The Investment Decision

- Capital allocation between risky portfolio and risk-free asset
- Asset allocation across broad asset classes
- Security selection of individual assets within each asset class

Diversification and Portfolio Risk

- Market risk
  - Systematic or non-diversifiable
- Firm-specific risk
  - Diversifiable or nonsystematic

Portfolio Risk as a Function of the Number of Stocks in the Portfolio

- Figure 7.1: Portfolio risk as a function of the number of stocks in the portfolio. Panel A: All risk is firm specific. Panel B: Some risk is systematic, or marketwide.

Portfolio Diversification

- Figure 7.2: Portfolio diversification. The average standard deviation of returns of portfolios composed of only one stock was 49.2%. The average portfolio risk fell rapidly as the number of stocks included in the portfolio increased. In the limit, portfolio risk could be reduced to only 19.2%.

Covariance and Correlation

- Portfolio risk depends on the correlation between the returns of the assets in the portfolio.

- Covariance and the correlation coefficient provide a measure of the way returns of two assets vary.

Two-Security Portfolio: Risk

- Another way to express variance of the portfolio:

\[ \sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_D w_E \text{Cov}(r_D, r_E) \]

- Covariance: \( \text{Cov}(r_D, r_E) = \rho_{DE} \sigma_D \sigma_E \)

- Correlation coefficient: \( \rho_{D,E} = \text{Correlation coefficient of returns} \)

- Standard deviations: \( \sigma_D = \text{Standard deviation of returns for Security D} \)

- Standard deviations: \( \sigma_E = \text{Standard deviation of returns for Security E} \)
Correlation Coefficients: Possible Values

Range of values for $\rho_{1,2}$

$$+1.0 \geq \rho \geq -1.0$$

If $\rho = 1.0$, the securities are perfectly positively correlated

If $\rho = -1.0$, the securities are perfectly negatively correlated

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Three-Asset Portfolio

$$E(r_p) = w_1E(r_1) + w_2E(r_2) + w_3E(r_3)$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2$$

$$+ 2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_2w_3\sigma_{2,3}$$

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Correlation Coefficients

- When $\rho_{DE} = 1$, there is no diversification

$$\sigma_p = w_E\sigma_E + w_D\sigma_D$$

- When $\rho_{DE} = -1$, a perfect hedge is possible

$$w_E = \frac{\sigma_d}{\sigma_D + \sigma_E} = 1 - w_D$$

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Portfolio Expected Return as a Function of Investment Proportions

![Portfolio expected return as a function of investment proportions](image)
**The Minimum Variance Portfolio**

- The minimum variance portfolio is the portfolio composed of the risky assets that has the smallest standard deviation, the portfolio with least risk.
- When correlation is less than +1, the portfolio standard deviation may be smaller than that of either of the individual component assets.
- When correlation is -1, the standard deviation of the minimum variance portfolio is zero.

**Correlation Effects**

- The amount of possible risk reduction through diversification depends on the correlation.
- The risk reduction potential increases as the correlation approaches -1.
  - If $r = +1$, no risk reduction is possible.
  - If $r = 0$, $\sigma_P$ may be less than the standard deviation of either component asset.
  - If $r = -1$, a riskless hedge is possible.
The Sharpe Ratio

- Maximize the slope of the CAL for any possible portfolio, $P$.
- The objective function is the slope:

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

- The slope is also the Sharpe ratio.

Markowitz Portfolio Selection Model

- Security Selection
  - The first step is to determine the risk-return opportunities available.
  - All portfolios that lie on the minimum-variance frontier from the global minimum-variance portfolio and upward provide the best risk-return combinations.
The Minimum-Variance Frontier of Risky Assets

Figure 7.10 The minimum-variance frontier of risky assets

The Efficient Frontier of Risky Assets with the Optimal CAL

Figure 7.11 The efficient frontier of risky assets with the optimal CAL

Markowitz Portfolio Selection Model

- Everyone invests in P, regardless of their degree of risk aversion.
- More risk averse investors put more in the risk-free asset.
- Less risk averse investors put more in P.

Capital Allocation and the Separation Property

- The separation property tells us that the portfolio choice problem may be separated into two independent tasks.
- Determination of the optimal risky portfolio is purely technical.
- Allocation of the complete portfolio to T-bills versus the risky portfolio depends on personal preference.
Risk Pooling and the Insurance Principle

- Risk pooling: merging uncorrelated, risky projects as a means to reduce risk.
- Increases the scale of the risky investment by adding additional uncorrelated assets.
- The insurance principle: risk increases less than proportionally to the number of policies insured when the policies are uncorrelated.
- Sharpe ratio increases

Risk Sharing

- As risky assets are added to the portfolio, a portion of the pool is sold to maintain a risky portfolio of fixed size.
- Risk sharing combined with risk pooling is the key to the insurance industry.
- True diversification means spreading a portfolio of fixed size across many assets, not merely adding more risky bets to an ever-growing risky portfolio.