrium. For example,
  - If $g_K > s/v - \delta$, there is excess demand. $\Rightarrow$ Firms have underinvested and would invest more. $\Rightarrow$ growth $\uparrow$, requiring even further investment. Result: explosive growth: $g_K$ continuously increases.
  - If $g_K < s/v - \delta$, there is excess capacity. Multiplier works in the opposite way, $g_K$ continuously decreases.

- This property is known as Harrod’s knife-edge.
Results Cont.d

Second main result:

- Suppose capital and labor levels are such that there is full employment in both:

\[
\frac{K}{L} = \frac{v}{\alpha} \Rightarrow K^* = \frac{v}{\alpha}L
\]

- For the full employment to remain intact: (DERIVE!)

\[
\frac{\dot{K}}{K} = \frac{\dot{L}}{L}
\]  

(9)

- (9)\Rightarrow(5)

\[
g_K = n_L
\]  

(10)
(10) brings another knife-edge condition

- If population grows faster, growing unemployment.
- If capital grows faster, labor becomes the scarce input.

If, by chance, \( n_L = g_K = s/v - \delta \), all markets are in equilibrium.

Inherent instability of the model was one of the biggest criticisms towards Harrod-Domar model.

Not a mistake: Harrod argued that stable growth periods are less likely to happen in a capitalist society.

Instead, there will be alternating cycles (of long periods of unemployment and fast growth).
Third main result (and the policy conclusion):

- Increase the saving rate (or find a way to reduce the coefficient $v$) and growth rate of the economy increases as well.
  $\Rightarrow$ Deeply influenced the central planning of economics of the time: India, Soviet Union and China.

**Criticism:**
Modern theories of growth criticized exogenous saving rate assumption

- Also, started a flow of foreign aid to poor economies to reduce the “financing gap” (See the optional reading by Easterly for details.)