EC 205 Macroeconomics I
Fall 2015

Problem Session 2 Solutions

Q1. Use the neoclassical theory of distribution to predict the impact on the real wage and the real rental price of capital of each of the following events:

a. A wave of immigration increases the labor force.

b. An earthquake destroys some of the capital stock.

c. A technological advance improves the production function.

A1.

a. According to the neoclassical theory of distribution, the real wage equals the marginal product of labor. Because of diminishing returns to labor, an increase in the labor force causes the marginal product of labor to fall. Hence, the real wage falls.

b. The real rental price equals the marginal product of capital. If an earthquake destroys some of the capital stock (yet miraculously does not kill anyone and lower the labor force), the marginal product of capital rises and, hence, the real rental price rises.

c. If a technological advance improves the production function, this is likely to increase the marginal products of both capital and labor. Hence, the real wage and the real rental price both increase.
Q2. Suppose that an economy’s production function is Cobb–Douglas with parameter $\alpha = 0.3$.

a. What fractions of income do capital and labor receive?

b. Suppose that immigration increases the labor force by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?

c. Suppose that a gift of capital from abroad raises the capital stock by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?

d. Suppose that a technological advance raises the value of the parameter $A$ by 10 percent. What happens to total output (in percent)? The rental price of capital? The real wage?

A2.

A Cobb–Douglas production function has the form $Y = AK^{\alpha}L^{1-\alpha}$. The text showed that the marginal products for the Cobb–Douglas production function are:

$$MPL = (1 - \alpha)Y/L.$$

$$MPK = \alpha Y/K.$$  

Competitive profit-maximizing firms hire labor until its marginal product equals the real wage, and hire capital until its marginal product equals the real rental rate. Using these facts and the above marginal products for the Cobb–Douglas production function, we find:

$$W/P = MPL = (1 - \alpha)Y/L.$$  

$$R/P = MPK = \alpha Y/K.$$  

Rewriting this:

$$(W/P)L = MPL \times L = (1 - \alpha)Y.$$  

$$(R/P)K = MPK \times K = \alpha Y.$$  

Note that the terms $(W/P)L$ and $(R/P)K$ are the wage bill and total return to capital, respectively. Given that the value of $\alpha = 0.3$, then the above formulas indicate that labor receives 70 percent of total output (or income), which is $(1 - 0.3)$, and capital receives 30 percent of total output (or income).
b. To determine what happens to total output when the labor force increases by 10 percent, consider the formula for the Cobb–Douglas production function:

\[ Y = AK^\alpha L^{1-\alpha}. \]

Let \( Y_1 \) equal the initial value of output and \( Y_2 \) equal final output. We know that \( \alpha = 0.3 \). We also know that labor \( L \) increases by 10 percent:

\[ Y_1 = AK^{0.3}L^{0.7}. \]

\[ Y_2 = AK^{0.3}(1.1L)^{0.7}. \]

Note that we multiplied \( L \) by 1.1 to reflect the 10-percent increase in the labor force.

To calculate the percentage change in output, divide \( Y_2 \) by \( Y_1 \):

\[
\frac{Y_2}{Y_1} = \frac{AK^{0.3}(1.1L)^{0.7}}{AK^{0.3}L^{0.7}}
\]

\[ = (1.1)^{0.7} \]

\[ = 1.069. \]

That is, output increases by 6.9 percent.

To determine how the increase in the labor force affects the rental price of capital, consider the formula for the real rental price of capital \( R/P \):

\[ R/P = MPK = \alpha AK^{\alpha-1}L^{1-\alpha}. \]

We know that \( \alpha = 0.3 \). We also know that labor \( (L) \) increases by 10 percent. Let \( (R/P)_1 \) equal the initial value of the rental price of capital, and \( (R/P)_2 \) equal the final rental price of capital after the labor force increases by 10 percent. To find \( (R/P)_2 \), multiply \( L \) by 1.1 to reflect the 10-percent increase in the labor force:

\[ (R/P)_1 = 0.3AK^{-0.7}L^{0.7}. \]

\[ (R/P)_2 = 0.3AK^{-0.7}(1.1L)^{0.7}. \]

The rental price increases by the ratio

\[
\frac{(R/P)_2}{(R/P)_1} = \frac{0.3AK^{-0.7}(1.1L)^{0.7}}{0.3AK^{-0.7}L^{0.7}}
\]

\[ = (1.1)^{0.7} \]

\[ = 1.069. \]

So the rental price increases by 6.9 percent.
To determine how the increase in the labor force affects the real wage, consider the formula for the real wage \( W/P \):

\[
W/P = MPL = (1 - \alpha)AK^\alpha L^{-\alpha}.
\]

We know that \( \alpha = 0.3 \). We also know that labor \((L)\) increases by 10 percent. Let \((W/P)_1\) equal the initial value of the real wage and \((W/P)_2\) equal the final value of the real wage. To find \((W/P)_2\), multiply \(L\) by 1.1 to reflect the 10-percent increase in the labor force:

\[
(W/P)_1 = (1 - 0.3)AK^{0.3}L^{-0.3}.
\]

\[
(W/P)_2 = (1 - 0.3)AK^{0.3}(1.1L)^{-0.3}.
\]

To calculate the percentage change in the real wage, divide \((W/P)_2\) by \((W/P)_1\):

\[
\frac{(W/P)_2}{(W/P)_1} = \frac{(1 - 0.3)AK^{0.3}(1.1L)^{-0.3}}{(1 - 0.3)AK^{0.3}L^{-0.3}}
\]

\[
= (1.1)^{-0.3}
\]

\[
= 0.972.
\]

That is, the real wage falls by 2.8 percent.

c. We can use the same logic as in part (b) to set

\[
Y_1 = AK^{0.3}L^{0.7}.
\]

\[
Y_2 = A(1.1K)^{0.3}L^{0.7}.
\]

Therefore, we have:

\[
\frac{Y_2}{Y_1} = \frac{A(1.1K)^{0.3}L^{0.7}}{AK^{0.3}L^{0.7}}
\]

\[
= (1.1)^{0.3}
\]

\[
= 1.029.
\]

This equation shows that output increases by about 3 percent. Notice that \( \alpha < 0.5 \) means that proportional increases to capital will increase output by less than the same proportional increase to labor.

Again using the same logic as in part (b) for the change in the real rental price of capital:

\[
\frac{(R/P)_2}{(R/P)_1} = \frac{0.3A(1.1K)^{-0.7}L^{0.7}}{0.3AK^{-0.7}L^{0.7}}
\]

\[
= (1.1)^{-0.7}
\]

\[
= 0.935.
\]
Q3- Consider an economy with Cobb-Douglas aggregate production function given by \( Y = F(K,L) = K^{1/4}L^{3/4} \)

a. Prove that this production function exhibits constant returns to scale.

b. Solve for the MPK and MPL.

c. Prove that this function exhibits diminishing MPK and diminishing MPL

A3- a. \( F(K,L) = (zK)^{1/4}(zL)^{3/4} \)

\[ = zK^{1/4}L^{3/4} \]
b. $\text{MPK} = \frac{dY}{dK} = \frac{1}{4}K^{-3/4}L^{3/4}$

$\text{MPL} = \frac{dY}{dL} = 3\frac{1}{4}K^{1/4}L^{-1/4}$

c. $\text{dMPK/dK} = -\frac{3}{16}K^{-7/4}L^{3/4} < 0$. Hence, MPK is diminishing.

$\text{dMPL/dL} = -\frac{3}{16}K^{1/4}L^{-5/4} < 0$. Hence, MPL is diminishing

Q4- Consider an economy described by the following equations:

\[ Y = C + I + G \]
\[ Y = 5000 \]
\[ G = 1000 \]
\[ T = 1000 \]
\[ C = 250 + 0.75(Y - T) \]
\[ I = 1000 - 50r \]

a. In this economy, compute private saving, public saving and national saving.

b. Find the equilibrium interest rate.


d. Find the new equilibrium interest rate. What happens to investment? What is the

A4 a. Private saving is the amount of disposable income, $Y - T$, that is not consumed:

\[ S_{\text{private}} = Y - T - C \]

\[ = 5000 - 1000 - (250 + 0.75(5000 - 1000)) \]

\[ = 750. \]

Public saving is the amount of taxes the government has left over after it
makes its purchases:
\[ S_{\text{public}} = T - G \]
\[ = 1,000 - 1,000 \]
\[ = 0. \]

Total saving is the sum of private saving and public saving:

\[ S = S_{\text{private}} + S_{\text{public}} \]
\[ = 750 + 0 \]
\[ = 750. \]

**b.** The equilibrium interest rate is the value of \( r \) that clears the market for loanable funds. We already know that national saving is 750, so we just need to set it equal to investment:

\[ S = I \]
\[ 750 = 1,000 - 50r \]

Solving this equation for \( r \), we find:

\[ r = 5\%. \]

**c.** When the government increases its spending, private saving remains the same as before (notice that \( G \) does not appear in the \( S_{\text{private}} \) above) while government saving decreases. Putting the new \( G \) into the equations above:

\[ S_{\text{private}} = 750 \]
\[ S_{\text{public}} = T - G \]
\[ = 1,000 - 1,250 \]
Thus,

\[ S = S_{\text{private}} + S_{\text{public}} \]

\[ = 750 + (-250) \]

\[ = 500. \]

\[ d. \] Once again the equilibrium interest rate clears the market for loanable funds:

\[ S = I \]

\[ 500 = 1,000 - 50r \]

Solving this equation for \( r \), we find:

\[ r = 10\%. \]

**Q5** - The government raises taxes by $100 billion. If the marginal propensity to consume is 0.6, what happens to the following? Do they rise or fall? By what amounts?

a. Public saving  
b. Private saving  
c. National saving
A5- The effect of a government tax increase of $100 billion on (a) public saving, (b) private saving, and (c) national saving can be analyzed by using the following relationships:

\[ \text{National Saving} = [\text{Private Saving}] + [\text{Public Saving}] \]

\[ = [Y - T - C(Y - T)] + [T - G] \]

\[ = Y - C(Y - T) - G. \]

a. **Public Saving**—The tax increase causes a 1-for-1 increase in public saving. \( T \) increases by $100 billion and, therefore, public saving increases by $100 billion.

b. **Private Saving**—The increase in taxes decreases disposable income, \( Y - T \), by $100 billion. Since the marginal propensity to consume (MPC) is 0.6, consumption falls by \( 0.6 \times $100 \) billion, or $60 billion. Hence,

\[ \Delta \text{Private Saving} = - $100b - 0.6 (- $100b) = - $40b. \]

Private saving falls $40 billion.

c. **National Saving**—Because national saving is the sum of private and public saving, we can conclude that the $100 billion tax increase leads to a $60 billion increase in national saving.

Another way to see this is by using the third equation for national saving expressed above, that national saving equals \( Y - C(Y - T) - G \). The $100 billion tax increase reduces disposable income and causes consumption to fall by $60 billion. Since neither \( G \) nor \( Y \) changes, national saving thus rises by $60 billion.