Outline of model

A closed economy, market-clearing model

- Supply side
  - factor markets
  - determination of output/income

- Demand side
  - determinants of C, I, and G

- Equilibrium
  - goods market
  - loanable funds market
Factors of production

\[ K = \text{capital: tools, machines, and structures used in production} \]

\[ L = \text{labor: the physical and mental efforts of workers} \]
The production function: \( Y = F(K,L) \)

- shows how much output (\( Y \)) the economy can produce from \( K \) units of capital and \( L \) units of labor
- reflects the economy’s level of technology
- exhibits constant returns to scale
Returns to scale: A review

Initially \( Y_1 = F(K_1, L_1) \)

Scale all inputs by the same factor \( z \):

\[
K_2 = zK_1 \quad \text{and} \quad L_2 = zL_1
\]

(e.g., if \( z = 1.2 \), then all inputs are increased by 20%)

What happens to output, \( Y_2 = F(K_2, L_2) \)?

- If **constant returns to scale**, \( Y_2 = zY_1 \)
- If **increasing returns to scale**, \( Y_2 > zY_1 \)
- If **decreasing returns to scale**, \( Y_2 < zY_1 \)
Returns to scale: Example

\[ F(K, L) = \sqrt{KL} \]

\[ F(zK, zL) = \sqrt{(zK)(zL)} \]

\[ = \sqrt{z^2KL} \]

\[ = z\sqrt{KL} \]

\[ = zF(K, L) \]

constant returns to scale for any \( z > 0 \)
Assumptions

1. Technology is fixed.

2. The economy’s supplies of capital and labor are fixed at

\[ K = \bar{K} \quad \text{and} \quad L = \bar{L} \]
Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

\[
\bar{Y} = F(\bar{K}, \bar{L})
\]
The distribution of national income

- determined by factor prices, the prices per unit firms pay for the factors of production
  - wage = price of $L$
  - rental rate = price of $K$
**Notation**

- $W$ = nominal wage
- $R$ = nominal rental rate
- $P$ = price of output
- $W/P$ = real wage
  (measured in units of output)
- $R/P$ = real rental rate
How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?
Demand for labor

- Assume markets are competitive: each firm takes $W$, $R$, and $P$ as given.
- Basic idea:
  A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor
Definition: The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

\[ MPL = F(K, L + 1) - F(K, L) \]
Example:
Calculate & graph MPL

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<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>L</td>
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<td>MPL</td>
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<tr>
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<td>10</td>
<td>55</td>
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a. Determine **MPL** at each value of **L**.

b. Graph the production function.

c. Graph the **MPL** curve with **MPL** on the vertical axis and **L** on the horizontal axis.
Example: Answers

Production function

Marginal Product of Labor

Output (Y)

Labor (L)

MPL (units of output)

Labor (L)
MPL and the production function

As more labor is added, MPL \downarrow

Slope of the production function equals MPL
Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).

- Intuition:
  Suppose $\uparrow L$ while holding $K$ fixed
  - fewer machines per worker
  - lower worker productivity
Example:

**MPL and labor demand**

Suppose $W/P = 6$.

- If $L = 3$, should firm hire more or less labor? Why?
- If $L = 7$, should firm hire more or less labor? Why?

<table>
<thead>
<tr>
<th>$L$</th>
<th>$Y$</th>
<th>$MPL$</th>
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<td>10</td>
<td>55</td>
<td>1</td>
</tr>
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</table>
**MPL and the demand for labor**

Each firm hires labor up to the point where $MPL = W/P$. 

- **Units of output**
- **Real wage**
- **MPL, Labor demand**
- **Units of labor, $L$**
- **Quantity of labor demanded**
The equilibrium real wage

The real wage adjusts to equate labor demand with supply.

Units of output

Labor supply

$\bar{L}$

Units of labor, $L$

**MPL**, Labor demand

**equilibrium real wage**
Determining the rental rate

- We have just seen that \( MPL = \frac{W}{P} \).
- The same logic shows that \( MPK = \frac{R}{P} \):
  - diminishing returns to capital: \( MPK \downarrow \) as \( K \uparrow \)
  - The \( MPK \) curve is the firm’s demand curve for renting capital.
- Firms maximize profits by choosing \( K \) such that \( MPK = \frac{R}{P} \).
The equilibrium real rental rate

The real rental rate adjusts to equate demand for capital with supply.

Units of output

Supply of capital

equilibrium \( R/P \)

MPK, demand for capital

Units of capital, \( K \)
The Neoclassical Theory of Distribution

- states that each factor input is paid its marginal product
- a good starting point for thinking about income distribution
How income is distributed to L and K

total labor income = \( \frac{W}{P} \overline{L} = MPL \times \overline{L} \)

total capital income = \( \frac{R}{P} \overline{K} = MPK \times \overline{K} \)

If production function has constant returns to scale, then

\[ \overline{Y} = MPL \times \overline{L} + MPK \times \overline{K} \]

national income

labor income

capital income
The ratio of labor income to total income in the U.S., 1960-2007

Labor’s share of total income is approximately constant over time. (Thus, capital’s share is, too.)
The Cobb-Douglas Production Function

- The Cobb-Douglas Production function satisfies the neoclassical properties (constant returns to scale, diminishing marginal returns, essentiality of inputs)

\[ Y = AK^\alpha L^{1-\alpha} \]

\[ MPK = \alpha AK^{\alpha-1} L^{1-\alpha} = \frac{\alpha Y}{K} \]

\[ MPL = (1-\alpha) AK^\alpha L^{-\alpha} = \frac{(1-\alpha)Y}{L} \]
The Cobb-Douglas Production Function

- The Cobb-Douglas production function has constant factor shares:

\[ \alpha = \text{capital's share of total income} : \]

\[ \alpha = \text{capital's share of total income}: \]

- capital income  =  \( MPK \times K = \alpha Y \)

- labor income  =  \( MPL \times L = (1 - \alpha)Y \)
Labor productivity and wages

- Theory: wages depend on labor productivity
- U.S. data:

<table>
<thead>
<tr>
<th>period</th>
<th>productivity growth</th>
<th>real wage growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-2007</td>
<td>2.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td>1959-1973</td>
<td>2.8%</td>
<td>2.8%</td>
</tr>
<tr>
<td>1973-1995</td>
<td>1.4%</td>
<td>1.2%</td>
</tr>
<tr>
<td>1995-2009</td>
<td>2.6%</td>
<td>2.3%</td>
</tr>
</tbody>
</table>
Outline of model

A closed economy, market-clearing model

Supply side

DONE ✓ factor markets (supply, demand, price)
DONE ✓ determination of output/income

Demand side

Next ➔ ❏ determinants of $C$, $I$, and $G$

Equilibrium

❏ goods market
❏ loanable funds market
Demand for goods & services

Components of aggregate demand:

\[ C = \text{consumer demand for goods & services} \]
\[ I = \text{demand for investment goods} \]
\[ G = \text{government demand for goods & services} \]

(closed economy: no \( NX \))
Consumption, $C$

- **Disposable income** is total income minus total taxes: $Y - T$.

- Consumption function: $C = C(Y - T)$
  Assumption: $\uparrow(Y - T) \rightarrow \uparrow C$

- **Marginal propensity to consume (MPC)** is the change in $C$ when disposable income increases by one dollar.

- **Autonomous consumption** is the part of $C$ that is independent of the level of disposable income.
The consumption function

\[ C = C(Y - T) \]

The slope of the consumption function is the **MPC**.

Autonomous consumption is the ‘intercept’
Investment, $I$

- The investment function is $I = I(r)$, where $r$ denotes the real interest rate, i.e. the nominal interest rate corrected for inflation.

- $r = i - \pi$

- The real interest rate is
  - the cost of borrowing
  - the opportunity cost of using one’s own funds to finance investment spending

So, $\uparrow r \Rightarrow \downarrow I$
The investment function

Spending on investment goods depends negatively on the real interest rate.

\[ I(r) \]

\( r \)
Government Spending, $G$

- **$G$:** Government spending on goods and services
- **$G$** excludes government transfers (unemployment benefits, social security transfers)
- Assume government spending and taxes are exogenous

\[ G = \bar{G} \quad \text{&} \quad T = \bar{T} \]
The market for goods & services

- Aggregate demand: \( C(Y - T) + I(r) + G \)

- Aggregate supply: \( Y = F(K, L) \)

- Equilibrium: \( Y = C(Y - T) + I(r) + G \)

The real interest rate adjusts to equate demand with supply.
The loanable funds market

- A simple supply-demand model of the financial system.

- One asset: “loanable funds”
  - demand for funds: investment
  - supply of funds: saving
  - “price” of funds: real interest rate
Demand for funds: Investment

The demand for loanable funds...

- **comes from investment:**
  Firms borrow to finance spending on plant & equipment, new office buildings, etc. Consumers borrow to buy new houses.

- **depends negatively on** \( r \),
  the “price” of loanable funds (cost of borrowing).
The investment curve is also the demand curve for loanable funds.
Supply of funds: Saving

- The supply of loanable funds comes from saving:
  - Households use their saving to make bank deposits, purchase bonds and other assets. These funds become available to firms to borrow to finance investment spending.
  - The government may also contribute to saving if it does not spend all the tax revenue it receives.
Types of saving

**private saving**  =  \((Y - T) - C\)

**public saving**  =  \(T - G\)

**national saving**, \(S\)

\[= \text{private saving} + \text{public saving}\]

\[= (Y - T) - C + T - G\]

\[= Y - C - G\]
Notation: \( \Delta = \text{change in a variable} \)

- For any variable \( X \), \( \Delta X = \text{“the change in } X \text{”} \)
  \( \Delta \) is the Greek (uppercase) letter \( \text{Delta} \)

Examples:

- If \( \Delta L = 1 \) and \( \Delta K = 0 \), then \( \Delta Y = MPL \).
  More generally, if \( \Delta K = 0 \), then \( MPL = \frac{\Delta Y}{\Delta L} \).

- \( \Delta(Y - T) = \Delta Y - \Delta T \), so
  \[ \Delta C = MPC \times (\Delta Y - \Delta T) \]
  \[ = MPC \times \Delta Y - MPC \times \Delta T \]
Example:

Calculate the change in saving

Suppose \( \text{MPC} = 0.8 \) and \( \text{MPL} = 20 \).

For each of the following, calculate \( \Delta S \):

a. \( \Delta G = 100 \)

b. \( \Delta T = 100 \)

c. \( \Delta Y = 100 \)

d. \( \Delta L = 10 \)
Example:

Answers

\[ \Delta S = \Delta Y - \Delta C - \Delta G = \Delta Y - 0.8(\Delta Y - \Delta T) - \Delta G = 0.2\Delta Y + 0.8\Delta T - \Delta G \]

a. \[ \Delta S = -100 \]

b. \[ \Delta S = 0.8 \times 100 = 80 \]

c. \[ \Delta S = 0.2 \times 100 = 20 \]

d. \[ \Delta Y = MPL \times \Delta L = 20 \times 10 = 200, \]
   \[ \Delta S = 0.2 \times \Delta Y = 0.2 \times 200 = 40. \]
Budget surpluses and deficits

- If $T > G$, **budget surplus** $= (T - G)$
  $= $ public saving.

- If $T < G$, **budget deficit** $= (G - T)$
  and public saving is negative.

- If $T = G$, “balanced budget,” public saving $= 0$.

- Governments finance their deficit by issuing Treasury bonds – *i.e.*, borrowing.
Loanable funds supply curve

National saving does not depend on \( r \), so the supply curve is vertical.

\[
\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}
\]
Loanable funds market equilibrium

\[ S = Y - C(Y - T) - G \]

Equilibrium real interest rate

Equilibrium level of investment
The special role of $r$

$r$ adjusts to equilibrate the goods market and the loanable funds market simultaneously:

If L.F. market in equilibrium, then

$$Y - C - G = I$$

Add $(C + G)$ to both sides to get

$$Y = C + I + G \quad (goods \ market\ eq’m)$$

Thus,

Eq’m in L.F. market $\iff$ Eq’m in goods market
CASE STUDY:
The Reagan deficits

- Reagan policies during early 1980s:
  - increases in defense spending: $\Delta G > 0$
  - big tax cuts: $\Delta T < 0$

- Both policies reduce national saving:

\[
\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}
\]

\[
\uparrow \bar{G} \Rightarrow \downarrow \bar{S} \quad \downarrow \bar{T} \Rightarrow \uparrow C \Rightarrow \downarrow \bar{S}
\]
CASE STUDY: The Reagan deficits

1. The increase in the deficit reduces saving…

2. …which causes the real interest rate to rise…

3. …which reduces the level of investment.
Are the data consistent with these results?

<table>
<thead>
<tr>
<th>variable</th>
<th>1970s</th>
<th>1980s</th>
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</thead>
<tbody>
<tr>
<td>$T - G$</td>
<td>-2.2</td>
<td>-3.9</td>
</tr>
<tr>
<td>$S$</td>
<td>19.6</td>
<td>17.4</td>
</tr>
<tr>
<td>$r$</td>
<td>1.1</td>
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</tr>
<tr>
<td>$I$</td>
<td>19.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>

$T - G$, $S$, and $I$ are expressed as a percent of GDP
All figures are averages over the decade shown.
Mastering the loanable funds model

Things that shift the investment curve:

- some technological innovations
  - to take advantage of some innovations, firms must buy new investment goods
- tax laws that affect investment
  - e.g., investment tax credit
An increase in investment demand

...raises the interest rate.

But the equilibrium level of investment cannot increase because the supply of loanable funds is fixed.
Saving and the interest rate

- Why might saving depend on \( r \) ?

- How would the results of an increase in investment demand be different?
  - Would \( r \) rise as much?
  - Would the equilibrium value of \( I \) change?
An increase in investment demand when saving depends on $r$

An increase in investment demand raises $r$, which induces an increase in the quantity of saving, which allows $I$ to increase.
Total output is determined by:
- the economy’s quantities of capital and labor
- the level of technology

Competitive firms hire each factor until its marginal product equals its price.

If the production function has constant returns to scale, then labor income plus capital income equals total income (output).
Chapter Summary

- A closed economy’s output is used for:
  - consumption
  - investment
  - government spending

- The real interest rate adjusts to equate the demand for and supply of:
  - goods and services
  - loanable funds
Chapter Summary

- A decrease in national saving causes the interest rate to rise and investment to fall.
- An increase in investment demand causes the interest rate to rise, but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.